TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol. 12, No. 10, October 2014, pp. 7533 ~ 7542
DOI: 10.11591/telkomnika.v12i8.5989

Recent Study on Distance Formula and Similarity Measures between Two Vague Sets

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Abstract

Firstly, two new distance formulas were put forward and proved to satisfy with known axioms. An appropriate distance formula may give Vague set regulations a more reasonable cluster, and reduce search scale of rule base and complexity of calculation. Then, by studying the deficiencies and reasons of similarity measures between Vague sets (values), a new definition, which is a certain number in open interval (0, 1), was put forward. Moreover, a new formula satisfying this definition was built according to this definition.

Keywords*: axioms of the distance formula, new distance formula, clustering of the Vague set rules*

1. Introduction

Since 1993, the Vague set theory [1] proposed by Gau and Buehrer has been widely used in intelligent systems. It has been studied in a variety of vague set reasoning [2-9]. Generally speaking, the system needs to search the entire rule base in order to find a suitable matching rule,; however, when the rule base is large, the searching scale also results in a large one, which greatly increases the computational complexity. A concept of clustering vague rules is presented in Reference [2] in order to achieve reduction in the size of the search and reduce the computational complexity reasoning. The approach is: Assuming the accuracy of clustering is ε before selecting the rules. If the first rule is selected, it is classified in the first category; if the selected rule is i (i>1), measure the each distance between the prerequisite in the rule i and other prerequisites in the known rules separately (indicated by d). If d≤ε always exists in any rule within a class, the selected rule can be included in the class, and the clustering calculation of the next rule can be continued. On the other hand, if the rule can not be classified in the cluster calculation of any existing class, it should be included in a new class. After the above process is redone, a cluster will be obtained with the accuracy of ε. By the same method, sub requisite clustering of the same rule base A can be constructed, and then the approximate reasoning on clustering Vague can be concluded.

It should be emphasized that a distance formula is needed to calculate the distance between Vague sets in this approximate reasoning of vague set. Relying on this formula, the clustering of the vague rules can be done. Reference [2] shows a formula for the distance between vague sets, while Reference [10] gives six distance formulae with the axioms that these six formulae should be followed. In this study, two new distance formulae between vague sets are presented. The aim is to prove that they are suitable with the known axioms, therefore, there will be more choices about selecting distance formulae in order to facilitate the search and reduce the computational complexity purposes in the process of clustering Vague rules.

Received March 17, 2014; Revised August 2, 2014; Accepted August 25, 2014

2. Preliminary Knowledge

2.1.The First Distance Formula for the Vague Sets (Values)

Assume the discrete domain : $A=\sum_{i=1}^{n}\left[t_A\left(u_i\right),1-f_A\left(u_i\right)\right]/u_i$, $B=\sum_{i=1}^{n}\left[t_B\left(u_i\right),1-f_B\left(u_i\right)\right]/u_i$,make $S(A(u_i))=t_A(u_i)-f_A(u_i),(i\in\{1,2,\cdots,n\})$.

Reference [2] shows the formula for Vague sets (values) as follows: definiteany:

$$d(A(u_i),B(u_i))=\left|S(A(u_i))-S(B(u_i))\right|/4+\left[\left|t_A(u_i)-t_B(u_i)\right|+\left|f_A(u_i)-f_B(u_i)\right|\right]/4$$

$$d(A,B)=\frac{1}{n}\sum_{i=1}^{n}d(A(u_i),B(u_i))+\frac{1}{n}\sum_{i=1}^{n}\{S(A(u_i))-S(B(u_i))/4$$
$$+\left[|t_A(u_i)-t_B(u_i)|+|f_A(u_i)-f_B(u_i)|\right]/4\}$$

The above two formulae referred to the first distance formula for the Vague sets (values).

2.2. The Axioms that the Distance Formula for the Vague Set (Value) to be Obeyed

Presented by Reference [10], the axioms that thedistance formula for theVague set (value) to be obeyed are as following:

Based on the domain X, D ∈V (U), any x, y, z ∈X, then the distance d (D (x), D (y)), d (D (x), D (z)), d (D (y), D (z)) betweenVague Sets D (x), D (y) and D (z) should follow the following four axioms.

