

Pilot-aided Joint Channel Estimation for OFDM based Cooperative Multi-cell Networks

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Abstract

Multi-cell joint channel estimation (JCE) is the basis for application of multi-cell cooperative processing. For weakness of the existing algorithms that power delay profile (PDP) knowledge of all multiple cells need be known or pilot sequence sets of all cells must be identical, in this paper, a pilot-aided time domain JCE algorithm is presented. And then, by using Generalized Akaike Information Criterion (GAIC) to estimate PDPs of all cells for reducing the signal space of channel estimation, the paper further optimizes this JCE algorithm. Simulation results show that the proposed algorithms have good mean square error (MSE) and corresponding space frequency block coded (SFBC) cooperative multi-cell transmission system has the good bit error rate (BER) performance too.

Keywords: Multi-cell cooperation, Pilot-aided, Joint channel estimation, GAIC

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1. Introduction

Recently, for solving the severe inter-cell interference (ICI) problem in deployment of Orthogonal Frequency Division Multiplexing (OFDM) based future wireless communication networks, the emerging technologies based on multi-cell cooperation and joint process are receiving more attention and research. Nevertheless, the employments of these technologies are highly dependent on the whole exact channel state information (CSI) among the cooperative BSs and the users. Therefore the optimum channel estimation is the base of future wireless communication networks.

In cooperative cellular systems, joint channel estimation (JCE), which is initially applied in a multi-user scheme, could be extended to a multi-cell environment and obtain better performance as depicted in publications [1–3]. However, multi-cell JCE needs the power delay profiles (PDP) knowledge of multiple cells known by receiver or pilot sequence sets of all cells being identical (in this paper, we exploit PDP to precisely represent multipath delays power and multipath tap locations [4], the below is same). In general, the signals received from different BSs or different users will not have the same PDP for different spatial locations of these BSs or users [5] and while the PDP knowledge will not be known a priori to receiver, these problems make the practical value of multi-cell JCE reduced severely. For this weakness, this paper presents a pilot-aided time domain JCE algorithm for cooperative multi-cell network and derives the corresponding Cramer-Rao bound (CRB) when PDPs of multiple cells are not same and unknown. Then, by using Generalized Akaike Information Criterion (GAIC) to estimate PDPs of all cells for reducing the signal space of channel estimation, the paper further optimizes this JCE algorithm.

The rest of the paper is organized as follows. Section 2 introduces a cooperative multi-cell OFDM system model. Section 3 presents a pilot-aided time domain multi-cell JCE algorithm and further optimizes it by exploiting GAIC. Numerical simulations are presented to verify performance of the proposed algorithms in section 4 and the paper will be concluded in last Section 5.

Notation: In this paper, Bold letters represent a matrix or a vector; $\lceil x \rceil$ denotes the nearest integer larger than or equal to x ; $(\cdot)^H$ stands for the conjugate transpose; $\text{diag}(\cdot)$ indicates the diagonal matrix; \mathbf{I}_N denotes $N \times N$ identity matrix; $\|\cdot\|$ is the Euclidean norm; $E[\cdot]$ represents expectation.

2. System Model

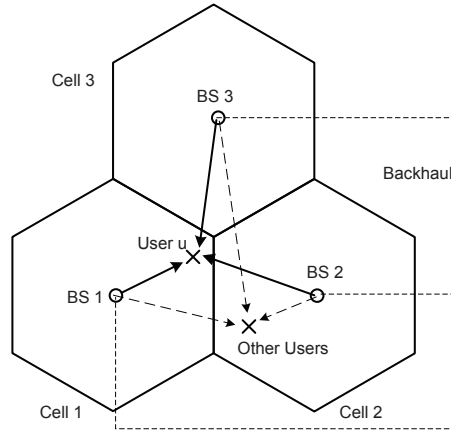


Figure 1. An OFDM based multi-cell cooperation scenario with frequency reuse 1.

