

## Channel Estimation on 60GHz Wireless System Based on Subspace Pursuit

Baokai Zu<sup>1</sup>, Xinyuan Xia<sup>2</sup>, Kewen Xia<sup>\*3</sup>, Chuanjian Bai<sup>4</sup>

School of Information Engineering, Hebei University of Technology, Tianjin 300401, China

\*Corresponding author, e-mail: zubaokai@163.com<sup>1</sup>, will\_9898@sina.com<sup>2</sup>, kwxia@hebut.edu.cn<sup>3</sup>, chuanjianbai888@163.com<sup>4</sup>

### Abstract

Due to the channel with characteristic of sparse multi-path in the 60GHz wireless communication system, the channel estimation problem can be attributed to that of sparse signals recovery. And with the consideration of the subspace pursuit (SP) algorithm is superior to the orthogonal matching pursuit (OMP) at reconstruction precision, the channel estimation technique based on the SP algorithm is presented in the 60GHz wireless communication system. First, design the OFDM multi-carrier modulation communication system. Then, establish the channel estimation mathematical model with indoor Line-of-sight. Finally, complete the reconstruction of sparse signals using SP algorithm. The experimental results and comparison analysis show that the presented technique based on the SP algorithm provides better channel estimation performances in the same pilot conditions, and it is superior to the technique based on OMP algorithm and the technique based on least square (LS) algorithm.

**Keywords:** channel estimation, 60GHz wireless communication system, subspace pursuit algorithm, orthogonal matching pursuit algorithm

**Copyright © 2014 Institute of Advanced Engineering and Science. All rights reserved.**

### 1. Introduction

The channel estimation is an important basis for channel equalization and signal detection technology in wireless communication. Therefore, the channel estimation in wireless communication has become an important research topic [1]. The wireless channel presents a strong sparsity with the development of wireless broadband communication systems. Therefore, the sparse channel estimation becomes a research hotspot. In recent years, the compressed sensing (CS) theory [2, 3] in the field of signal processing breaks the limitation of the Nyquist sampling theorem, it can sample signals at low sampling rate in accordance with the structure characteristics of signals and can effectively complete reconstruction of sparse signals and can achieve the purpose of saving resources and improving efficiency. Now the CS theory is applied in different systems for sparse channel estimation, such as ultra-wideband (UWB) systems [4] and underwater acoustic communication system [5], etc.

The 60GHz wireless communication system has become an important part of the fourth generation communication since it can provide a data transfer rate with several Gbps. For there is a free frequency interval with 7GHz band in the system [6, 7] the wireless channel shows diffuse multi-path characteristics in transmission [7]. Since there is only a few non-zero taps in 60GHz wireless channel, the traditional LS algorithm [8-10] cannot accurately interpolate channel response without making full use of the sparse priori knowledge of the channel by sampling the zero taps. And thus the estimation accuracy and effectiveness is not high enough. Therefore, this method attributes the channel estimation problem as the reconstruction of sparse signal by exploiting the sparse characteristics of the channel with CS theory [9]. It can accomplish sparse channel estimation effectively by using a very limited pilot without getting the impulse response of the sub-carriers by interpolation method, which can reduce the error of channel estimation and improve spectrum efficiency [10].

In addition, the greedy algorithm is mainly used among numerous reconstruction algorithm of the CS theory. A typical class of the greedy algorithm is matching pursuit (MP) and its derivative algorithms, such as orthogonal matching pursuit (OMP), etc [11]. However, the disadvantage of this kind algorithm is that it still didn't get the support in theory and the reconstruction quality is not high. Therefore, a special kind of greedy algorithm-subspace pursuit

algorithm (SP) was proposed [12]. The reconstruction accuracy of SP is higher than OMP, and the theoretical proves is abundant. Therefore, in order to improve the quality of the 60GHz wireless channel estimation technique further, channel estimation based on SP algorithm is studied, and the simulation experiment and comparative analysis is conduct.