Axiom 1: Boundedness $0\le d(D(x),D(y))\le 1$;

Axiom 2: Boundary conditions $d(D(x),D(x))=0$,when $D(x)=D(y)$;

Axiom 3: Symmetry $d(D(x),D(y))=d(D(y),D(x))$;

Axiom 4: Triangle inequality $d(D(x),D(y))+d(D(x)+D(z))\ge d(D(y),D(z))$.

Assume the discrete domain is U, A、B、C ∈V(U), and $U=\{u_1,u_2,\cdots,u_n\}$,

$$A=\sum_{i=1}^{n}\left[t_A\left(u_i\right),1-f_A\left(u_i\right)\right]/u_i,\ B=\sum_{i=1}^{n}\left[t_B\left(u_i\right),1-f_B\left(u_i\right)\right]/u_i,\ C=\sum_{i=1}^{n}\left[t_C\left(u_i\right),1-f_C\left(u_i\right)\right]/u_i,$$

the distance formula for the Vague sets A and B is defined as $d(A,B)=\frac{1}{n}\sum_{i=1}^{n}d(A(u_i),B(u_i))$, then the distance d(A,B), d(A,C) and d(B,C) among the Vague sets A, B and C should obey the four axioms as follows:

Axiom 5: Boundedness $0\le d(A,B)\le 1$

Axiom 6: Boundary conditions $d(A,A)=0$,when $A=B$

Axiom 7: Symmetry $d(A,B)=d(B,A)$

Axiom 8: Triangle inequality $d(A,B)+d(A,C)\ge d(B,C)$

Theorem 1: The first distance formula for the Vague set (value) accords to axioms1-8.

3. The New Distance Formulae for the Vague Set (Value)

3.1. The Second Distance Formula for the Vague Set (Value)

The discrete domain U and the vague sets A and B appeared in the part of preliminary knowledge, and the second distance formula for the Vague set (value) is defined as:

For any $i \in \{1, 2, \cdots, n\}$,

$$d = (A(u_i), B(u_i)) = \left|S(A(u_i)) - S(B(u_i))\right| / 2 \tag{1}$$

$$d(A,B) = \frac{1}{n}\sum_{i=1}^{n} d(A(u_i), B(u_i)) = \frac{1}{n}\sum_{i=1}^{n}\{\left|S(A(u_i)) - S(B(u_i))\right| / 2\} \tag{2}$$

Theorem 2: The second distance formula for the Vague set (value) accords to axioms 1-8.

3.2. The Third Distance Formula for the Vague Set (Value)

The discrete domain U and the Vague sets A and B appeared in the part of preliminary knowledge, and the third distance formula for the Vague set (value) is defined as: for any $i \in \{1, 2, \cdots, n\}$,

$$d(A(u_i), B(u_i)) = \alpha\left|S(A(u_i)) - S(B(u_i))\right|,$$
$$d(A,B) = \frac{1}{n}\sum_{i=1}^{n} d(A(u_i), B(u_i)) = \frac{1}{n}\sum_{i=1}^{n}\alpha\,|\,d(A(u_i), B(u_i))\,|$$

In the above formulae, parameter α satisfies the condition: $0 \le \alpha \le 1/2$.

Theorem 3: When $\alpha = 1/2$, the third distance formula for the Vague set (value) equals to the second distance formula for the Vague set (value).

Theorem 4: The third distance formula for the Vague set (value) accords to axioms 1-8.

4. Analysis of Examples

Example 4.1: Assume $U = \{u_1, u_2, u_3\}$, A、B、$C \in V(U)$ and $A = [0.5, 0.8]/u_1 + [0.6, 0.9]/u_2 + [0.7, 0.9]/u_3$, $B = [0.4, 0.7]/u_1 + [0.5, 0.8]/u_2 + [0.6, 0.9]/u_3$, $C = [0.3, 0.6]/u_1 + [0.4, 0.7]/u_2 + [0.5, 0.9]/u_3$.