Consider an OFDM system operating with a bandwidth of $Bw = 1/THz$ (T is the sampling period) and consisting a total of N subcarriers using QPSK modulation over frequency selective Rayleigh fading channels. A cyclic prefix (CP) of length L_{CP} is inserted before each symbol. Comb-type pilot pattern is exploited to perform channel estimation i.e., in each symbol, N_p tones used as pilots to assist channel estimation are evenly distributed over N subcarriers with equal power. Channel impulse response (CIR) has L multipath components, where each path is characterized by a complex gain factor $h_l(t)$ and a corresponding delay $\tau_l(t)$:

$$h(t, \tau) = \sum_{l=0}^{L-1} h_l(t) \delta(\tau - \tau_l(t) \cdot T) \quad (1)$$

In practice, PDP of channel is considered to be constant over a frame, so t in (1) can be ignored and channel frequency response (CFR) vector for the k -th subcarrier of the n -th OFDM symbol can be expressed as

$$H(n, k) = \sum_{l=0}^{L-1} h_l(n) \exp(-j \frac{2\pi}{N} k \tau_l), \quad k = 1, \dots, N \quad (2)$$

Since JCE does not need to distinguish downlink or uplink of system [3], an application scenario of cooperative Multi-cell OFDM system downlink is illustrated as an example in Figure 1. N_{BS} BSs each equipped with N_T transmit antennas cooperatively transmit to U users distributed around N_{BS} cells. Based on cooperation, pilot sets of multiple cells are known by every user and exploited to estimate corresponding CSI, while all BSs could share these information. Therefore joint process technique can be used to further improve system performance.

Without loss of generality, we discuss channel estimation for case of only one user, because, from signal processing point of view, multiple antenna wireless channels and multi-user wireless channels have no intrinsic difference, if the channels are assumed to be independent from each other both can be reduced to a Multiple-Input and Single Output (MISO) channel model [3]. So the system diagram is depicted as Figure 2.

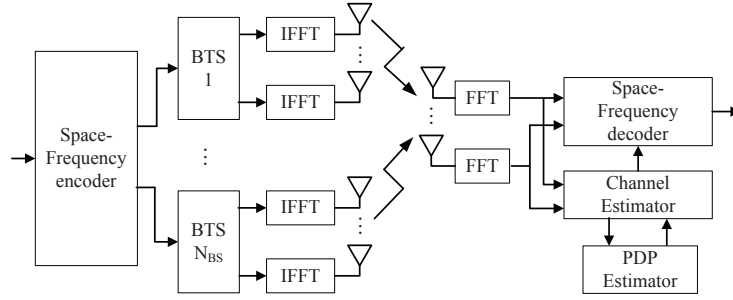


Figure 2. Pilot-aided time domain JCE model for multi-cell cooperation system.

3. Pilot-aided time domain joint channel estimation

Because of the difference on location for multiple cells, the PDPs of these cells are not same [5]. So, set channel order as L , we represent the condition that PDPs of different cells are not same with the maximum time delay set $\{\tau_L^{(1)}, \dots, \tau_L^{(N_{BS})}\}$. Call for each OFDM symbol, N_p pilot symbols $\{p(s); s = 1, \dots, N_p\}$ are evenly inserted into the N subcarriers at positions $\{k_s; k_s = (s-1) \cdot D_f, D_f = N/N_p, s = 1, \dots, N_p\}$ with equal power $E_p = |p(s)|^2$ and equal distance D_f between two adjacent pilots, where k_s indicates the subcarrier index of pilot.