## 2. Channel Estimation of the 60GHZ Communication System

### 2.1. The Channel Model

We use the channel model proposed by the IEEE 802.15.3c group (IEEE TG3c) for living environment communication [6]. The model has obvious direct path components under the condition of line-of-sight (LOS) and directional antenna. The channel can be shown as follows.

$$\mathbf{h}(t) = \beta\delta(t) + \sum_{l=0}^{L-1} \sum_{m=0}^{M_l-1} \alpha_{l,m} \delta(t - T_l - \tau_{l,m}) \delta(\varphi - \Psi_l - \psi_{l,m}) \quad (1)$$

$$|\alpha_{l,m}|^2 = \Omega_0 e^{-T_l/\Gamma} e^{-\tau_{l,m}/\gamma - k[1-\delta(m)]} \sqrt{G_r(0, \Psi_l + \psi_{l,m})} \quad (2)$$

Where,  $t$  is the time (ns),  $\delta(\cdot)$  is the Dirac delta function,  $\beta\delta(t)$  is the direct path component,  $L$  is the number of clusters,  $m$  is the number of the arriving multipath components of the  $l$  cluster,  $M_l$  is the total number of the arriving multipath components of the  $l$  cluster,  $T_l$  is the arrival time of the first multipath component of the  $l$  cluster,  $\tau_{l,m}$  is the arrival time delay of the  $m$  multipath component with the  $l$  cluster relative to the first multipath component,  $\Omega_0$  is the average power of the first multipath of the direct-path component,  $\Psi_l$  is the arrival angle of the first multipath of the direct-path component,  $\psi_{l,m}$  is the angle of the  $m$  multipath component with the direct-path component relative to the arrival angle of the first multipath component. In the formula (2), the Angle  $\alpha_{l,m}$  follows uniform distribution.

In this paper, the channel configuration and simulation parameters of the channel, i.e., CM1.1 proposed by IEEE 802.15.3c Working Group is shown in Table 1 and Table 2.

Table 1. Channel Configuration of CM1.1 TSV Channel Models

Channel Model	Environment	Antenna Model	Rx antenna HPBW	Sample rate
CM1.1	Residential LOS TSV	Gaussian-Distributed Antenna Model	30 (Deg)	1 (GHz)

Table 2. Channel Parameters for CM1.1 TSV Channel Models

Parameter	$\Lambda$ [1/ns]	$\lambda$ [1/ns]	$\Gamma$ [ns]	$\gamma$ [ns]	$\sigma$ cluster	$\sigma$ ray	$\sigma$ $\phi$	$\Delta k$ [dB]	$\Omega(d)$ [dB]	$n_d$	$A_{NLOS}$
CM1.1	0.191	1.22	4.46	6.25	6.28	13.0	49.8	18.8	-88.7	2	0

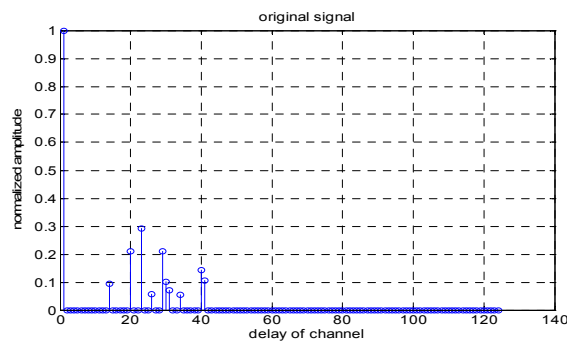


Figure 1. Impulse Response of the CM1.1 Channel

Figure 1 shows the schematic diagram of the channel's impulse response, where the multipath of the smaller impulse response is negligible. It can be seen that the 60GHz channel communication system is typically sparse channels.

## 2.2. Channel Estimation Model

Through the channel of 60 GHz system, the output of signal is:

$$y(n) = \mathbf{h}^T \mathbf{s}(n) + w(n) \quad (3)$$

Where,  $\mathbf{s}(n) = [s(n), s(n-1), \dots, s(n-L)]^T$  is the training sequence vector,  $s(n)$  is the pilot sequence of transmitting end,  $w(n)$  represents the Gaussian white noise which is independent and identical distribution with the input signal,  $y(n)$  is the measurement vector. The matrix form of the formula (3) is:

$$\mathbf{y} = \mathbf{S}\mathbf{h} + \mathbf{W} \quad (4)$$

$$\mathbf{S} = \begin{bmatrix} s(n) & s(n-L) & \dots & s(n-1) \\ s(n-1) & s(n) & \dots & s(n-2) \\ \vdots & \vdots & \ddots & \vdots \\ s(n-L) & s(n-L-1) & \dots & s(n) \end{bmatrix} \quad (5)$$

In above formula,  $\mathbf{S}$  is a training sequence matrix of  $L \times M$ ,  $\mathbf{W}$  is the noise vector.

