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Study on Quasi-linear Stochastic Programming Model and Its Solution

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Abstract

In this paper, we first analyze the features and shortcomings of existing stochastic programming methods. For the bottleneck of higher computational complexity, we give the concept of reliability coefficient and a quasi-linear processing pattern for chance-constrain based on mathematic expectation and variance, analyze the relationship between reliability coefficient and reliability (probability), and give the selecting strategy of reliability coefficient. Then, we establish the quasi-linear stochastic programming model, and discuss the performance of this model by an example.

Keywords: *stochastic programming, quasi-linear programming, chance-constrained programming, reliability coefficient*

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1. Introduction

Stochastic programming is a problem to be faced and solved in many areas. Constructing an operable stochastic programming model and its solving method is very important in both theory and applications. And lots of results have been obtained. Yang et al. [1] established the expected value model, chance-constrained programming and dependent-chance programming of transportation problems. [2] presented a new chance constrained programming method for the optimal transmission system for several uncertain factors such as the locations and capacities of new power plants as well as demand growth. [3] used expected value model to consider the problem of producing and transporting a unique product directly from a origin to a destination where demands are stochastic. [4] studied disaster prevention flood emergency logistics planning problems using chance-constrained programming.

At present, there are three generally acknowledged stochastic programming models: Expected value model [5]; Chance-constrained programming [6]; Dependent-chance programming [7, 8]. Although the above three models have been widely used in different fields, a couple of deficiencies still exist: 1) Expected value model can't effectively solve the decision problem under different risk consciousnesses, and the quality of decision can't be guaranteed for extreme uncertainty; 2) Chance-constrained programming has the ability to control decision quality in advance, but the probability distributions of objective and constraints are often difficult to determine or precisely known, therefore, it is difficult to establish operable analytic method under complex random environment; 3) Dependent-chance programming also involves the problem of calculating the probability of events, thus it is difficult to achieve the solving problem; 4)When the distribution of random variable is incomplete, all the above three methods (especially for the chance-constrained programming and dependent-chance programming) cannot give operable solution. Many scholars had many specific discussions on these deficiencies, for instance, [9-14] constructed some solution methods through integrating random simulation and some intelligent algorithms, but random simulation must involve lots of tests, so these methods are only suitable for small-scale stochastic programming problems.

Based on chance-constrained programming model, the main contributions of this paper are as follows. First, we propose a quasi-linear chance-constrain processing pattern based on expectation and variance by transforming equivalently the chance-constrain. Second, we put forward the concept of reliability coefficient, and analyze the relationship between reliability coefficient and reliability (probability), and give the selecting strategy of reliability coefficient.

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Third, we establish a quasi-linear stochastic programming model. Finally, we analyze the features and validity of the model by an example.

In what follows, for the random variable ¶ and event *A* on a probability space $(\Omega, \mathcal{B}, \Pr)$, let $E(¶)$ and $D(¶)$ denote the mathematical expectation and variation of ¶, respectively, $\pm(¶) = (D(¶))^{0.5}$ the standard variation of ¶, and Pr(*A*) the probability of *A*.

2. Quasi-linear Programming Model of Stochastic Programming

2.1 Quasi-linear Processing Stochastic Constraints

For the constraints of stochastic programming can't be often absolutely satisfied, Charnes and Cooper [5] dealt with the constraints and objective functions by reliability, and then put forward chance-constrained programming:

$$\begin{cases} \min \bar{f}(x), \\ \text{s.t. } \Pr(f(x,¶) \le \bar{f}(x)) \ge \sim, \\ \quad \Pr(g_j(x,¶) \le 0) \ge \ _j, \ j = 1, 2, \cdots, m. \end{cases} \tag{1}$$

Here, $x = (x_1, x_2, \cdots, x_n)$ is decision vector, $¶ = (¶_1, ¶_2, \cdots, ¶_n)$ is the given random variable on a probability space $(\Omega, \mathcal{B}, \Pr)$, $f(x,¶)$ is objective function, $g_j(x,¶) \le 0 \ (j = 1, 2, \cdots, m)$ are random constraints, $_j, \sim \in [0,1]$ represent the reliability for the constraints and objective functions, respectively. The key to the chance-constrained programming model is to transform the random constraints and objective function into ordinary ones through some given reliability, which has good explanation. At present, chance-constrained programming is the common method to solve stochastic programming, but this method involves a lot of probability calculations, thus the high complexity make it difficult to realize the solution. In this section, we mainly discuss the simplification of objective function and chance-constrains.

