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The NSCT-NLmeans Based CS Reconstruction for Noisy Image

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Abstract

Sparsity was prior condition in compressed sensing which had been widely concerned in signal reconstruction. Meanwhile nonsubsampled contourlet proposed as a development to contourlet, not only provided flexible multi-scale, multi-direction sparse image decomposition but also featured with shift-invariance property which was beneficial to image denoising. This paper combined threshold operator in nonsubsampled contourlet domain with non local means filter for image denoising in the compressed sensing framework. Therefore, NSCT-NLmeans based compressed sensing reconstruction was proposed for noisy image. The experiment results showed that NSCT- NLmeans based algorithm outperformed the other multi-resolution and multi-directional transforms in recovering and denoising image simultaneously.

***Keywords**: compressed sensing, compressive sampling, image reconstruction, denoising*

1. Introduction

Over the past few years, a new sampling and recovery framework called compressed sensing (CS) [1-6] has emerged. There are many applications in compressed sensing such as signal processing [1], imaging [7, 8] and communication [9]. This framework is described as follows:

Let $x \in \mathbb{R}^N$ be an unknown signal ones want to recover. It has only K non-zero components with $K = N$. The compressed sample denoted by y can be expressed by the following equation:

$$y = \Phi x + \mathrm{n} \tag{1}$$

Where Φ denotes a $m \times N$ random measurement matrix with $m < N$. And n is the noise perturbation. The word “compressed” means the length of sample y is smaller than that of x.

Since Φ has fewer rows than columns, the reconstruction is an ill-posed problem. However, Candes [4] states that if a signal x is sparse in some transform domain, then the signal will be recovered via some convex optimizations methods. The algorithms based on some transforms were proposed for CS reconstruction, such as [7, 10]. But these algorithms performs well only in the noiseless case.

When CS is applied to image reconstruction, it has some noisy challenges from the random projection and environment. Therefore our contributions are summarized as following:

(1) Propose nonsubsampled contourlet (NSCT) [11] transform to CS reconstruction and denoising.

(2) Incorporate the threshold operator [12] in NCST domain and non local means (NL-means) filter [13] in spatial domain into CS recovering and denoising operation.

(3) Propose a method that accomplishes image reconstruction and denoising simultaneously in the CS framework.

The simulations demonstrate that the NSCT-NLmeans method outperforms the other multi-resolution and multi-directional transform methods. The rest of this paper is organized as

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follows. Section 2 briefly reviews the CS framework, NSCT transform and Non Local means (NLmeans) filter. Section 3 presents the NSCT-NLmeans based CS framework for recovering and denosing the noisy image. Section 4 shows the simulation results and is followed by conclusion in section 5.

2. Preliminary

2.1. A Review of the CS Framework

CS is a very efficient signal recovery framework. CS model [5] is described as:

$$\min \|x\|_0 \quad \text{subject to} \quad y = \Phi x \tag{2}$$

Unfortunately solving Equation (2) is an NP hard problem. For the computational tractability, ℓ_1 optimization problem is used as an alternative [5].

$$\min \|x\|_1 \quad \text{subject to} \quad y = \Phi x \tag{3}$$

The sparsity and the restricted isometric property (RIP) of measurement matrix Φ are important prior information for CS reconstruction.

Sparsity means signal has a much smaller number of non-zero elements than its original length. In Equation (1), the recovery of signal x from the sample y is a linear inverse problem. And the number of rows of random measurement matrix Φ is less than its columns number. Therefore, Equation (1) will find infinite solutions to meet $y = \Phi x$ if the sparsity constraint doesn't exist.

To guarantee accurate reconstruction of each K-sparse signal, the random measurement matrix Φ should satisfy Restricted Isometry Property (RIP). Many types of random measurement matrices satisfy the RIP [1, 3, 6, 14], such as Uniform Spherical Ensemble [1, 6], Partial Fourier Ensemble [1, 3].

2.2. A Review of NSCT Transform

Realized by pyramid directional filter bank (PDFB) [15 16], contourlet transform [17] can effectively capture the intrinsic geometrical structure and provide images with sparse representations. However, the contourlet transform is a shift variant transform as downsampling and upsampling. Then Nonsubsampled Contourlet is proposed as a development of the contourlet. It not only provides flexible multi-scale, multi-direction image decomposition but features with shift-invariance property. The property is beneficial to image denosing. As stated in [11], NSCT is an efficient transform in image denoising and enhancement. Due to the nonsubsampled pyramid structure and nonsubsampled directional filter banks (NSFB), the NSCT can provide a shift-invariant, multi-resolution and directional expansion. Figure 1. describes the example of nonsubsampled contourlet transform on image-'barbara'.