Our aim is to obtain  $\mathbf{h}$  by the measurement vector  $\mathbf{y}$  and the matrix  $\mathbf{S}$ . Since  $\mathbf{h}$  in formula (4) is sparse, the problem can be understood as the reconstruction of the sparse signal by the observed signal with noise and then solved by CS theory.

There are some characteristics such as high data transmission capacity, efficient spectrum efficiency and resistance to multipath interference in the orthogonal frequency division multiplexing (OFDM) system. It became the best solutions of the 60 GHz wireless communication system [7]. Herein the OFDM multi-carrier modulation method is used. The diagram of the OFDM system is provided in Figure 2, where,  $x_g(n)$  is the  $\mathbf{s}(n)$  in the formula (3) and the channel is the channel model mentioned in formula (1).

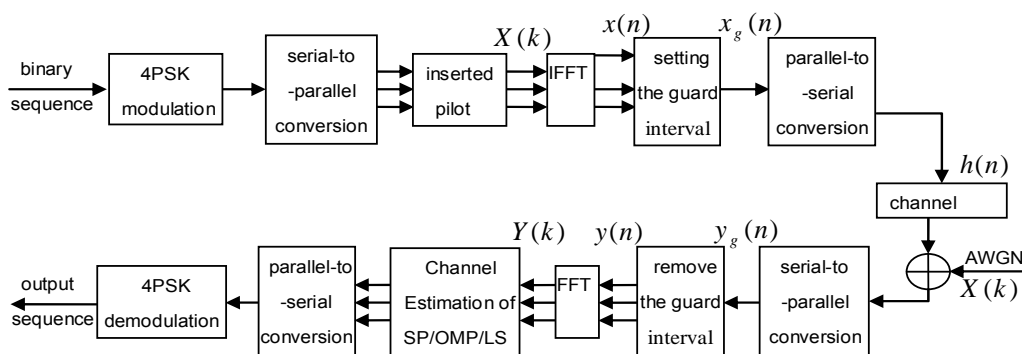


Figure 2. The OFDM System Diagram

## 3. Method and Principle

### 3.1. Compressed Sensing Theory

The CS theory suggests that the original signal can reconstruct from a small amount of projection with high probability as long as the signal is sparse or present sparse features in a

transform domain. The core of the theory mainly includes sparse representation of signals, the design of the measurement matrix and signal reconstruction three key issues. The research content is the problem of solving the solution of underdetermined equations as follows [2, 3]:

$$\mathbf{y} = \Phi \mathbf{x} \quad (6)$$

Here,  $\mathbf{x}$  is the  $N$ -dimensional data vector,  $\mathbf{y}$  is the  $M$ -dimensional measurement vector,  $\Phi$  is the measurement matrix of  $M \times N$ .

And any signal can be represented as a sparse form under the sparse matrix. If the signal itself is not sparse then there must exist a set of transformation base  $\Psi \in R^{M \times M}$  which makes the projection of  $x$  based on the transformation base is sparse, i.e.,

$$\mathbf{x} = \Psi \boldsymbol{\theta} \quad (7)$$

Where  $\boldsymbol{\theta}$  is  $K$ -sparse. Thus the measurement process can be expressed as:

$$\mathbf{y} = \Phi \mathbf{x} = \Phi \Psi \boldsymbol{\theta} = \Theta \boldsymbol{\theta} \quad (8)$$

Where  $\Theta = \Phi \Psi$  is sensing matrix. When  $\Theta(N \times M)$  satisfies the Restricted Isometry Property (RIP),  $\boldsymbol{\theta}$  can be reconstructed accurately by solving the minimum 0 norm [9], i.e.,

$$\begin{cases} \hat{\boldsymbol{\theta}} = \arg \min \|\boldsymbol{\theta}\|_0 \\ \text{s.t. } \Theta \boldsymbol{\theta} = \mathbf{y} \end{cases} \quad (9)$$

The purpose of reconstructing signal is obtained sparse representation  $\boldsymbol{\theta}$  by the formula (8) under the condition of known  $\mathbf{y}$ . It is NP-hard to look for the sparse solution of the underdetermined equation. However, due to the characteristics of sparse signal, the definite solution can be identified as long as finding the position of non-zero elements in  $\boldsymbol{\theta}$ .

