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Variable-weight Combination Prediction of Thermal Error Modeling on CNC Machine Tools

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Abstract

Due to the thermal error modeling of CNC machine tools has characters of small sample and discrete data, the variable-weight combined modeling method was presented by integrating time series analysis and least squares support vector machines. Taking minimum sum of error square of prediction model as the optimization criterion, optimal weights in different time were calculated. Using grey GM (1, 1) model to predict the variable weights, the prediction result of thermal error was obtained as well. Application of the grey variable-weight combined model on a five axis vertical machining center indicated that it can get higher prediction accuracy than single modeling method. Therefore online error compensation to CNC machine tool becomes more effective.

Keywords: numerical control machine tool, thermal error, combination prediction, grey forecast, variable weight

1. Introduction

With the application of five-axis CNC machine tools in the industries of aerospace, automotive and molds becoming more and more widely, the prediction and compensation of machining accuracy also gain extensive attentions. Thermal error is one of the main sources of error in five axis NC machine tools, which accounts for 60%~70% of total processing errors [1]. In order to reduce machine tool error, we can use hardware and software compensation method. The former is aim to reduce the thermal deformation by thermal symmetrical design, pre-stretch, forced cooling and other ways, while the latter is to create a new kind of error artificially to offset original error by establishing comprehensive error prediction model. Compared with hardware compensation, software compensation can improve machining precision with lower price. Therefore it's becoming an important research direction of analyzing thermal error of precision CNC machine tools [2].

In order to realize software compensation of thermal error, the key point is to establish thermal error model. Domestic and foreign scholars have studied deeply in the field of thermal error compensation and evolved a series of modeling method, such as multi-body system theory [3], grey theory [4], neural network [5], least squares support vector machines(LS-SVM) [6] and so on. These single modeling methods have made some successful applications, but they are difficult to establish accurate mathematical model of thermal error, and compensation effect is still not ideal. The combination prediction method can make full use of information of single prediction model [7-8], by which we can set up a more comprehensive, effective, systematic thermal error model. As a result, combination prediction is applied more and more extensively at present in thermal error modeling of machine tools. In traditional combination prediction model, the optimize weight coefficient is constant. But in fact, the performance of a single prediction method is not the same in different time, so the weight coefficient should be optimized dynamically.

For solving above problem, the variable-weight combination modeling method is presented in this paper by integrating time series analysis and least squares support vector machines. Using grey GM (1, 1) model to predict the variable weights, we can set up a more comprehensive, effective, systematic thermal error model.

2. Variable-weight Combination Model of CNC Machine Thermal Error

Thermal error prediction model of CNC machine tool is the simulation model of complicated relationship between the temperature field and the thermal error of machine tool. It is influenced by temperature field, geometry error, shape of parts and other factors. In order to establish the thermal error model, the basic premise is to determine appropriate learning algorithm. Time series analysis and LS-SVM method have applied successfully in the area of modeling prediction as effective learning methods. In the present work, a combined prediction method of time series analysis and LS-SVM is proposed. Taking minimum sum of error square of prediction model as the optimization criterion, optimal weights in different time are calculated. The model can be expressed as:

$$\hat{x} = \alpha \hat{x}_{\text{AR(P)}}(t) + \beta \hat{x}_{\text{LS-SVM}}(t) \tag{1}$$

Where $\hat{x}_{\text{AR(P)}}(t)$ is thermal error prediction value of time series analysis, $\hat{x}_{\text{LS-SVM}}(t)$ is thermal error prediction value of LS-SVM, t is temperature of measured point, and α, β is weighted coefficient.

2.1. Modeling of Time Series Analysis Method

Let $\{x_t\} = \{x_1, x_2, \cdots, x_n\}$ be a precision testing sequence. After smoothing and zero mean processing, the difference equation can be obtained as:

$$x_t - \varphi_1 x_{t-1} - \varphi_2 x_{t-2} - \cdots - \varphi_p x_{t-p} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} \tag{2}$$

Where φ_t is autoregressive coefficient, θ_t is moving average coefficient and a_t is residuals with zero mean normal random series of independent distribution. Equation (2) is p order autoregressive moving average q order model, marked as AMRA(p,q) model. It can be converted into two exceptions MA(q) and AR(p). Time series analysis model is set up by followed steps: time series data pretreatment, model identification, parameter estimation, model test and prediction model obtained finally [9]. Due to the autoregressive *AR* model modeling simple, small amount of calculation and model parameter estimation more accurate [10], it's suitable for thermal error compensation model of CNC machine tool. The autoregressive AR (p) model is as follows:

$$x_t(s) = \varphi_1 x_t(l - 1) + \varphi_2 x_t(l - 2) + \cdots + \varphi_p x_t(l - p) + a_t \tag{3}$$

Where l=1,2,3… is predicting step.