3.1. Pilot-aided time domain channel estimation algorithm

Assuming the perfect synchronization in multiple cells, the received signal vector on pilot subcarriers of the n -th OFDM symbol between the j -th transmit antenna of i -th BS and the q -th receive antenna of user u can be expressed as:

$$\begin{aligned} \mathbf{Y}_{p,u}^q(n) &= \sum_{i=1}^{N_{BS}} \sum_{j=1}^{N_T} \mathbf{X}_{p,u,j,i}(n) \mathbf{H}_{p,u,j,i}^q(n) + \mathbf{W}_{p,u}^q(n) \\ &= \sum_{i=1}^{N_{BS}} \sum_{j=1}^{N_T} \mathbf{X}_{p,u,j,i}(n) \mathbf{F}_p \mathbf{h}_{u,j,i}^q(n) + \mathbf{W}_{p,u}^q(n) \\ &= \mathbf{G}_{p,u}(n) \mathbf{h}_u^q(n) + \mathbf{W}_{p,u}^q(n) \end{aligned} \quad (3)$$

Where, $\mathbf{Y}_{p,u}^q(n) = [Y_{p,u}^q(n, k_1), \dots, Y_{p,u}^q(n, k_{N_p})]^T$ is the frequency-domain received vector of N_p dimension. $\mathbf{X}_{p,u,j,i}(n) = \text{diag}\{p_{u,j,i}(n, 1), \dots, p_{u,j,i}(n, N_p)\}$ indicates the pilot matrix of dimension $N_p \times N_p$ with element $p_{u,j,i}(n, k)$ representing the k -th pilot symbol of n -th OFDM symbol between the j -th transmit antenna of i -th base station and the user u . $\mathbf{H}_{u,j,i}^q(n) = [H_{u,j,i}^q(n, 0), \dots, H_{u,j,i}^q(n, N_p - 1)]^T$ denotes the CFR vector between the j -th transmit antenna of i -th BS and the q -th receive antenna of user u . While $\mathbf{h}_{u,j,i}^q(n) = [h_{u,j,i}^q(n, 0), \dots, h_{u,j,i}^q(n, N - 1)]^T$ is the corresponding CIR vector. \mathbf{F}_p is the $N_p \times N$ dimension submatrix derived from the N -point FFT matrix whose (s, l) -th element is denoted by $[\mathbf{F}_p]_{s,l} = \exp(-j2\pi k_s l / N)$, $s = 1, \dots, N_p$, $l = 1, \dots, N$. So the corresponding equivalent matrix for the total pilots of multiple cells can be expressed as

$$\mathbf{G}_{p,u}(n) = [\mathbf{X}_{p,u,1,1}(n) \mathbf{F}_p, \dots, \mathbf{X}_{p,u,N_T,1}(n) \mathbf{F}_p, \dots, \mathbf{X}_{p,u,1,N_{BS}}(n) \mathbf{F}_p, \dots, \mathbf{X}_{p,u,N_T,N_{BS}}(n) \mathbf{F}_p]$$

with dimension of $N_p \times N_{BS} N_T N$. While the equivalent CIR matrix can be shown as

$$\mathbf{h}_u^q(n) = [\mathbf{h}_{u,1,1}^q(n), \dots, \mathbf{h}_{u,N_T,1}^q(n), \dots, \mathbf{h}_{u,1,N_{BS}}^q(n), \dots, \mathbf{h}_{u,N_T,N_{BS}}^q(n)]^T$$

of $N_{BS} N_T N \times 1$ dimension. $\mathbf{W}_{p,u}^q(n)$ is a noise vector of dimension $N_p \times 1$, whose entries are

assumed to be i.i.d. and complex Gaussian distributed with zero-mean and equal variance σ_w^2 .

Since PDPs remain constant over a frame, n of (3) can be ignored. Thereby ignoring the index of u and q , (3) can be rewritten as

$$\mathbf{Y}_p = \mathbf{G}_p \mathbf{h} + \mathbf{W}_p \quad (4)$$

Obviously, \mathbf{G}_p is an ill-conditioned matrix. According to least squared (LS) criteria, there is no certain solution of (4). Fortunately, CIR is always much shorter than OFDM symbol length, i. e., $\max\{\tau_L^{(i)}\} \ll N, i = 1, \dots, N_{BS}$. Letting $L_b = \lceil N_p / (N_T \cdot N_{BS}) \rceil$ and considering $\max\{\tau_L^{(i)}\} \leq L_b$, (4) can be rearranged as