### 3.2. The Subspace Pursuit Algorithm

The biggest problem of signal reconstruction is to find a sub-space of  $K$  columns from  $\Phi$ , and then get the coefficient of the signal by calculating the pseudo-inverse coefficients. OMP is a greedy algorithm, which starts with an empty list, identifies one candidate during the each iteration, and adds them to the already existing list. Once a coordinate is deemed to be reliable and is added to the list, it is not removed from it until the algorithm terminates [11]. While the subspace pursuit algorithm (SP) which uses the idea of back pursuit is a special kind of greedy algorithm. The defining character of the SP algorithm is the method used for finding the  $K$  columns that span the correct subspace: SP tests subsets of  $K$  columns in a group, for the purpose of refining at each stage an initially chosen estimate for the subspace. More specifically, the algorithm maintains a list of  $K$  columns from  $\Phi$ , performs a simple test in the spanned space, and then refines the list. If  $\mathbf{y}$  does not lie in the current estimate for the correct spanning space, one refines the estimate by retaining reliable candidates, discarding the unreliable ones while adding the same number of new candidates. As a consequence, OMP algorithm is overly restrictive, since candidates have to be selected with extreme caution. In contrast, the SP algorithm uses a simple method for reevaluating the reliability of all candidates at the process of each iteration [12]. The main steps of the SP algorithm are summarized below.

Step1. Input:  $K, \Theta, \mathbf{y}$ . Support set:  $\hat{T} = \{K \text{ indices corresponding to the largest magnitude entries in the vector } \Theta^* \mathbf{y}\}$ , The residue vector:  $\mathbf{y}_r = \text{resid}(\mathbf{y}, \Theta_{\hat{T}})$ , accuracy control:  $\epsilon$ , Maximum iterations:  $n$ , iterations:  $t$ , Initialization  $t=1$ .

Step2. Iteration: At the  $t$ th iteration, go through the following steps.

- 1) Merge the subscript set  $\hat{\mathbf{T}}$  to the Support set  $\mathbf{T}'$ ,  $\mathbf{T}' = \hat{\mathbf{T}} \cup \{K \text{ indices corresponding to the largest magnitude entries in the vector } \mathbf{\Theta}^* \mathbf{y}_r\}$ .
  - 2) Calculate  $\mathbf{\theta}_p'$  restricted to  $\mathbf{\Theta}_{\mathbf{T}'}$ : Set  $\mathbf{\theta}_p' = \mathbf{\Theta}_{\mathbf{T}'}^+ \mathbf{y}$ . Where,  $\mathbf{\Theta}_{\mathbf{T}'}^+ = (\mathbf{\Theta}_{\mathbf{T}'}^* \mathbf{\Theta}_{\mathbf{T}'})^{-1} \mathbf{\Theta}_{\mathbf{T}'}^*$ .
  - 3) Update the support set as  $\tilde{\mathbf{T}}: \tilde{\mathbf{T}} = \{K \text{ indices corresponding to the largest magnitude elements of } \mathbf{\theta}_p'\}$ .
  - 4) Update the residue vector  $\tilde{\mathbf{y}}_r: \tilde{\mathbf{y}}_r = \text{resid}(\mathbf{y}, \mathbf{\Theta}_{\tilde{\mathbf{T}}})$ .
- Step3. Judge:  
 If  $\|\tilde{\mathbf{y}}_r\| \geq \|\mathbf{y}_r\|$  or  $\|\tilde{\mathbf{y}}_r\| \leq \epsilon$  or  $t > n$ , quit the iteration, go to Step4.  
 Else, go to Step2.
- Step4. Output:  
 The estimated signal  $\hat{\theta}$ ,  $\mathbf{\theta}_{\hat{\mathbf{T}}} = \mathbf{\Theta}_{\hat{\mathbf{T}}}^+ \mathbf{y}$ .

### 3.3. Channel Estimation Method Based on SP Algorithm

The steps of Channel estimation method based on the SP are listed as follows.