Then, the calculation based on the first distance formula is $d(A,B) = 0.083$, $d(A,C) = 0.17$

Calculation based on the second distance formula is $d(A,B) = 0.083$, $d(A,C) = 0.17$

Calculation based on the third distance formula ($\alpha = 0.4$) is $d(A,B) = 0.067$, $d(A,C) = 0.13$.

Example 4.2: assume $X = \{x_1, x_2, x_3\}$, A、B、$C \in V(X)$, and $A = [0.4, 0.6]/x_1 + [0.5, 0.7]/x_2 + [0.7, 0.9]/x_3$, $B = [0.3, 0.4]/x_1 + [0.5, 0.5]/x_2 + [0.7, 0.8]/x_3$, $C = [0.3, 0.5]/x_1 + [0.4, 0.7]/x_2 + [0.6, 0.7]/x_3$,

Then, the calculation based on the first distance formula is $d(A,B) = 0.10$, $d(A,C) = 0.083$.

The calculation based on the second distance formula is $d(A,B)=0.10$; $d(A,C)=0.10$.

The calculation based on the third distance formula is ($\alpha=0.4$) is $d(A,B)=0.08$; $d(A,C)=0.08$.

From the above two examples, it indicates that different distance formulae are applied separately for the different vague sets. Under this way, generally, the calculating results will not be the same. Select a suitable distance formula, and a much more reasonable clustering of the vague set could be obtained.

In this study, two new distance formulae for thevague set are proposed. By proving the accordance with the known axioms, there are more choices of the distance formulae when the clustering of the vague set is processing. Then, in the reasoning of vague rules, it is beneficial for decreasing the searching scope and reducing the complexity in the process of reasoning and calculating. Similarity measures between Vague set(value) play an important role in the Vague set of applications and has become an important part of the Vague set theory. However, it has been noted that the similarity measures between vague sets are inadequate for a long time. For example, the references [2-4] have pointed out the Vague X=Y= [0-1]. X and Y the similarity measure M(X,Y)=1 is unscientific. This study aims to research and analyse this issue and try to put forward the method of correction inadequacies.

5. Inadequacies of the Existing Similarity between Vague Value Metrics

Assume is Vague (value):

$$M_1(x,y)=1-\frac{|t_x-t_y|+|f_x-f_y|}{2} \tag{3}$$

$$M_2(x,y)=1-\frac{|(t_x-t_y)-(f_x-f_y)|}{2} \tag{4}$$

$$M_3(x,y)=1-\frac{|(t_x-t_y)-(f_x-f_y)|}{4}-\frac{|t_x-t_y|+|f_x-f_y|}{4} \tag{5}$$

$$M_4(x,y)=1-\frac{|S_x-S_y|}{2}-\frac{|K_x-K_y|}{2} \tag{6}$$

Among $S_x=t_x-f_x, S_y=t_y-f_y, K_x=t_x+f_x, K_y=t_y+f_y$.

$$M_5(x,y)=1-\frac{|t_x-t_y|+|f_x-f_y|+|\pi_x-\pi_y|}{2} \tag{7}$$

$$M_6(x,y)=1-\frac{|t_x-t_y|}{1+t_x+t_y}-\frac{|f_x-f_y|}{1+f_x+f_y} \tag{8}$$

Example 5.1. Uses formula (3)-(8) to calculate Table 1 Vague set date, got vague similarity between vague values.