$$\mathbf{Y}_p = \mathbf{G}'_p \mathbf{h} + \mathbf{W}_p \quad (5)$$

where $\mathbf{G}'_p = [\mathbf{X}_{p,1,1} \mathbf{F}_{p,L_b}, \dots, \mathbf{X}_{p,N_T,1} \mathbf{F}_{p,L_b}, \dots, \mathbf{X}_{p,1,N_{BS}} \mathbf{F}_{p,L_b}, \dots, \mathbf{X}_{p,N_T,N_{BS}} \mathbf{F}_{p,L_b}]$ is a equivalent matrix with dimension of $N_p \times N_{BS} N_T L_b$, \mathbf{F}_{p,L_b} stands for the matrix \mathbf{F}_p retaining the first L_b columns. Because $\mathbf{h}_{j,i}$ is modified as an L_b -dimensional vector, the equivalent CIR matrix can be shown as $\mathbf{h} = [\mathbf{h}_{1,1}^T, \dots, \mathbf{h}_{N_T,1}^T, \dots, \mathbf{h}_{1,N_{BS}}^T, \dots, \mathbf{h}_{N_T,N_{BS}}^T]^T$ with dimension of $N_{BS} N_T L_b \times 1$. So we can obtain the corresponding LS estimator for \mathbf{h} :

$$\hat{\mathbf{h}} = (\mathbf{G}'_p{}^H \mathbf{G}'_p)^{-1} \mathbf{G}'_p{}^H \mathbf{Y}_p \quad (6)$$

From (6), the MSE of single tap of \mathbf{h} with multi-cell JCE can be given by

$$MSE_{LS} = \frac{1}{N_{BS} N_T L_{CP}} E[|\hat{\mathbf{h}} - \mathbf{h}|^2] = \frac{\sigma_w^2}{N_{BS} N_T L_{CP}} \text{tr}[(\mathbf{G}'_p{}^H \mathbf{G}'_p)^{-1}] \quad (7)$$

3.2. Pilots Design

According to [6], if $(\mathbf{G}'_p{}^H \mathbf{G}'_p)^{-1} = \mathbf{I}_{N_{BS} N_T L_b} / (N_p E_p)$ can be satisfied, (6) will be simplified as $\hat{\mathbf{h}} = \mathbf{I}_{N_{BS} N_T L_b} \mathbf{G}'_p{}^H \mathbf{Y}_p / (N_p E_p)$ with computational complexity being greatly reduced; While minimum MSE value $\sigma_w^2 / (N_p E_p)$ will be obtain. We exploit Chu sequence to design pilots which has perfect orthogonality as [7] to meet this condition.

First, we arbitrarily choose a length of N_p Chu sequence [8] $\{c(s); s = 1, \dots, N_p\}$ as the basis pilot sequence for further design. The s -th element of a length of N_p Chu sequence is expressed as:

$$c(s) = \begin{cases} e^{j(\pi r(s-1)^2)/N_p}, & \text{for even } N_p \\ e^{j(\pi r(s-1)s)/N_p}, & \text{for odd } N_p \end{cases}, s = 1, \dots, N_p \quad (8)$$

in which r and N_p are relatively prime.

Then, let $\Delta = L_b$, the pilot value $p_{j,i}(s)$ sent from the j -th transmit antenna of i -th BS can be described as:

$$p_{j,i}(s) = A \cdot c(s) \cdot e^{j \frac{2\pi s \cdot \Delta((i-1)N_T + (j-1))}{N_p}} \quad (9)$$

where $A = \sqrt{E_p}$ is the amplitude of pilot symbol. $i = 1, \dots, N_{BS}$ indicates the index of cell or BS, $j = 1, \dots, N_T$ denotes the index of transmit antenna, $s = 1, \dots, N_p$ represents the index of pilot symbol.