- (1) Design the OFDM communication system. We adopt the OFDM multi-carrier modulation method which mainly includes the sub-carrier modulation, serial-to-parallel conversion, the implementation of DFT, setting the guard interval, the cyclic prefix and the number of sub-carrier selection, etc, which is depicted in Figure 2.
- (2) Establish the mathematical model of channel estimation. Establish the mathematical model according to the channel model of 60GHZ indoor living environment communication with LOS proposed by 802.15.3c workgroup.
- (3) Achieve reconstruction of the sparse signal based on the SP algorithm. After the sender signals  $s(n)$  passed the channel  $h$  under the OFDM multi-carrier modulation mode, the received signal  $y$  was obtained on the receiving end. Then complete the reconstruction of sparse signals  $h$  by adopting the SP algorithm in the formula (4).

## 4. The Simulation Experiment and Comparative Analysis

In order to test the superiority of the SP algorithm, simulation experiments were conducted. The simulation experiment is conducted in MATLAB7.11.0 (R2010b) environment. In the simulation, we adopt 4PSK modulation mode in the OFDM system, the total subcarrier number  $N = 864$ , cyclic prefix  $CP = 10$ . To combat the noise, repeat 100 times for each algorithm. We put the average value of the estimated results as the channel estimation result.

For comparison, we adopt the mean square error (MSE) and bit error rate (BER) to compare channel estimation performance of the SP, OMP algorithm and traditional LS algorithm. The equation for the mean square error is as follows.

$$MSE = \frac{E[\sum_k |\mathbf{h}(k) - \hat{\mathbf{h}}(k)|^2]}{E[\sum_k |\mathbf{h}(k)|^2]} \quad (10)$$

In the simulation experiments, compared the mean square error (MSE) and bit error rate (BER) of the above three kinds of algorithms at different SNR (Signal to Noise Ratio) and the pilot number  $P = 216$ ,  $P = 144$ ,  $P = 72$  respectively. The pilot is inserted in uniform distribution in the transmission symbol. The simulation results are shown in Figure 3, Figure 4 and Figure 5. Figure 3 shows the channel estimation results of the 60GHZ communication system based on the SP algorithm estimates when the pilot number is 72. Figure 4 and Figure 5 are the MSE and BER curve of the SP, OMP and the LS algorithm in different SNR and pilot number respectively. The precise value of Mean square error (MSE) and bit error rate (BER) is shown in Table 3 and Table 4 respectively.

The simulation results, shown in Figure 3, indicate that there is a good reconstruction performance of the SP algorithm. Even in a small number of pilot cases, it can also estimate accurately. Obviously shown in Figure 4 and Figure 5, no matter how much the pilot is, the estimation performance of SP and OMP algorithm are far higher than the traditional LS algorithm. Meanwhile, the estimation performance of the SP algorithm is better than the OMP algorithm. Even when the pilot number is small, the BER of SP algorithm can achieve zero in a certain SNR. As the Table 4 indicates, all of the three BER values of the SP algorithm reach 0 when the SNR is 15dB, 20dB and 25dB under the pilot  $P = 216$ ,  $P = 144$ ,  $P = 72$  respectively, which gets a good estimated performance. The two BER values of OMP algorithm reach 0 when the SNR is 15dB and 25dB under the pilot  $P = 216$ ,  $P = 144$  respectively, while the BER value of OMP algorithm is small when the SNR is 30 dB under the pilot  $P = 72$ . Thus, SP algorithm has higher estimation performance than OMP algorithm. And yet for the traditional LS algorithm, even when the SNR is 30dB the BER is 0.2297Db, 0.2749dB, 0.3288dB under the pilot  $P = 216$ ,  $P = 144$ ,  $P = 72$  respectively, which presents a greater BER.

Shown in Figure 4 and Figure 5, with the increasing of the SNR the MSE and BER of the SP and OMP algorithm decreased rapidly, while the traditional LS algorithm's tends to a horizontal line. In Table 3 and Table 4, with the SNR increases from 0 to 30dB, the MSE of the SP algorithm rolls down from 0.4279dB to 7.357e-005dB, while the OMP algorithm rolls down from 0.698dB to 0.698dB and the LS algorithm from 2.256dB to 2.256dB. Meanwhile, the BER of the SP algorithm rolls down from 0.1406dB to 0, while the OMP algorithm rolls down from 0.2288dB to 0 and the LS algorithm from 0.2889dB to 0.2303dB.

