TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol. 12, No. 10, October 2014, pp. 7114 ~ 7122
DOI: 10.11591/telkomnika.v12i8.6421

Optimal Design of a 3-Phase Core Type Distribution Transformer Using Modified Hooke and Jeeves Method

Raju Basak[1], Arabinda Das*[2], Amar Nath Sanyal[3]
[1,2]Electrical Engineering Department, Jadavpur University,
188, Raja S.C. Mallick Road, Kolkata - 700032, India
[3]Calcutta Institute of Engineering and Management, Kolkata - 700040, India
*Corresponding author, e-mail: basak.raju@yahoo.com[1], adas_ee_ju@yahoo.com[2],
ansanyal@yahoo.co.in[3]

Abstract

Hooke and Jeeves method is de facto a pattern search technique, which can be employed for getting an optimal solution. In this paper the method, in a modified form, has been applied for the design optimization of a distribution transformer. It is a constrained multi-variable optimization problem. The solution is obtained by choosing an initial point in the world map of the key variables and by making a local search (exploratory in all directions in the hyper surface formed by the variables. After recognizing the pattern, its advantage is taken by moving towards a lower cost point, using an acceleration factor for faster convergence. The step length is adjusted as we proceed to expedite improvement. The method has been applied to two different cost functions: the cost of production and the cost against production plus capitalized running losses. In both the cases, the problem has converged to a solution and the results are both interesting and illuminating.

Keywords*: distribution transformer, cost function, design variables, pattern search, constraints*

1. Introduction

There are four different approaches for solving a design problem [1] viz. analytical design, synthetic design, optimal design and standard design. There is no loop or feedback from the results obtained in the analytic procedure. Hence there is no provision for any constraint satisfaction in this method. The synthetic design is better than the analytic design as it provides for constraint satisfaction. However, this method gives only a feasible solution not the best possible one. Technical persons aim at optimal design- it gives the best possible out of different feasible solutions, satisfying given constraints. Standard design methods are followed by the bulk manufacturers which are based on selection of standard stampings, standard core size etc. All these methods are applied to transformer design.

2. The 3-phase Core-type Oil-immersed Distribution Transformer

The 3-limbed core construction is employed for 3-phase distribution transformers as it is more economic compared to shell type [1, 2]. These are invariably of the oil-immersed type with natural or forced cooling depending on the size. The core is made by stacking varnished laminations of high grade silicon steel. Either copper or aluminium is used as conductor material. The core-coil structure is placed on a soft bed in the oil-filled tank having a protruding conservator along with a breather. The conservator takes care of the expansion of oil under loading and the breather is used to stop the ingress of moisture into the oil tank. Cooling tubes or radiators are to be added to keep the temperature rise of oil within statutory limits. For large rating, forced air or forced oil-cooling has to be augmented. Other auxiliaries for protection like Buchholtz relay, indicators etc. are added. The construction, principle and design considerations for distribution and power transformers have been elucidated in several text-books on electrical machine design [2-4].

Received June 25, 2014; Revised July 19, 2014; Accepted July 28, 2014

2.1. Design Considerations

The load factor of distribution transformers is much less than that of a power transformer. So it is designed for maximum efficiency at its probable load factor (0.4 - 0.6), keeping iron loss relatively less. So a lower flux-density is used compared to that for the power transformer. CRS-type cores are invariably used for all applications. Aluminium is used as conductor in distribution transformers up to a size of about 500 KVA, for economic reasons. But copper is a far better material for larger rating, particularly if there be constraint on the bulk of the transformer as is usual in densely populated urban places. As the voltage regulation has to be kept at a low value for a distribution transformer, the gap between L.T. and H.T. coils is kept at its minimum allowable value and the window height: width ratio is kept at a relatively large value compared to the power transformer to reduce the leakage reactance [3, 4]. Admissible values of design variables are obtained from data-book [5].