2.3. A Review of NL-means filter for Image Denoising

Taken full advantage of the image redundant information, NL-means filter corrects the noisy image instead of separating the noise from original un-noisy image. The main feature of non local means filter is that it gives larger weights to those similar pixels in a whole neighborhood.

$$NL(u)(i) = \sum_{j \in I} w(i,j)u(j)$$

Where the weight $w(i,j)$ is determined by the similarity between the pixels i and j. The similarity is determined by the gray level of $u(N_i)$ and $u(N_j)$. The weight is defined by:

$$w(i, j) = \frac{1}{Z(i)} e(-\frac{\| u(N_i) - u(N_j) \|_{2,\sim}^{2} \ ^{2}}{h^2})$$

Where the normalized constant $Z(i) = e^{(-\frac{\|u(N_i)-u(N_j)\|_{2,\sim}{}^{2}}{h^2})}$, and h masters the decay rate of exponential function.

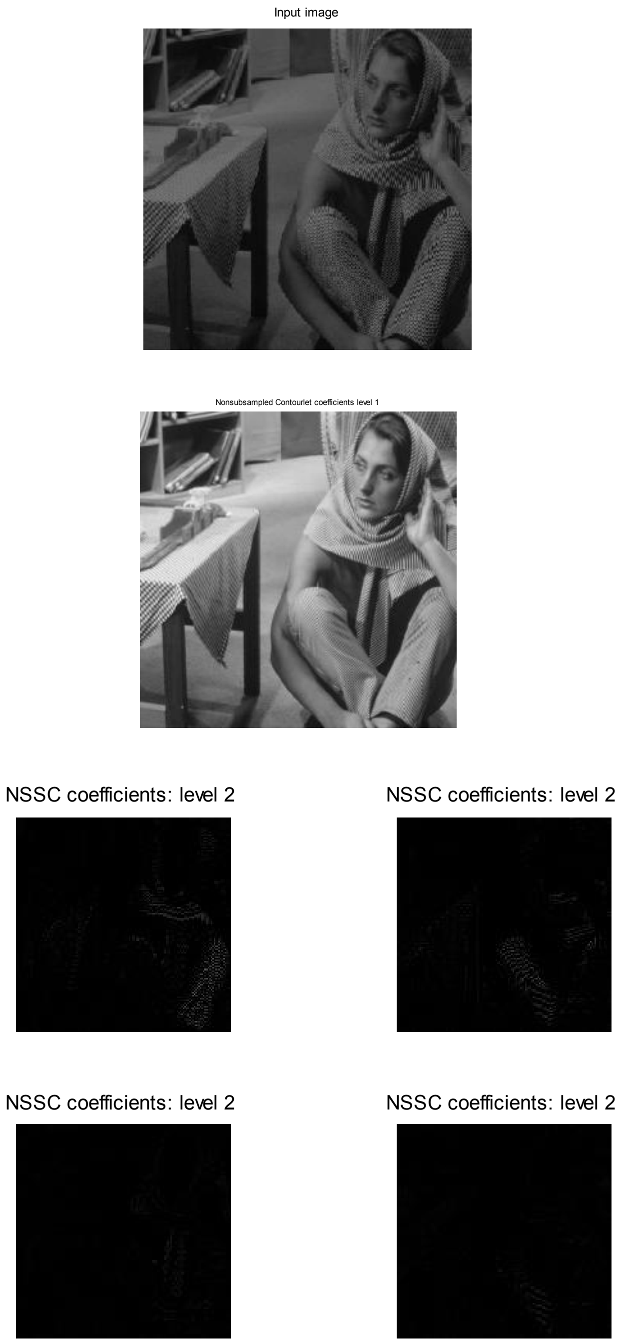

Figure 1. Presents the Example of Nonsubsampled Contourlet Transform on Image-'barbara'

3. NSCT-NLmeans Based Algorithm for Recovering and Denoising Image

There are a lot of reconstruction algorithms for CS reconstruction such as Basis Pursuit (BP) [18] algorithm, greedy pursuit algorithm [19, 20] and iterative bound-optimization algorithm [21]. The last one proposed by J. Haupt and R. Nowak is an effective method in signal reconstruction. The most useful feature of the iterative bound-optimization algorithm is that it can combine with orthonormal transform in iterations procedure. Therefore, it has been combined with contourlet [17] and complex-valued dual-tree wavelets [22] to recover the noiseless images. In this paper, NSCT-NLmeans algorithm is also combined with the iterative bound-optimization algorithm to implement image reconstruction and denoising simultaneously in the CS framework.

Let x be the unknown noisy image and Φ be the random measurement matrix. y is a measurement sample obtained by $y = \Phi x$.The reconstruction process consists of the following steps.

a) Initialization: $\tilde{x}^{(0)} = \Phi^T \times y$ {the approximation denoted by $\tilde{x}$ is initialized, Φ^{T} denotes the transposition matrix of Φ}; $i = 1$ {iteration index}.

b) Main iteration:

(1) Primary estimation is done based on iterative bound-optimization: $\hat{x}^{(i)} = \tilde{x}^{(i)} + \Phi^T (y - \Phi \tilde{x}^{(i)})$

(2) Primary estimation $\hat{x}^{(i)}$ is transformed by nonsubsampled contourlet $\hat{x}_{NS}^{(i)} = NSCT(\hat{x}^{(i)})$

(3) Threshold operator is performed in nonsubsampled contourlet domain to denoising : $\hat{x}_{NS}^{(i)} = thr(\hat{x}_{NS}^{(i)})$

(4) After thresholding, the coefficients $\hat{x}_{NS}^{(i)}$ are obtained. Then inverse NSCT is implemented on them: $\hat{x}^{(i)} = NSCT^{-1}(\hat{x}_{NS}^{(i)})$.