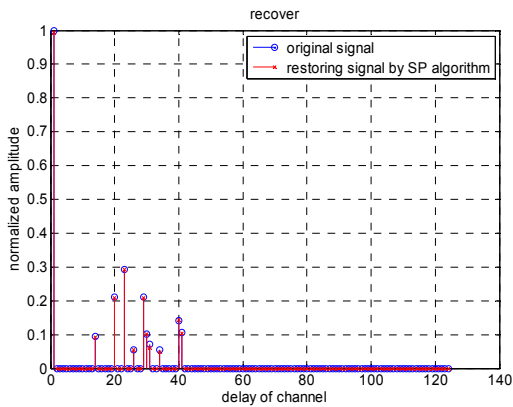


Figure 3. The Impulse Response of Original Signal and Recovered Signal Based on SP Algorithm

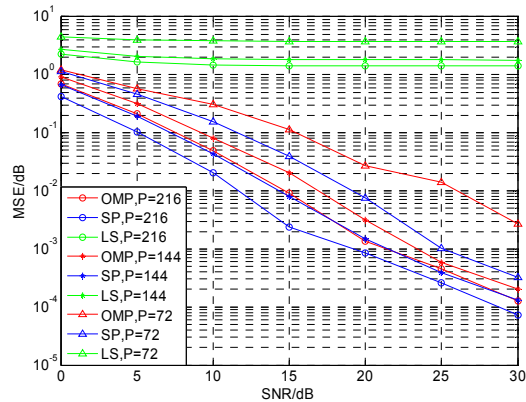


Figure 4. Mean Square Error (MSE) for the Three Algorithms

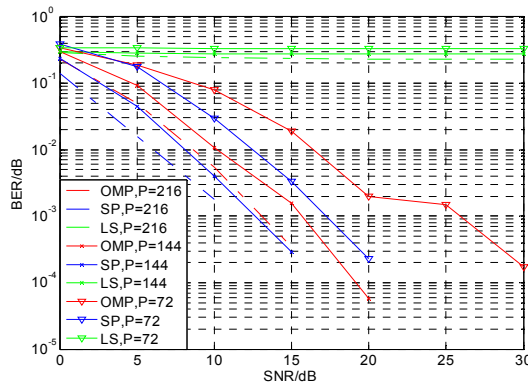


Figure 5. Bit Error Rate (BER) for the Three Algorithms

Table 3. Mean Square Error (MSE) Values for the Three Algorithms

Algorithms	Pilot number $P$	Mean square error (MSE)/dB						
		0dB SNR	5dB SNR	10dB SNR	15dB SNR	20dB SNR	25dB SNR	30dB SNR
OMP	$P=216$	0.698	0.2097	0.0487	0.009185	0.001349	0.0004625	0.0001256
SP	$P=216$	0.4279	0.1038	0.02027	0.002361	0.00086	0.0002574	7.357e-005
LS	$P=216$	2.256	1.667	1.478	1.424	1.41	1.407	1.404
OMP	$P=144$	0.9307	0.319	0.08022	0.02054	0.003134	0.0005882	0.0002023
SP	$P=144$	0.6786	0.1905	0.04297	0.008091	0.001483	0.0003914	0.0001324
LS	$P=144$	2.746	2.07	1.881	1.882	1.806	1.802	1.797
OMP	$P=72$	1.233	0.5828	0.3137	0.1122	0.02743	0.01408	0.0002657
SP	$P=72$	1.156	0.4608	0.1544	0.0392	0.007539	0.001021	0.0003244
LS	$P=72$	4.528	4.01	3.813	3.782	3.747	3.754	3.753