Discussion: in the Table 1, $M_j(x_i, y_i) = 1, (j = 1, 2, 3, 4, 5, 6; i = 1, 2)$. Because $x_1 = y_1$ and $x_2 = y_2$ are ordinary collection. Their uncertainty is $\pi_{x_i} = \pi_{y_i} = 0 (i = 1, 2)$. Therefore, they measure the similarity max is 1, which is consistent with people's intuition. But $M_j(x_3, y_3) = 1$, $(j = 1, 2, 3, 4, 5, 6)$. Because $x_3 = y_3$ is fuzzy set.Set $\pi_{x_3} = \pi_{y_3} = 0$. Thus, they measure the similarity max is 1, which is consistent with people's intuition. Because $M_j(x_4, y_4) = 1, (j = 1, 2, 3, 4, 5, 6)$. It is counterintuitive with people.It is because [2-4] $x_4 = y_4 =$[0,1] is particular Vague, which is characterized by its uncertainty, the max $\pi_{x_4} = \pi_{y_4} = 1$. People know nothing about. this element, Calculate the similarity measure is equal to 1. It means the two Vague answer is $x_4 = y_4 =$[0,1] is the most similarity, which is contrary to intuition. But for the normal Vague,such as $x_5 = y_5 = [0.4, 0.8], x_6 = y_6 = [0.7, 0.9]$. Their characteristics are uncertainty 0: $\pi_{x_5} = \pi_{y_5} = 0.4, \pi_{x_6} = \pi_{y_6} = 0.2$ .For the reason, work out the answer is 1, which means that they are most similary and uncertain. Thus,.a more reasonable number of similarity metrics should be in the open interval (0-1). The uncertainty in the similarity of vague value plays a pivotal role.

Table 1. Comparisonof the Similarity Measures between Vague Values

x_i	x_1 =[1,1]	x_2 =[0,0]	x_3 =[0.2,0.2]	x_4 =[0,1]	x_5 =[0.4,0.8]	x_6 =[0.7,0.9]
y_i	y_1 =[1,1]	y_2 =[0,0]	y_3 =[0.2,0.2]	y_4 =[0,1]	y_5 =[0.4,0.8]	y_6 =[0.7,0.9]
$M_1(x_i, y_i)$	1	1	1	1	1	1
$M_2(x_i, y_i)$	1	1	1	1	1	1
$M_3(x_i, y_i)$	1	1	1	1	1	1
$M_4(x_i, y_i)$	1	1	1	1	1	1
$M_5(x_i, y_i)$	1	1	1	1	1	1
$M_6(x_i, y_i)$	1	1	1	1	1	1
$M_{(2)}(x_i, y_i)$	1	1	1	0	0.64	0.008

Example 5.2. Uses formula (3)-(8) to calculate Table 2 Vague set date, got vague similarity between vague values.

Observed from the Table 2, although the resolution of formula (3)-(7) is not high, it can be intuitivelydetermined thatformula (8) is of higher resolution than formula (3)-(7) .

Table 2. Comparison of Accuracy of the Similarity Measures between Vague Value

x_i	x_1 =[0.4,0.8]	x_2 =[0.4,0.8]	x_3 =[0.4,0.8]	x_4 =[0.4,0.8]
y_i	y_1 =[0.3,0.8]	y_2 =[0.4,0.9]	y_3 =[0.5,0.8]	y_4 =[0.4,0.7]
$M_1(x_i, y_i)$	0.950	0.950	0.950	0.950
$M_2(x_i, y_i)$	0.950	0.950	0.950	0.950
$M_3(x_i, y_i)$	0.950	0.950	0.950	0.950
$M_4(x_i, y_i)$	0.900	0.900	0.900	0.900
$M_5(x_i, y_i)$	0.900	0.900	0.900	0.900
$M_6(x_i, y_i)$	0.941	0.923	0.947	0.933
$M_{(2)}(x_i, y_i)$	0.837	0.799	0.902	0.868

6. The New Vague Value of the Similarity and New Formula

The data is given to the new formula.

Definition 1 [11]. For Vague $x=[t_x,1-f_x],$: define $t_x^{(0)}=t_x$, $f_x^{(0)}=f_x$, $\pi_x^{(0)}=\pi_x=1-t_x-f_x$,and $t_x^{(m)}=t_x(1+\pi_x+\pi_x^2+\cdots+\pi_x^m)$, $f_x^{(m)}=f_x(1+\pi_x+\pi_x^2+\cdots+\pi_x^m)$, , $\pi_x^{(m)}=\pi_x^{m+1}$ ($m=0,1,2,\cdots.$).

Lemma 1[11]. $x_x^{(m)}=[t_x^{(m)},1-f_x^{(m)}]$ ($m=0,1,2,\cdots.$) is the vague value.