For model (1), by:

$$\Pr(f(x,¶) \le \bar{f}(x)) \ge \sim \Leftrightarrow \Pr\left\{ \frac{f(x,¶) - \bar{f}(x) - E(f(x,¶)) + \bar{f}(x)}{\pm(f(x,¶))} \le \frac{-E(f(x,¶)) + \bar{f}(x)}{\pm(f(x,¶))} \right\} \ge \sim$$

$$\Leftrightarrow \Pr\left\{ \frac{f(x,¶) - E(f(x,¶))}{\pm(f(x,¶))} \le \frac{-E(f(x,¶)) + \bar{f}(x)}{\pm(f(x,¶))} \right\} \ge \sim . \tag{2}$$

We can know, if we set $\Psi = [f(x,¶) - E(f(x,¶))] / \pm(f(x,¶))$, and let $\Psi_\sim$ denote the $\sim$-quantile of Ψ (that is, $\Pr(\Psi \le \Psi_\sim) \ge \sim$, $\Pr(\Psi > \Psi_\sim) \le 1 - \sim$), then $E(\Psi) = 0, D(\Psi) = 1$ (that is, $\Psi_\sim$ is the standard $\sim$-quantile of $f(x,¶)$), and:

$$\Pr(f(x,¶) \le \bar{f}(x)) \ge \sim \ \Leftrightarrow \frac{-E(f(x,¶)) + \bar{f}(x)}{\pm(f(x,¶))} \ge \Psi_\sim$$

$$\Leftrightarrow E(f(x,¶)) + \Psi_\sim \pm(f(x,¶)) \le \bar{f}(x). \tag{3}$$

Similarly, for constraints,

$$\Pr(g_j(x,¶) \le 0) \ge \ _j \Leftrightarrow -E(g(x,¶)) / \pm(g_j(x,¶)) \ge \Phi^{(j)}_j$$

$$\Leftrightarrow E(g_j(x,¶)) + \Phi^{(j)}_j \pm(g_j(x,¶)) \le 0 . \tag{4}$$

Here, $\Phi_j = [g_j(x,¶) - E(g_j(x,¶))] / ±(g_j(x,¶))$, and $\Phi^{(j)}_j$ denote the $_j$-quantile of Φ_j (that is, $\Pr(\Phi_j \le \Phi^{(j)}_j) \ge \ _j$, $\Pr(\Phi_j \ge \Phi^{(j)}_j) \le 1- \ _j$), then $E(\Phi_j) = 0,\quad D(\Phi_j) = 1$ (that is, $\Phi^{(j)}_j$ is the standard $_j$-quantile of $g_j(x,¶)$), $j = 1, 2, \cdots, m$.

2.2. Quasi-linear Stochastic Programming Model

To correspond with reliability $_j, \sim \in [0,1]$, we call standard quantile $\Psi_{\sim}$, $\Phi^{(j)}_j$ reliability coefficient. Easily to know, $\Psi_{\sim}$, $\Phi^{(j)}_j$ is the function of *x* and only depends on the distribution of $f(x,¶)$ and $g_j(x,¶)$. The reliability coefficient of common distributions has regularity [15] (Table 1 lists the reliability coefficient of common distributions), so when the distributions of $f(x,¶)$ and $g_j(x,¶)$ are basically determined, we can regard $\Psi_{\sim}$, $\Phi^{(j)}_j$ as constants, then chance-constrained programming (1) can be converted into following linear model based on expectation and standard deviation:

$$\begin{cases} \min\ \bar{f}(x), \\ \text{s.t.}\ \ E(f(x,¶)) + \Psi_{\sim} ±(f(x,¶)) \le \bar{f}(x), \\ \quad\ E(g_j(x,¶)) + \Phi^{(j)}_j ±(g_j(x,¶)) \le 0,\ j = 1, 2, \cdots, m. \end{cases} \tag{5}$$

Above analysis indicates the reliability coefficient is the parameter reflecting decision consciousness, but the same reliability coefficient represents different reliability (probability) in different distributions, therefore, we should select reliability coefficient combining with the distribution feature of objective and constraints in the reality.