(5) Candidate estimation: $\tilde{x}^{(i+1)} = \hat{x}^{(i)} + \Phi^T (y - \Phi \hat{x}^{(i)})$.

If the stopping condition is true, break the iterations.

c) Nonlocal-means filter is applied into $\tilde{x}$: $x_{rec} = NLM(\tilde{x})$

4. Results and Discussion

Some experiments are performed to show the performance of the proposed algorithm. The images in simulations are contaminated by additive wile Gaussian noise (AWGN) with noise levels ± . The parameters of nonlocal means filter are set as: h equals ± , and the radio of search window and similarity window is 5 and 2 respectively. The images are divided into 32×32 sub-blocks. The proposed method is compared with other CS reconstruction algorithms based on complex-valued dual-tree wavelets and contourlet [10].

The perceptual quality of the recovered results are presented in Figure 2. The tested image 'Lena' is corrupted by AWGN with the standard deviation $\pm = 10$. The tolerance used in iterations is 10^{-4} and compression ratio denoted by $r = m/N$ is 0.3. Figure 2(a) shows the noisy image. Figure 2(b) and Figure 2(c) show the recovered results based on contourlet and complex-valued dual-tree wavelets respectively. Figure 2(d) presents the result of CS based on NSCT-NLmeans.

(a) 512×512 noisy image: 'Lena' $\pm =10$

(b) The CS recovered result based on contourlet , PSNR=28.81

(c) The CS recovered result based on complex-valued dual-tree wavelets, PSNR=29.38

(d) The CS recovered result based on NSCT , PSNR=31.40

Figure 2. Reconstruction and Denoising Results using Different Methods when Standard Deviation Equals 10

Table 1. Comparison of the Reconstruction and Denoising Performance in Different Compression Ratios using Different Methods when Noise Levels Equals 10

Lena, $\pm =10$	$r=0.1$	$r=0.2$	$r=0.3$	$r=0.4$	$r=0.5$
NSCT-NLmeans	**27.23**	**29.82**	**31.40**	**32.46**	**33.29**
SPL-CT	25.93	28.17	28.81	28.98	28.96
SPL-DDWT	27.02	28.87	29.38	29.49	29.39

Table 2. Comparison of the Reconstruction and Denoising Performance in Different Compression Ratios using Different Methods when Noise Levels Equals 20

Lena, $\pm =20$	$r=0.1$	$r=0.2$	$r=0.3$	$r=0.4$	$r=0.5$
NSCT-NLmeans	**25.80**	**27.77**	**28.91**	**29.73**	**30.37**
SPL-CT	23.86	24.30	24.21	23.93	23.62
SPL-DDWT	25.46	25.54	25.13	24.64	24.19

Table 3. Comparison of the Reconstruction and Denoising Performance in Different Compression Ratios using Different Methods when Noise Levels Equals 10

Peppers, $\pm =10$	$r=0.1$	$r=0.2$	$r=0.3$	$r=0.4$	$r=0.5$
NSCT-NLmeans	**27.83**	**29.95**	**31.24**	**32.08**	**32.72**
SPL-CT	26.28	28.11	28.59	28.74	28.71
SPL-DDWT	27.37	28.83	29.24	29.27	29.19

Table 4. Comparison of the Reconstruction and Denoising Performance in Different Compression Ratios using Different Methods when Noise Levels Equals 20

Peppers, $\pm = 20$	$r = 0.1$	$r = 0.2$	$r = 0.3$	$r = 0.4$	$r = 0.5$
NSCT-NLmeans	**25.84**	**27.66**	**28.83**	**29.62**	**30.26**
SPL-CT	23.67	24.25	24.04	23.75	23.49
SPL-DDWT	25.02	25.36	24.98	24.52	24.07

Meanwhile two tested images, "Lena" and "Peppers", are used in the experiment. In Table 1-4, the performance of these benchmark images is quantified across different noise levels by PSNR. $PSNR(x,\hat{x}) = 10\log_{10}(255^2/MSE)$. The best results are shown in bold for easy comparison. The results exhibit that the proposed method outperforms the other methods in recovering and denoising images.

5. Conclusion

CS is a very useful sampling and recovery framework. Featured with shift-invariance property, NSCT is beneficial to image denoising. Therefore, CS is combined with NSCT to recover and denoise in this paper. Meanwhile, threshold operator in NSCT domain and non local means filter in spatial domain are also introduced to denoise. The experiment results show that NSCT-NLmeans based algorithm outperforms the other multi-resolution and multi-directional transforms in recovering and denoising image simultaneously. Some work needs to be done in future. The threshold operator shall be improved based on image structure and the extension of this work to CS videos and astronomy images will be considered.

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