2.2. Optimal Design of Transformers- various Methods

The first foot-steps to computer-aided design of electrical machines started long before in fifties. The concept of optimization was established long before in history but its application to machine design came in a much later stage. The optimizing programs developed much later on. One such method has been reported by O.W. Anderson [6] in 1967. Since then the work is in progress- several authors have proposed newer and newer techniques and advanced papers towards realization of optimal design. J.C. Olivares *et al* have described a technique for optimal design of shell-type transformers [7]. He has also highlighted on core lamination selection and choice of conductor materials for distribution transformers [8, 9]. Pavlos *et al* have proposed a heuristic solution to cost-optimization problems for transformer design [10]. Breslin and Hurley have proposed a web-based design of transformer taking help from the internet [11]. Some authors have used recently developed soft-computing techniques for reaching the optimal solution [12]. Hermandez and Aurora have developed an intelligent assistant for designing distribution transformer [13]. Subranian and Padma have proposed a method for optimization of transformer design using bacterial foraging algorithm [14]. A. K. Jadav has advanced a method for optimizing the design of power transformer using simulated annealing [15, 16]. R.A. Jabr has applied geometric programming to transformer design [17].

3. Procedure for Optimization

For reaching an optimal solution, one has to formulate the problem at first, choose the design variables, fix up the constraints and frame the objective function [18]. Maximum and minimum bounds are imposed on the design variables by the experienced designer and the optimal solution is sought in the world map of the variables, either by classical techniques or by recently developed intelligent techniques. There are several techniques to reach an optimal solution, for a constrained or an unconstrained design problem by the classical method. These are broadly classified into methods based on: i) exhaustive search, ii) random search, iii) pattern search, iv) gradient search [19, 20]. The exhaustive search is simple but it is time-consuming, particularly if there be a large number of variables and chosen step lengths are small. The random search gives only a quasi-optimal solution, not the optima. Gradient or pattern search techniques are better mathematical tools which can be used efficiently to find out the global optima in a much less no. of steps. The constraints can be accounted for and the step-length can be varied as the problem converges to its final solution. There are a variety of techniques based on pattern or gradient search. Hooke and Jeeves method is one amongst them [18-20]. It is a direct method based on pattern search, applicable to multivariable problems.

3.1. Hook and Jeeves Method of Pattern Search

The method uses a set of search directions which spans the entire search space defined by the bounds of the design variables [18],[20]. In an n-dimensional problem, there must be n number of linearly independent search directions. These directions and the corresponding step lengths are to be judiciously chosen in order to reach the solution by smaller no. of iterations. In the Hooke and Jeeves method, a combination of exploratory move and heuristic pattern search is used. Firstly, an initial point is chosen in the search space from designer's experience or by consulting a design data-book. Then a local search is made in all the directions to find out the best point around the chosen point.

3.2. Algorithm

The algorithm [20, 21] has been framed with slight modification over the original Hook and Jeeves method, in order to ensure convergence. Let the base point be $\mathbf{x^0}$, where $\mathbf{x}$ is a n-vector; n is the number of design variables. This has to be judiciously chosen for faster convergence. Let the variable x_i (for each iteration) be perturbed to $x_i(1+\lambda_i)$, where λ is the step length. The steps to be followed are as given below:

```
Step 1: read $n$ = number of design variables; $\varepsilon$ = convergence factor for the objective function;
        $k_{\max}$ = maximum number of iterations; $c_{\max}$ = maximum number of iterations for
        Changing step length
Step 2:  for i =1 to n
Step 3:  read $x_i$ , $\lambda_i$ , $\alpha_i$  ' $x \to$ design variable, $\lambda \to$ step length; $\alpha \to$ acceleration factor
Step 4   end for
Step 5:  set $k \leftarrow 0$ 'exploratory move
Step 6:  find $f_i^k = f(x_i)$          'obtained from transformer design subroutine
Step 7:  if $k$ = 0 then go to step 16
Step 8:  If $f_i^{k-1} - f_i^k > 0$ then step 15
Step 9:  for i =1 to n
Step 10:  $\lambda_i \leftarrow s\lambda_i$ 'reduction of step length
Step 11:  end for
Step 12:  $c \leftarrow c+1$
Step 13; if $c > c_{\max}$ then go to step 24
Step 14:  $k \leftarrow k-1$ : go to step 6
Step 15:  if $f_i^{k-1} - f_i^k < \varepsilon$ then go to step 22
Step 16:  For i=1 to n
Step 17:  Find $f_i^+ \leftarrow f[x_i(1+\lambda_i)]$
Step 18:  Find $G_i \leftarrow (f_i^+ - f_i^k) / f_i^k$
Step 19:  end for
Step 20:  set $x_i \leftarrow x_i - G_i\lambda_i\alpha_i$ 'pattern move
Step 21:  $k \leftarrow k+1$ : if $k > k_{\max}$ then go to step 23 else to step 6
Step 22:  print "Success- the solution has converged." print out results: go to step 24
Step 23:  print "Failure, the solution is not obtained within $k_{\max}$ no. of iterations."
Step 24:  stop
Step 25:  end
Step 26: print "Change in step length does not make any improvement. Initialize once again": go
         to step 24
```