$\min\ \bar{f}(x)$ and $E(f(x,¶)) + \Psi_{\sim} ±(f(x,¶)) \le \bar{f}(x)$ in model (5) can be integrated into $\min\left[E(f(x,¶)) + \Psi_{\sim} ±(f(x,¶))\right]$, so (6) can be simplified as:

$$\begin{cases} \min\left[E(f(x,¶)) + \Psi_{\sim} ±(f(x,¶))\right], \\ \text{s.t.}\ \ E(g_j(x,¶)) + \Phi^{(j)}_j ±(g_j(x,¶)) \le 0,\ j = 1, 2, \cdots, m. \end{cases} \tag{6}$$

Obviously, when the expectation and variance of $f(x,¶)$, $g_j(x,¶)$ are easily to be calculated, the computation complexity of (6) is much lower than that of (1), thus (6) greatly improve the operability of stochastic programming under a certain condition. Because this model is linear on mathematical expectation and standard deviation, we call (1) quasi-linear stochastic programming model in the following.

Easily to see: 1) when $\Psi_{\sim} = 0$, $\Phi^{(j)}_j \equiv 0$, (6) is the expected value model; 2) when $f(x,¶)$ and $g_j(x,¶)$ have no randomness (that is, $D(f(x,¶)) = 0, D(g_j(x,¶)) = 0)$, (6) has the same solution as the ordinary programming; 3) when $f(x,¶)$ or $g_j(x,¶)$ has randomness, different $\Psi_{\sim}$, $\Phi^{(j)}_j$ will get different optimal schemes.

Here, $N(«, ±^2)$ denotes the normal distribution with parameters « and $±^2$, $U(a,b)$ is the uniform distribution with parameters *a* and *b*, $Exp(ª)$ is the exponential distribution with parameter ª .

Table 1. Reliability Coefficients of Common Distributions

Distribution	Expectation	Variance	Parameter	Reliability	Reliability coefficient
$N(«, ±^2)$	$«$	$±^2$	$«, ±^2$	0.50	0
				0.55	0.13
				0.60	0.25
				0.65	0.39
				0.70	0.52
				0.75	0.68
				0.80	0.84
				0.85	1.04
				0.90	1.28
				0.95	1.65
$U(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	a, b	0.50	0
				0.55	0.17
				0.60	0.35
				0.65	0.52
				0.70	0.69
				0.75	0.87
				0.80	1.04
				0.85	1.21
				0.90	1.39
				0.95	1.56
$Exp(¯)$	$-$	$-_2$	$-$	0.50	-0.31
				0.55	-0.20
				0.60	-0.08
				0.65	0.05
				0.70	0.20
				0.75	0.39
				0.80	0.61
				0.85	0.90
				0.90	1.30
				0.95	1.99

3. Example Analysis

3.1. Example 1 [16] Consider the Following Stochastic Programming

$$\begin{cases} \max ¶_1 x_1 + ¶_2 x_2 + ¶_3 x_3 \\ \text{s.t. } ¦_1 x_1^2 + ¦_2 x_2^2 + ¦_3 x_3^2 \le 8, \\ \quad ²_1 x_1^3 + ²_2 x_2^3 + ²_3 x_3^3 \le 15, \\ \quad x_1, x_2, x_3 \ge 0. \end{cases} \tag{7}$$

Here, $¶_1, ¦_1, ²_1$ follow uniform distributions $U(1,2)$, $U(2,3)$, $U(3,4)$, $¶_2, ¦_2, ²_2$ follow normal distributions $N(1,1)$, $N(2,1)$, $N(3,1)$, and $¶_3, ¦_3, ²_3$ follow exponential distributions $Exp(1), Exp(2), Exp(3)$.

Using chance-constraint programming model (1) and quasi-linear programming (6), we can convert (7) into the following programming (8) and (9):

$$\begin{cases} \max \overline{f} \\ \text{s.t. } \Pr\{¶_1 x_1 + ¶_2 x_2 + ¶_3 x_3 \ge \overline{f}\} \ge \sim, \\ \quad \Pr\{¦_1 x_1^2 + ¦_2 x_2^2 + ¦_3 x_3^2 \le 8\} \ge {}_1, \\ \quad \Pr\{²_1 x_1^3 + ²_2 x_2^3 + ²_3 x_3^3 \le 15\} \ge {}_2, \\ \quad x_1, x_2, x_3 \ge 0. \end{cases} \tag{8}$$

$$\begin{cases} \max\left[1.5x_1 + x_2 + x_3 + \Psi_{\sim}\sqrt{x_1^2/12 + x_2^2 + x_3^2}\right] \\ \text{s.t. } 2.5x_1^2 + 2x_2^2 + 2x_3^2 + \Phi_1^{(1)}\sqrt{x_1^4/12 + x_2^4 + 4x_3^4} \le 8, \\ \quad 3.3x_1^3 + 3x_2^3 + 3x_3^3 + \Phi_2^{(2)}\sqrt{x_1^6/12 + x_2^6 + 9x_3^4} \le 15, \\ \quad x_1, x_2, x_3 \ge 0. \end{cases} \tag{9}$$