Due to the perfect periodic autocorrelation property of the polyphase Chu sequence, the pilot sequences designed as (9) will be orthogonal to each other and leads to the condition of $(\mathbf{G}'_p{}^H \mathbf{G}'_p)^{-1} = \mathbf{I}_{N_{BS} N_T L_b} / (N_p E_p)$ being satisfied. Then the minimum MSE of channel estimator will be obtained. Cramer-Rao Bound (CRB) reflects a lower bound on the error variance for an unbiased estimator. The CRB for an arbitrary estimator $\hat{\mathbf{h}}_{j,i}$ of \mathbf{h} with signal model as (5) is given

by [9]:

$$CRB(\hat{\mathbf{h}}_{j,i}) = (L\sigma_w^2)/(N_p E_p) \quad (10)$$

While MSE of $\hat{\mathbf{h}}_{j,i}$ is defined as:

$$MSE = E[||\mathbf{h}_{j,i} - \hat{\mathbf{h}}_{j,i}||^2] \geq CRB(\hat{\mathbf{h}}_{j,i}) \quad (11)$$

3.3. Optimum algorithm

In broadband wireless communications, multipath channel is sparse in general. The channel estimation accuracy can be improved by once knowing the PDP knowledge of channel. In [10], GAIC criterion is applied to MIMO sparse channel estimation problem to estimate PDP in an iterative manner and obtain better performance. We extend this algorithm to cooperative multiple cells scheme. Because the PDPs of different cells are not same and the PDPs of different transmit antennas in a cell are identical, for each cell only the PDP of channel estimator between the first transmit antenna and user, i.e. $\hat{\mathbf{h}}_{1,i}$, $i = 1, \dots, N_{BS}$, needs to estimate. Then, for estimator $\hat{\mathbf{h}}_{1,i}$, the GAIC cost function for a test channel order l has the form:

$$GAIC_{1,i}(l) = L_b/2 \ln(\hat{\sigma}_{1,i;l}^2) + \gamma \ln(\ln(L_b))(l+1) \quad (12)$$

γ is a user specified parameter, $\hat{\sigma}_{1,i;l}^2$ denotes the estimation of noise variance for channel order l and is given by

$$\begin{aligned} \hat{\sigma}_{1,i;l}^2 &= (\hat{\mathbf{h}}_{1,i} - \hat{\mathbf{h}}_{1,i;l})^H \mathbf{F}_{p;L_b}^H \mathbf{X}_{p;1,i}^H \mathbf{X}_{p;1,i} \mathbf{F}_{p;L_b} (\hat{\mathbf{h}}_{1,i} - \hat{\mathbf{h}}_{1,i;l}) / L_b \\ &= (\hat{\mathbf{h}}_{1,i} - \hat{\mathbf{h}}_{1,i;l})^H N_p I_{L_b} (\hat{\mathbf{h}}_{1,i} - \hat{\mathbf{h}}_{1,i;l}) / L_b \end{aligned} \quad (13)$$

where $\hat{\mathbf{h}}_{1,i;l}$ is the channel estimator for channel order l and padded with $L_b - l$ zeros. $\mathbf{X}_{p;1,i} \mathbf{F}_{p;L_b}$ is the submatrix from \mathbf{G}'_p of (5) with $N_p \times L_b$ dimension.

The GAIC test is executed as following steps:

Algorithm 1 Optimization for JCE with GAIC test

- 1: Initially set $P = L_{CP}$;
 - 2: Calculate the cost function $GAIC_{1,i}(L)$ for $L = 1, \dots, P$;
 - 3: Obtain the GAIC estimator as $\hat{L} = \arg \min_L \{GAIC_{1,i}(L)\}$;
 - 4: Remove the effect of the newly estimated tap by setting $\hat{\mathbf{h}}_{1,i}(\hat{L}) = 0$;
 - 5: Set $P = \hat{L} - 1$ and repeat 1-3 to estimate the next significant tap positions;
 - 6: If $\hat{L} \neq 1$ go to step 4.
-