4. The Design Variables

The solution of an optimization problem starts by identification of design variables [18, 19]. The objective function may be highly sensitive to certain variables. These are the key variables. For some other variables, the sensitivity may be less. They are given less importance. The key variables to be chosen to optimize a design problem depend on the objective function-whether it is the cost of production or a weighted combination of the cost of production and the lost energy units during its operating life or something else. The variables may be decision variables or continuous variables. For a transformer, the variables have been identified:

a. Decision variables:
   (1) The choice of core material- costlier CRGOS may be more economic than cheaper CRNOS considering the over-all cost including that of copper.
   (2) The choice of conductor materials- costlier copper may have to be used considering over-all performance and cost, particularly if there be space constraints.

b. Continuous variables:
   (1) The emf constant K (in eqn. $E_t = K\sqrt{S}$, where E_t = emf per turn, S =KVA rating).
   (2) The ratio of window height to window width: $R_w = H_w / W_w$
   (3) The maximum flux-density B_m
   (4) The maximum current-density, δ
   (5) The ratio of iron loss to copper loss: P_i / P_c

4.1. The Bounds on Design Variables

From designer's experience and from design data book [19] the following values of design variables have been suggested for a 3-phase core type distribution transformer with copper as conductor material:

E.M.F. constant, K = 0.45 (somewhat smaller for Aluminium)

Window height/width, R_w :3.0-4.0 for distribution transformer

The following choice of materials has been recommended:
Core material: CRNOS for smaller ratings, CRGOS for larger ratings.
Conductor materials: Aluminium for smaller ratings, Copper for larger ratings

After choosing the conductor and the core material judiciously, our task is to choose such values of $K, R_w, B_m \,\&\, \delta$ which gives minimality of the objective function without violating the design constraints. Parallelly, we have to check the iron loss: ohmic loss ratio P_i / P_c .

4.2. Constraints

The design constraints appear due to statutory rules imposed by the regulatory authorities or by the customer. The following constraints have been identified for a distribution transformer [1-2], [4]:

(1) The efficiency should not fall below the specified limit
(2) The voltage regulation should be kept within the specified limit - so their leakage reactance should be relatively low.
(3) The maximum allowable temperature rise must not be exceeded- this has to be accomplished by using cooling tubes/ radiators. Forced cooling may have to be added.

The design variables should be chosen with a look to these points.

4.3. The Objective Function

The next step is to frame the objective function [18, 19] in terms of the design variables and other parameters. If the cost of production of the transformer is taken as the objective function, the iron loss and copper loss are kept at their maximum possible values. Accordingly, the flux density and the current density are kept at their maximum possible values without violating the design constraints. Only the emf constant K and window height: width ratio R_w are considered to be key variables. But if we take into consideration the over-all economy of the customer and the manufacturer then the running cost towards lost energy units must also be included in the objective function. Therefore, the flux density and the current density are also to be chosen as design variables to find the minimality conditions for the chosen objective function. In this paper, we have taken both types of objective functions and made two case-studies to get a clear picture.