Definition 2. Assume Vague $x=[t_x,1-f_x],\ y=[t_y,1-f_y]$, if the formula suit the $M(x,y)$:

1) Normative $0\le M(x,y)\le 1$;

2) Symmetry $M(x,y)=M(y,x)$;

3) Mix when the $x=[0,0], y=[1,1]$ or $x=[1,1], y=[0,0]$, $M(x,y)=0$;

4) Max $M(x,y)=1 \Leftrightarrow x=y$ and $\pi_x=\pi_y=0$;

5) ParticularityFor $x=y=[0,1]$, $M(x,y)=0.$ $z=[t_z,1-f_z]$;

6) Resolution when $x\neq y,$ and $z=[t_z,1-f_z]$ is of arbitrary Vaguevalue , $M(x,z)\neq M(y,z)$.

So $M(x,y)$ is Vague value X and Y the similarity.

Explanation:most formula has the lemma, normative, particularity and so on. Max and particularity is trying to resolve the example 5.1 cases of an existing discussion vague value established by the inadequacies similarity measures . Resolution is established when trying to overcome the drawbacks of existing similarity measures in the example 5.2 similarity ,based on paper [12].

Theorem 5 it's the Vague value $x=[t_x,1-f_x]$ and $y=[t_y,1-f_y]$: similarity measures,

$$M_{(m)}(x,y)=1-\frac{\left|t_x^{(m)}-t_y^{(m)}\right|+\left|f_x^{(m)}-f_y^{(m)}\right|+\pi_x^{(m)}+\pi_y^{(m)}}{2} \tag{9}$$

Among $m=0,1,2,\cdots$.

For example 5.1 Vague in the value data,the application of the formula (9) (parameter m=2) calculation calculated the similarity measures between the obtained vague value is also shown in Table 1, in thelastrow. We look forward to these results exactly, seen Table 1, the formula (9) has been overcome vague in example 5.2.

Definition 3. If $X=\{x_1,x_2,\cdots,x_n\}$, which has Vague:

$$G=([t_G(x_1),1-f_G(x_1)],[t_G(x_2),1-f_G(x_2)],\cdots,[t_G(x_n),1-f_G(x_n)]),$$
$$S=([t_S(x_1),1-f_S(x_1)],[t_S(x_2),1-f_S(x_2)],\cdots,[t_S(x_n),1-f_S(x_n)])$$

Labeled

$$G=([t_{G1},1-f_{G1}],[t_{x2},1-f_{G2}],\cdots,[t_{Gn},1-f_{Gn}]),B=([t_{S1},1-f_{S1}],\cdots,[t_{Sn},1-f_{Sn}]).$$

If the formula $M(G,S)$ suits it:

1) Lemma $0\le M(G,S)\le 1$;

2) Particularity $M(G,S)=M(G,S)$;

3) Mix formula $G=([0,0],[0,0],\cdots,[0,0])$, $S=([1,1],[1,1],\cdots,[1,1])$ or $G=([1,1],[1,1],\cdots,[1,1]),S=([0,0],[0,0],\cdots,[0,0])$时,$M(G,S)=0$;

4) Max $M(G,S)=1\Leftrightarrow A=B$,and $\pi_{Gk}=\pi_{Sk}=0,k=1,2,\cdots n$;

5) Particularity when $G=S=([0,1],[0,1],\cdots,[0,1])$, $M(G,S)=0$;

6) Resolution when $G\ne S$, but R is anyone value, so $M(G,R)\ne M(S,R)$. $M(G,S)$ is the similarity Vague between G and M.

Theorem 6. This is $M(G,S)$ vague similarity between G and M.

$$M_{(m)}(G,S)=1-\frac{1}{2n}\sum_{i=1}^{n}\left(\left|t_{Gi}^{(m)}-t_{Si}^{(m)}\right|+\left|f_{Gi}^{(m)}-f_{Si}^{(m)}\right|+\pi_{Gi}^{(m)}+\pi_{Si}^{(m)}\right) \tag{10}$$

$m=0,1,2,\cdots$.