Then the set $\{\hat{L}\}$ gives the positions of significant channel taps and it could be utilized to improve performance of the estimator as

$$\hat{\mathbf{h}}'_{j,i}(l) = \begin{cases} \hat{\mathbf{h}}_{j,i}(l), & l \in \{\hat{L}\} \\ 0, & l \notin \{\hat{L}\} \end{cases}; j = 1, \dots, N_T; i = 1, \dots, N_{BS} \quad (14)$$

In addition, this algorithm must be satisfied with $L_b > L_{CP} > \max\{\tau_L^{(i)}\}, i = 1, \dots, N_{BS}$.

4. Numerical Simulation results

Simulations were carried out to demonstrate MSE and bit error rate (BER) performance of the proposed algorithms and the MSE is defined as (11).

An OFDM system is simulated with following parameters: center frequency $f_c = 2.2GHz$, bandwidth $Bw = 5MHz$, number of overall subcarriers $N = 512$, CP length $L_{CP} = 24$. Each OFDM frame consists of $N = 50$ OFDM symbols. Comb-type pilot pattern is employed in system with number of pilot subcarriers $N_p = 128$ and pilot interval $D_f = 4$. The mobile speed is 30km/h.

Assuming that the cells number $N_{BS} = 2$ in simulation model, one BS per cell and each BS equipped with N_T transmit antennas, each user has receive antennas $N_R = 2$. The system exploits SFBC and QPSK modulation.

Multipath Rayleigh fading channels are considered as 3GPP-TR-25.996 SCM Case II channel model [11], with order $L = 6$. The time delay powers in PDPs of multiple cells is assumed to be exponential with pdf(t) $\propto \exp(-t)$. While the multipath tap locations in PDPs of multiple cells contain different and are uniformly distributed over $[0, 2510]ns$.

For the convenience of comparison, let $L_b = 32 \leq \lceil N_p / (N_T \cdot N_{BS}) \rceil$, $E_p = 1$ and define E_b/N_0 as $(N_T \cdot E_p) / \sigma_w^2$. Parameter γ is chosen as 2.5.

Figure 3 shows the MSE performance of the proposed pilot-aided multi-cell JCE algorithm (denoted as PA-MC-JCE) and its optimum algorithm with GAIC (denoted as GAIC-MC-JCE) compares with the corresponding CRB. From the figure, we observe that the MSE of multi-cell JCE (indicated as MC-JCE) with PDPs knowledge known accurately coincides with the curve of CRB and it means the estimator is unbiased. At the same time, because of using L_b length to replace the accurate PDPs knowledge, the MSE performance of PA-MC-JCE losses about 7.5 dB compared with CRB. However, due to optimization of GAIC, the MSE curve of GAIC-MC-JCE is close to CRB with E_b/N_0 increasing.

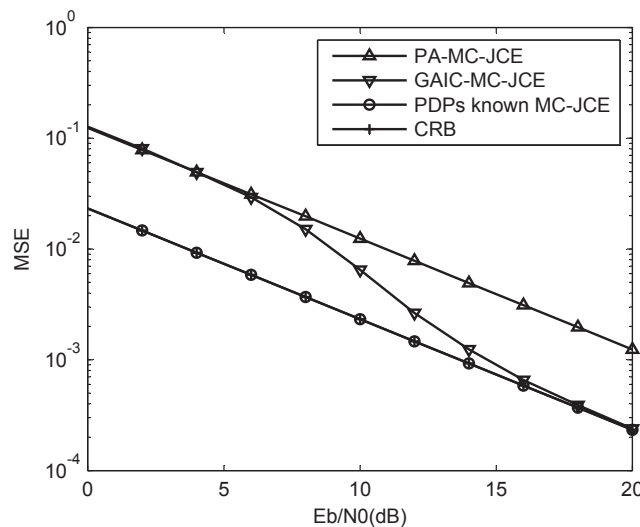


Figure 3. MSE performance comparison for various multi-cell JCE algorithms.