5. Results and Discussions

The cost optimal design of a 3-phase oil-filled distribution transformer has been taken up using modified Hooke and Jeeves method. Two case-studies have been made viz.

a) Optimization of the cost of production, based on the current market price of materials and labour.
b) Dual optimization with a look to the interest of the customer and the manufacturer- the objective function is a weighted combination of the cost of production and the price for annual energy loss.

The common elements in the two case-studies are given below:

Specifications:
KVA-rating of the machine = 1000
Nominal power factor = 0.8; Nominal frequency = 50 Hz.
Rated line voltage in L.T. / H.T.: 433 V/ 11000 V
Connection: Delta/ Star; Conductor material: Copper
No. of taps = 5; % turns between taps = 2.5

Materials:
Conductor material: Copper
Helical winding has been chosen for L.T. and cross-over winding for the H.T.
Core material: Laser-Core; Stacking factor =0.92
3-stepped core has been used.

Constraints:
Efficiency $\geq$ 0.98
No-load current $\leq$ 1%
Voltage regulation $\leq$ 4%
Temperature rise $\leq 40^{o}$ C

The specific cost of materials/BOT unit:
Cost of copper = Rs. 600/- per Kg
Cost of iron = Rs. 150/- per Kg
Cost of steel tank = Rs. 90/- per Kg
Cost of oil = Rs. 80/- per liter
Cost of BOT unit = Rs. 4/-

Case-I: In this case, the cost function is the selling cost which includes the direct cost for materials and labour and the indirect cost towards overheads. Two key variables which affect the cost function have been identified. They are: i) the emf constant, K ; ii) the window height: width ratio, R_w. The minimality condition is obtained for the following values of key variables:

The EMF- constant, K = 0.49366 the window height: width ratio, R_w = 3.8816. The following values have been chosen for two other design variables:

Maximum flux-density in the Laser-core = 1.55 Tesla
Current density in the copper conductor = 3.0 A/mm^2

Higher the values of these two variables lower will be the cost of production. Therefore, maximum possible values have been chosen for these two variables without violating the design constraints. Results obtained on convergence are:

Dimensions:
Current in Primary/ Secondary, A: 30.303 / 1333.4
Cross section of primary/ Secondary, mm^2 : 10.101 / 444.46
Number of nominal turns of the primary = 704
Number of additional turns of the primary for tapping = 36

Total number of turns of the primary = 740
Number of nominal turns of the secondary = 16
Net area of core iron = 4.5407E-02 m^2
Gross area of core iron = 4.9355E-02 m^2
Diameter of the core circle = 0.27346 m
Length of the core sides in mm: 247 / 193 / 116
Area of the window = 0.11663 m^2
Window height/width, m: 0.67283 / 0.17334
Distance between core centers = 0.42082 m
Width/height of yoke, m: 0.24748 / 0.19943
Total length of core = 1.1411 m ; Total height of core = 1.0717 m
Mean length of turn of Primary/ secondary, m: 0.96802 / 1.2403
Resistance of Primary/ Secondary, Ω: 1.4168 / 9.3763E-04
The tank length * width * height: 0.561 * 1.414 * 1.222
The number of tubes (50 mm dia.) required = 186

Performance evaluation:
Iron loss = 2674 W / % Iron loss= 0.2674
Copper loss = 8904 W ; % Copper loss = 0.8904 ; Total % loss = 1.1578
Efficiency at full load & 0.8 lagging p.f = 0.98573
Maximum efficiency of 0.99034 occurs at a load of 54.8 %
The magnetizing current = 0.5882 %; Core loss current = 0.26739 %
No load current = 0.64614 %
Leakage reactance = 3.4618 %
Voltage regulation at rated power & p.f. = 2.7894 %

Cost:
The weight/ cost of tank: 454.57 Kg / Rs. 40911/-
The volume/ cost of oil: 0 .96908 liter / Rs. 77526/-
Volume of iron = 0.19528 m^3; Weight of iron = 1493.9 Kg
Cost of iron = Rs. 224083/-
Volume of copper = 4.816734E-02 m^3; Weight of copper = 428.69 Kg
Cost of copper = Rs. 257214/-
Direct cost allowing 25 % labour charge = Rs. 749667/-
Selling cost allowing 35% overhead = Rs. 1012051/-