Theorem 7. This is $WM(G,S)$ Vague between G and M:

$$WM_{(m)}(G,S)=1-\frac{1}{2}\sum_{i=1}^{n}w_i\cdot\left(\left|t_{Gi}^{(m)}-t_{Si}^{(m)}\right|+\left|f_{Gi}^{(m)}-f_{Si}^{(m)}\right|+\pi_{Gi}^{(m)}+\pi_{Si}^{(m)}\right) \tag{11}$$

$m=0,1,2,\cdots$, and $w_i(0\le w_i\le 1)$ is the weight of x_i ,and suffice $\sum_{i=1}^{n}w_i=1$.

Example 6.1. Let the domain $X=\{x_1,x_2,x_3\}$, in it's standard modeVague set G_1,G_2,G_3 and preparation recognition mode S :

$$G_1=([0.2,0.4],[0.3,0.6],[0.6,0.7]),G_2=([0.5,0.6],[0.7,0.9],[0.6,0.8]),$$
$$G_3=([0.4,0.7],[0.3,0.8],[0.5,0.9]);S=([0.3,0.5],[0.4,0.7],[0.7,0.9]).$$

Application of the formula (10), take parameters $m=2$ calculate, get:

$$M_{(2)}(G_1,S)=0.83,M_{(2)}(G_2,S)=0.79,M_{(2)}(G_3,S)=0.86$$

Application Vaguecriterion of pattern recognition, getthe preparation recognition mode S itis vested in the standard mode G_3.

Application of the formula (8) calculate, get:

$$M_6(G_1,S)=0.86,M_6(G_2,S)=0.81,M_6(G_3,S)=0.87$$

The application of pattern recognition criteria Vague, getthe preparation recognition mode S Should be vested in the standard mode G_3. And application of the formula (10) the results obtained is the same as.

If the two results are different, then the application of the formula (10) the results obtained are more credible. Because formula (10) satisfies the definition of 2, its structure is more reasonable. Thus, the conclusion that it is more efficient inference.

7. Practical application

Example 7.1. Assume $X=\{x_1,x_2,x_3\}$, among it. Vague G1, G2, G3 and S:

$$G_1=([0.2,0.4],[0.3,0.6],[0.6,0.7]),G_2=([0.5,0.6],[0.7,0.9],[0.6,0.8]),$$
$$G_3=([0.4,0.7],[0.3,0.8],[0.5,0.9]),S=([0.3,0.5],[0.4,0.7],[0.7,0.9]).$$

So (8), and $m=2$ got : $M_{(2)}(G_1,S)=0.83,M_{(2)}(G_2,S)=0.79,M_{(2)}(G_3,S)=0.86$

Based on the Vague pattern recognition criterion, the resultant pattern S should be attributed to the standard pattern G3, which is identical to that obtained by formula (10).

$$M_6(G_1,S)=0.86,M_6(G_2,S)=0.81,M_6(G_3,S)=0.87$$

Note that results yielded by formula (10) is more reliable, for formula (10) satisfies definition 2 and features a better structure.

8. Conclusion

Distance between Vague sets is always the study focus, applications also more widely in recent years, such as references [14-21]. According to application cases, different distance formulas should be applied for different Vague sets, while the calculated distance values are also different. With appropriate distance formula, the more reasonable cluster for Vague set rules, which may be conductive to reduce search scale and calculation complexity in Vague rule-based reasoning, can be obtained. In addition, new definition of similarity measures between Vague sets reserves the basic nature of similarity measures between Vague sets and effectively excludes some deficiencies of similarity measures between Vague sets. In particular, this definition points out that the value should be a certain number in open interval (0, 1), rather than 1, which is more accurate and innovative. The sustainable improvement of definition for similarity measures between Vague sets is not the need for Vague set application, but also the new trend of Vague set theory.

Acknowledgements

The work was supported by the 2013 Hainan education science study topics of the "12th Five-Year Plan (No. QJY125119), the Hainan Planning projects of philosophy and Social Sciences (No. HNSK14-50), the hainan key scientific and technological plan projects (No. zdxm2014087), the 2013 innovation and entrepreneurship training college national college projects (No. 201311100061).

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