Figure 4 and Figure 5 illuminate the BER performance comparison for various multi-cell JCE algorithms with transmit antenna of one or two, respectively.

In Figure 4, with BS each equipped with one transmit antenna the BER of MC-JCE with PDPs knowledge known has slight performance loss as compared to case of perfect channel knowledge. And for PA-MC-JCE, the loss is about 2 dB. While for GAIC-MC-JCE the BER curve is close to the BER of MC-JCE with PDPs known gradually.

In Figure 5, with BS each equipped with two antennas the BER performance of both PA-MC-JCE and GAIC-MC-JCE get rapid and significant improvement and the curves have been

close to 10^{-4} at low E_b/N_0 (less than 12dB). But with E_b/N_0 increasing, due to the existence of Doppler frequency shift, the BER curves of both algorithms encounter error floors at about 20dB of E_b/N_0 .

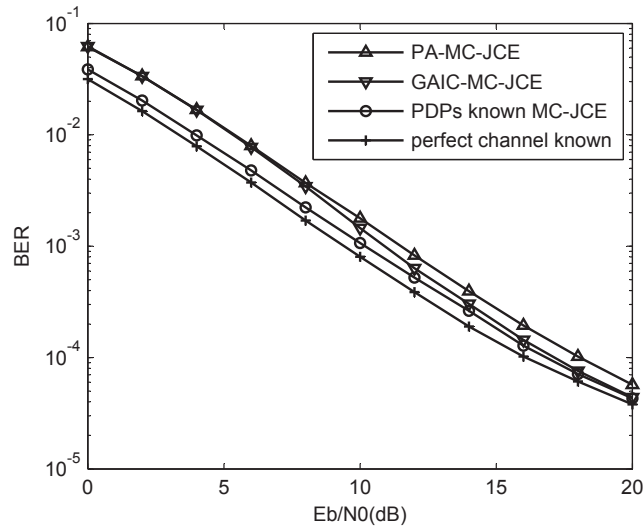


Figure 4. BER performance comparison for various multi-cell JCE algorithms with $N_T = 1$.

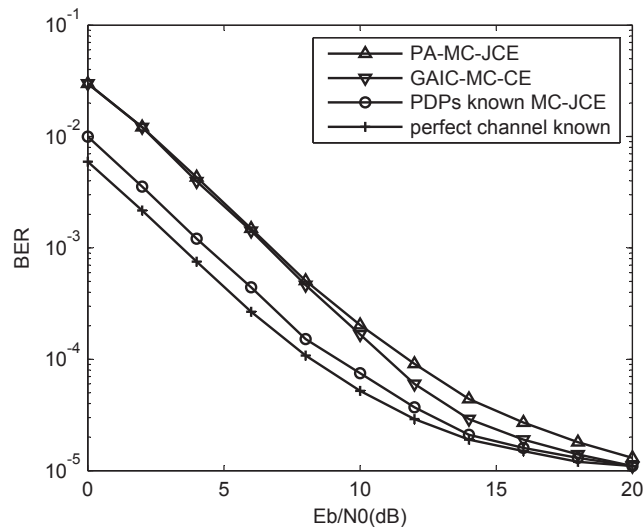


Figure 5. BER performance comparison for various multi-cell JCE algorithms with $N_T = 2$.

5. Conclusion

Based on multi-cell cooperation OFDM systems, this paper firstly presents a pilot-aided time domain JCE algorithm when PDPs of multiple cells are not same and unknown by receiver. Then, by exploiting GAIC, the paper further optimizes the algorithm. From simulation results we observe that, compared with the corresponding CRB, the MSE performance of proposed multi-cell JCE algorithms has a certain loss. This performance gap provides a possibility to optimize the multi-cell JCE algorithms and it is a further need to study.

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