Table 4. Bit Error Rate (BER) Values for the Three Algorithms

Algorithms	Pilot number $P$	Bit error rate(BER)/dB						
		0dB SNR	5dB SNR	10dB SNR	15dB SNR	20dB SNR	25dB SNR	30dB SNR
OMP	$P=216$	0.2288	0.05017	0.00544	0.0003472	0	0	0
SP	$P=216$	0.1406	0.01591	0.001794	0	0	0	0
LS	$P=216$	0.2889	0.2584	0.2416	0.2348	0.231	0.2311	0.2303
OMP	$P=144$	0.3015	0.09172	0.01071	0.001562	5.787e-005	0	0
SP	$P=144$	0.2373	0.04508	0.003993	0.0002894	0	0	0
LS	$P=144$	0.3019	0.2822	0.2738	0.2737	0.2759	0.2742	0.2738
OMP	$P=72$	0.3417	0.1872	0.07998	0.01881	0.001968	0.001505	0.001736
SP	$P=72$	0.3898	0.1765	0.02922	0.003299	0.0002315	0	0
LS	$P=72$	0.3418	0.336	0.3315	0.3299	0.3278	0.3281	0.3275

The SP channel estimation method can provide good channel estimation performance under the condition of less pilot number, which can significantly improve the spectrum efficiency of the system. Therefore, the SP algorithm is significantly better than the OMP algorithm and traditional LS algorithm.

## 5. Conclusion

In this paper, channel estimation of the 60GHz wireless communication system based on the SP algorithm of compressed sensing is proposed, i.e., we transform channel model of the communication system into CS solvable reconstruction model for its sparse features and accurately estimate the sparse channel by using the SP algorithm. The experimental results show that, the estimation performance based on the SP algorithm is superior to the OMP and least square (LS) algorithm and it can acquire an accurate estimation with a limited pilot number.

## Acknowledgements

This work was supported by the National Natural Science Foundation of China (No. 51208168), Tianjin Natural Science Foundation (No. 11JCYBJC00900, No. 13JCYBJC37700), Hebei Province Natural Science Foundation (No. F2013202254, No. F2013202102) and Hebei Province Foundation for Returned Scholars (No. C2012003038). The corresponding author is Prof. Xia Kenwen.

**References**

- [1] Sohail MS, Al-Naffouri TY. An EM based frequency domain channel estimation algorithm for multi-access OFDM systems. *Signal Processing*. 2010; 90(5): 1562-1572.
- [2] Donoho DL. Compressed sensing. *IEEE Transactions on Information Theory*. 2006; 52(4): 1289-1306.
- [3] Candès EJ, Wakin M B. An introduction to compressive sampling. *IEEE Signal Processing Magazine*. 2008; 25(2): 21-30.
- [4] Paredes JL, Arce GR, Wang Z. Ultra-wideband compressed sensing: channel estimation. *IEEE Journal of Selected Topics in Signal Processing*. 2007; 1(3): 383-395.
- [5] Berger CR, Zhou S, Preisig JC, et al. Sparse channel estimation for multicarrier underwater acoustic communication: From subspace methods to compressed sensing. *IEEE Transactions on Signal Processing*. 2010; 58(3): 1708-1721.
- [6] Zhang H, Xu N, Wang J, et al. *On the capacity of 60 GHz wireless communications over IEEE 802.15.3c channel models*. IEEE Pacific Rim Conference on Communications, Computers and Signal Processing (PacRim). Victoria. 2011: 680-684.
- [7] Fang WJ, Huang XG, Li G. A full duplex radio-over-fiber transmission of OFDM signals at 60GHz employing frequency quintupling optical up-conversion. *Optics Communications*. 2013; 294(1):118-122.
- [8] Yang L, Tian R, Jia M, et al. A Modified LS Channel Estimation Algorithm for OFDM System in Mountain Wireless Environment. *Procedia Engineering*. 2012; 29: 2732-2736.
- [9] Zhang AH, Sh Y, Gui G. Sparse Channel Estimation for MIMO-OFDM Two-Way Relay Network with Compressed Sensing. *International Journal of Antennas and Propagation*. 2013; 2013(2013).
- [10] Haupt J, Bajwa WU, Raz G, et al. Toeplitz compressed sensing matrices with applications to sparse channel estimation. *IEEE Transactions on Information Theory*. 2010; 56(11): 5862-5875.
- [11] Tropp JA, Gilbert AC. Signal recovery from random measurements via orthogonal matching pursuit. *IEEE Transactions on Information Theory*. 2007; 53(12): 4655-4666.
- [12] Dai W, Milenkovic O. Subspace pursuit for compressive sensing signal reconstruction. *IEEE Transactions on Information Theory*. 2009; 55(5): 2230-2249.