Average load is assumed to be 100% for 6 hours, 75% for 12 hours and 50% for 6 hours. For a life-span of 7 years, the cost of lost units = Rs. 1952613/-

The selling cost plus the cost of lost units = Rs. 2964664/-

Table 1. Steps in Pattern Search (values at every 10th step)

No	Cost	K	Gr-K	R	Gr-R	δ	Gr- δ	B_m	Gr-B_m
1	2769591/-	0.55	-7.375e-4	3	-2.757e-4	2.3	-3.102e-4	1.4	-5.291e-4
11	2758500/-	0.574	-3.369e-4	3.064	-2.016e-4	2.327	-2.866e-4	1.419	-2.974e-4
21	2754217/-	0.5864	-1.258e-4	3.123	-1.570e-4	2.356	-2.394e-4	1.431	-1.597e-4
31	2752643/-	0.5904	-5.204e-5	3.163	-1.369e-4	2.376	-2.002e-4	1.437	-1.040e-4
41	2751484/-	0.5921	-1.208e-5	3.206	-1.203e-4	2.396	-1.575e-4	1.442	-6.733e-5
51	2750777/-	0.5923	2.908e-6	3.240	-1.097e-4	2.410	-1.254e-4	1.445	-4.853e-5
61	2750305/-	0.5920	8.272e-6	3.269	-1.020e-4	2.421	-1.018e-4	1.447	-3.781e-5
71	2749937/-	0.5916	1.045e-5	3.296	-9.500e-5	2.429	-8.255e-5	1.448	-3.000e-5
81	2749613/-	0.591	1.164e-5	3.323	-8.819e-5	2.437	-6.519e-5	1.449	-2.328e-5
91	2749356/-	0.5904	1.146e-5	3.349	-8.822e-5	2.442	-5.147e-5	1.450	-1.855e-5
101	2749147/-	0.5899	1.109e-5	3.373	-7.648e-5	2.447	-4.056e-5	1.451	-1.464e-5
111	2748975/-	0.5893	1.037e-5	3.395	-7.148e-5	2.451	-3.210e-5	1.452	-1.173e-5
121	2748832/-	0.5889	9.822e-6	3.416	-6.657e-5	2.454	-2.537e-5	1.452	-9.095e-6
131	2748712/-	0.5884	9.095e-6	3.435	-6.239e-5	2.456	-2.001e-5	1.453	-7.276e-6
140	2748640/-	0.5881	8.823e-6	3.446	-5.930e-5	2.457	-1.692e-5	1.453	-6.185e-6
141	2748630/-	0.5880	8.459e-6	3.450	-5.921e-5	2.457	-1.674e-5	1.453	-6.003e-6

Case-II: In this case the cost function has been taken as the selling cost plus capitalized cost for the lost BOT units for an anticipated life of 7 years. The key variables which affect the cost function have been identified. They are:

i) emf constant, K ii) window height:width ratio, R_w iii) flux-density, B_m iv) current density, δ.
Results of pattern search at intermediate steps have been shown in Table 1 to show the problem converges to a solution (Gr- stands for gradients).
The initial values for the design variables have been chosen either from designer's experience or from design data-book. It may be noted that the cost differential comes down rapidly at first and then slowly. The same is true about the individual gradients.

After 140th step, changes have become very small. The cost differential has come down below Rs. 10/-. So this point has been taken as the point of convergence. At convergence, we get the following variables for the design variables:

K = 0.588; R_w = 3.452; δ = 2.458 A/mm^2; B_m = 1.453 Tesla

Based on these values of variables, the design details of the optimal machine are given below:

Dimensions:
Number of nominal turns of the primary = 572
Number of additional turns of the primary for tapping = 28
Total number of turns of the primary = 600
Number of nominal turns of the secondary = 13
Current in Primary/ Secondary: 30.303 / 1333.4 A
Chosen current density = 2.4575 A/mm^2
Cross section of primary/ secondary, mm^2: 12.331 / 542.6
Net area of core iron = 5.960788E-02 m^2
Gross area of core iron = 6.479118E-02 m^2
Diameter of the core circle = 0.31332 m
Length of the core sides in mm: 284; 222; 133
Area of the window = 9.639534E-02 m^2
Window height/width ratio = 3.452 Ω
Window height/width, m: 0.5768 / 0.1671
Distance between core centres = 0.45067 m
Width/height of yoke, m: 0.28355 / 0.22850
Total length/ height of core, m: 1.2444 / 1.0338 m
Inside/outside diameter of L.T. winding, m: 0.319 / 0.375
Inside/outside diameter of H.T. winding, m: 0.405 / 0.455
Mean length of turn of primary/ secondary, m: 1.089324 1.351832
Resistance of Primary/ Secondary: 1.0611 Ω ; 6.8015E-04 Ω
The tank length, width * height: 0.591 * 1.512 * 1.184
The number of tubes (50 mm dia.) required = 141

Performance evaluation:
Iron loss = 2854.4 W; % Iron loss = 0.2854
Copper loss = 6550.4 W; % Copper loss = 0.6551 ; Total % loss = 0.9405
Efficiency at full load and 0.8 lagging p.f. = 0.9884
Maximum efficiency of 0.9914 occurs at a % load of 66.01
The magnetizing current = 0.5466 %; The core loss current = 0.2854 %;
The number load current = 0.6167 %
The % leakage reactance = 2.886
The % voltage regulation at rated power & p.f = 2.2557

Cost:
The weight / cost of tank: 360.6 / Rs. 32454/-
The volume / cost of oil: 1.0569 / Rs. 84554/-
Volume of iron = 0.2515 m$^{3;}$ Weight of iron = 1924 Kg; Cost of iron = Rs. 250120 /-
Volume of copper = 5.278595E-02 m$^{3;}$ Weight of copper = 469.8 kg.

Cost of copper = Rs. 281877/-
Direct cost allowing 25 % labour charge = Rs. 811256/-
Selling cost allowing 35 % overhead = Rs. 1095196/-

Average load is assumed to be 100% for 6 hours, 75% for 12 hours and 50% for 6 hours.

The lost BOT units/annum = 59077
For a life-span of 7 years, the cost of lost units = Rs. 1654168/-
The cost function = Rs. 2749364/-

A comparison of the cost functions of case-I and case-II, reveals that dual optimization is more gainful.

6. Comparison with other Methods

There are several methods for optimizing a design problem. The easiest method is based on exhaustive search. It is simple and straight-forward but it takes large computer run-time, particularly if the steps chose are small. There are methods based on random search, gradient search and pattern search. The random search technique does not give the exact solution; it gives a solution close to it. The closeness depends on no of iterations. The gradient serach techniques are good; they yield the solution in smaller no of steps. However with gradient search, there is a chance of being trapped in local minima, provided the hypersurface in the serach space is not concave. The gradient serach methods are effective but are not computationally efficient. On the other hand, the direct serach method uses an optimization algorithm based on only the function values, not the gradients. The evolutionary method, the simplex search method, Hook and Jeeves pattern search method, Powell's conjugate direction method etc. falls in this category. These methods are computationally more efficient compared to the gradient-based methods.

7. Conclusion

This paper has dealt with design optimization based on modified Hook and Jeeves method. This method is based on pattern search applied to a properly chosen objective function. It is, in essence, a combination of an exploratory move and a pattern move to quickly reach the optimality (in this case minimality) criterion. The design variables are chosen according to the objective function and bounds are imposed on it to define the search space. An initial point is chosen in the search space. In the exploratory move, a local search is made around this point to find out the best point around the current point. Two such points are used in the pattern move in the original work of Hook and Jeeves, but we have used only one based on gradient along with an acceleration factor for faster convergence. Provision has been kept for reducing step-length for exactly reaching the minimal point.