Both (8) and (9) are nonlinear programming, and difficult to solve by analytic method, but there exist essential differences between them. Since the distributions of stochastic variables $\P_1 x_1 + \P_2 x_2 + \P_3 x_3$, $¦_1 x_1^2 + ¦_2 x_2^2 + ¦_3 x_3^2$ and ${}^2{}_1 x_1^3 + {}^2{}_2 x_2^3 + {}^2{}_3 x_3^3$ are difficult to determine and have close connection with x_1, x_2, x_3, model (8) needs stochastic simulation and intelligent algorithm to solve. But model (9) is an ordinary programming problem which can be solved by LINGO. The results are listed in Table 2.

Table 2. The Results of Chance-constrained Programming and Quasi-linear Programming

Model	Reliability	Method	(x_1, x_2, x_3)	($E(f(x))$, $\sigma(f(x))$)	Time (s)
(8)	$\sim = 0.90,\ {}_1 = 0.80,\ {}_1 = 0.85$	Stochastic simulation and genetic algorithm	(1.5296, 0.4226, 0.6608)	(2.6130, 0.9001)	1142.382
	$\Psi_{\sim} = \Phi_1^{(1)} = \Phi_2^{(2)} = 0$	LINGO9.0	(1.2312, 1.0259,1.0259)	(3.8990, 1.4937)	2.532
	$\Psi_{\sim} = \Phi_1^{(1)} = \Phi_2^{(2)} = 0.5$	LINGO9.0	(1.4621, 0.7604,0.6656)	(2.8881, 1.0952)	2.423
	$\Psi_{\sim} = \Phi_1^{(1)} = 0.8, \Phi_2^{(2)} = 0.85$	LINGO9.0	(1.5354, 0.6115, 0.5393)	(2.6862, 0.9280)	2.491
(9)	$\Psi_{\sim} = 0.9, \Phi_1^{(1)} = \Phi_2^{(2)} = 1.0$	LINGO9.0	(1.5377, 0.5749, 0.5060)	(2.6186, 0.7836)	2.289
	$\Psi_{\sim} = 1.2, \Phi_1^{(1)} = \Phi_2^{(2)} = 1.1$	LINGO9.0	(1.5362, 0.5530, 0.4998)	(2.5890, 0.8673)	2.135
	$\Psi_{\sim} = \Phi_1^{(1)} = \Phi_2^{(2)} = 1.3$	LINGO9.0	(1.5333, 0.5248, 0.4755)	(2.5336, 0.8351)	2.103
	$\Psi_{\sim} = 1.5, \Phi_1^{(1)} = \Phi_2^{(2)} = 1.4$	LINGO9.0	(1.5386, 0.4715, 0.4380)	(2.4481, 0.7819)	2.122

Note: In model (8), the times of stochastic simulation are 2000, the parameter of genetic algorithm are as follows: 1) the length is 36, population size is 40, the max executing generations are 200; 2) using optimum retention mechanism; 3) scale selection operator; 4 crossover probability is 0.6, mutation probability is 0.001.

From Table 2 we can see: 1) The complexity of model (8) is larger than (9); 2) The optimal solution of model (8) and (9) are different, but the distributions of stochastic objective are basically the same (for instance, the Case 3~5 of model (8) and (9)). In addition, for appropriate parameter setting, the quality of (9) is better than that of (8). For case 4 of (9), the corresponding expectation of random objective is basically same with that of (8), but the variance is smaller than that of (8); 3) With the increasing of reliability coefficient of model (9), the corresponding expectation and variance of stochastic objective are reducing.

4. Conclusion

By analyzing the essence of stochastic programming and the shortcomings of existing methods, we propose a quasi-linear pattern based on expectation and variance. Moreover, we establish a quasi-linear stochastic programming model and discuss its performance by an example. The results indicate that the quasi-linear programming model can effectively solve the stochastic programming problem under complex environment or with incomplete information, and has the advantages of simpler operability, better explanation, and lower computation. This method not only integrates subjective consciousness into decision, but also can realize the optimal performance of the system.

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