In this work, two case-studies have been made on the same transformer design. The machine is an oil-filled distribution transformer of rating 11000/433 V, 50 Hz., 1000 KVA, with 5% additional turns for tapping's on the H.V. side. In the first case, the objective function is the cost of production and in the second it is a weighted sum of the cost of production and the lost energy units. In the first case, the minimal cost of production has been found out to be Rs. 1012051/-. It is less than that for the second which is Rs. 1095196/-. But the price for lost energy units for an estimated life of 7 years is Rs. 1952613/- for the first case. This is much more than that in the second case which is Rs. 1654168/-. For a saving of a small amount of Rs. 83145/- in the cost of production, the additional cost of lost energy units is Rs. 298445/-. So the advantage of lower cost of production is totally offset due to additional losses. This proves that the objective function should be framed for dual optimization of cost of production and lost energy units.

References

[1] AK Shawney. A course in electrical machine design. Dhanpat Rai & Sons. 2003.
[2] I Dasgupta. Design of Transformers. TMH, New Delhi. 2002.
[3] MG Say. Performance and design of A.C. machines. CBS Publishers & Distributors.

[4] A Still. Principles of Transformer Design. John Wiley and Sons Inc, 2007.

[5] Shanmugasundaram, G Gangadharan, R Palani. *Electrical machine design data book.* Wiley eastern Ltd. ISBN 0 85226 813 0

[6] OW Anderson. Optimum design of electrical machines. *IEEE Trans. (PAS).* 1967; 86: 707-11.

[7] JC Olivares-Galvan, Pavlos et al. Optimal design of single-phase shell-type distribution transformers based on a multiple design method validated by measurements. *Electrical Engg.* 2011; 93: 237-246.

[8] JC Olivares-Galvan et al. Core lamination selection for distribution transformers based on sensitivity analysis. *Electrical Engg*. 2013; 95: 33-42.

[9] JC Olivares-Galván. Selection of copper against aluminium windings for distribution transformers. *IET Electrical Power Application*. 2010: 4: 474-485.

[10] Pavlos S Georgilakis et al. A heuristic solution to the transformer manufacturing cost optimization problem*. Journal of Material Process Technology.* 2007; 181: 260-266.

[11] JG Breslin, WG Hurley*. A web-based system for transformer design*. KES 2003, LNAI. Springer Berlin. 2003; 2773: 715-721.

[12] H Omranpour, M Ebadzadeh, S Shiry, S Barzegar. Dynamic Particle Swarm Optimization for Multimodal Function. *International Journal of Artificial Intelligence (IJ-AI*). 2012; 1(1): 1~10, ISSN: 2252-8938.

[13] C Hernandez, MA Arora. An intelligent assistant for designing distribution transformers. *Expert System Application*. 2008; 34; 1931-1937.

[14] S Subramanian, S Padma. Optimization of transformer design using bacterial foraging algorithm. *International Journal of Computer Applications*. 2011; 19(3): 52-57.

[15] Amit Kumar Yadav et al. Optimization of power transformer design using simulated annealing technique. *International Journal of Electrical Engineering.* ISSN 0974-2158. 2011; 4(2): 191-198.

[16] Amit Kr Yadav, OP Rahi, Hasmat Malik, Abdul Azeem. Design Optimization of High-Frequency Power Transformer by Genetic Algorithm and Simulated Annealing. *International Journal of Electrical and Computer Engineering (IJECE).* 2011; 1(2): 102~109, ISSN: 2088-8708.

[17] RA Jabr. Application of geometric programming to transformer design. *IEEE Trans Magnetics*. 2005; 41: 4261-4269.

[18] K Deb. Optimization for engineering design. PHI. 2010. ISBN 978-81-203-0943-2.

[19] SS Rao. Engineering optimization- theory and practice. New Age International. 2011; ISBN 978-81-224-2723-3.

[20] M Ramamoorty. Computer-aided design of electrical equipment. Affiliated East-West press. 1987. ISBN: 81-85095-57-4.

[21] NS Kambo. Mathematical programming techniques. Revised edition. New Delhi. 1991, 1984. 110 001. Affiliated East-West Press Pvt. Ltd. ISBN 81-85336-47-4.