2.2. Modeling of LS-SVM

LS-SVM is a machine learning method developed from support vector machine (SVM) proposed by Vapnik [11]. It puts the inequality constraints into equality constraints. Using LS-SVM, the machine tool thermal error model is established as Equation (4):

$$\hat{x}_{\text{LS-SVM}} = \sum_{i=1}^{l} a_i K(x, x_i) + b \tag{4}$$

Where a_i, b are model parameters. The values of a_i and b can be obtained by Equation (5).

$$\begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & K(x_1, x_1) + \frac{1}{\gamma} & \cdots & K(x_1, x_l) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & K(x_l, x_1) & \cdots & K(x_l, x_l) + \frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} b \\ a_1 \\ \vdots \\ a_l \end{bmatrix} = \begin{bmatrix} 0 \\ y_1 \\ \vdots \\ y_l \end{bmatrix} \tag{5}$$

Kernel function $K(x_i, x_j)$ is arbitrary symmetric function satisfying the *Mercer* condition. The commonly used kernel function is linear, polynomial kernel and radial basis kernel, etc. In this work the radial basis kernel is used as the kernel function of LS-SVM, as shown in Equation (6).

$$K(x_i, x_j) = e^{\frac{-(x_i - x_j)^2}{2\sigma^2}} \tag{6}$$

From Equation (5) and (6) we can see the learning and predictive ability of LS-SVM model are depended on parameters (adjustable parameter) and   (radial basis kernel parameter). Parameter   and   can be determined by dynamically adaptive algorithm.

3. Grey Pprediction of Variable Weight Coefficient
3.1. Weight Determination of Variable Weight Prediction Model

In order to improve the precision and generalization ability of combined prediction model by comprehensive utilization of AR (p) and the LS-SVM thermal error prediction information, the weight coefficient of   and   must be optimized. Taking minimum sum of error square of prediction model as the optimization criterion, optimal weights in different time can be calculated. Let e_t be the predicting error at time t :

$e_{t =}$ (t) e_{1t}+ (t) e_{2t} $\qquad\qquad\qquad$ (7)
e_{1t}—Error of AR (p) model at time t;
e_{2t}—Error of AR (p) model at time t;

Assuming the weight coefficient is a continuous function of time t, (t) and (t) can be approached with sum of appropriate order polynomials, which expressed as followed equation:

 (t)= $_0$+ $_1 t$+ $_2 t^2$+…+ $_p t^p$ $\qquad\qquad$ (8)

 (t)= $_0$+ $_1 t$+ $_2 t^2$+…+ $_p t^p$ $\qquad\qquad$ (9)

Considering the number of detection data, we take p=2 for more convenient application. Therefore e_t can be expressed as:

$$e_t = \begin{bmatrix} e_{1t} & e_{2t} \end{bmatrix} \begin{bmatrix} \alpha_0 & \alpha_1 & \alpha_2 \\ \beta_0 & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \end{bmatrix} \tag{10}$$

The sum of squared errors of variable-weight combination prediction model is represented as:

$$e_m{}^2 = \sum_{t=1}^{n} \left(\begin{bmatrix} e_{1t} & e_{2t} \end{bmatrix} \begin{bmatrix} \alpha_0 & \alpha_1 & \alpha_2 \\ \beta_0 & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \end{bmatrix} \right)^2 \tag{11}$$

So the problem of solving variable weights is converted into optimization problem as follows:

min $e_m{}^2$
st. (t)+ (t)=1,t=1,2,…,n $\qquad\qquad\qquad$ (12)

Let $k = \begin{bmatrix} \alpha_0 & \alpha_1 & \alpha_2 & \beta_0 & \beta_1 & \beta_2 \end{bmatrix}^T$, constraints can also be written as Equation (13):

$$\begin{bmatrix} 1 & t & t^2 & 1 & t & t^2 \end{bmatrix} . k = 1 \tag{13}$$

Defining K and Y as follows:

$$K = \begin{bmatrix} x_{11} & x_{11} & x_{11} & x_{21} & x_{21} & x_{21} \\ x_{12} & 2x_{12} & 4x_{12} & x_{22} & 2x_{22} & 4x_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{1n} & nx_{1n} & n^2 x_{1n} & x_{2n} & nx_{2n} & n^2 x_{2n} \end{bmatrix}, Y = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}$$

x_i—the measured value in period i;

x_{1i}—the predicting value of AR(p) model in period i;
x_{2i}—the predicting value of LS-SVM model in period i;

Then the sum of squared errors can be expressed as follows:

$$e_m{}^2 = (Y - Kk)^T(Y - Kk) \tag{14}$$

Defining T, T_0 and R_n as follows:

$$T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2^2 & 1 & 2 & 2^2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 1 & n & n^2 & 1 & n & n^2 \end{bmatrix}, T_0 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2^2 & 1 & 2 & 2^2 \\ 1 & 3 & 3^2 & 1 & 3 & 3^2 \end{bmatrix}, R_n = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$$

The normalized constraints of combined weights can be expressed as follows:

$$Tk = R_n \tag{15}$$

By the linear algebra theory, we know that Equation (15) is consistent linear system with three linearly independent equations. And the rest of the equations can be represented linearly by the three equations. Let $R_3 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$, and one of its largest independent linear equations can be expressed as follows:

$$T_0 k = R_3 \tag{16}$$

According to the calculation method of the literature [12], assuming K is column full rank matrix, the constrained least squares estimate of k is written as follows:

$$\hat{k} = k^* - (K^T K)^{-1} T_0{}^T [T_0 (K^T K)^{-1} T_0{}^T]^{-1} [T_0 k^* - R_3] \tag{17}$$

3.2. Predicting Variable Weights with GM(1,1) Model
After calculating combined weights of each single predicting model at time $t(t = N)$, we should determine combined weights at time $t+1$ so as to obtain combined prediction values. Due to non-negativity and isometry of the weight vector, we use GM(1, 1) model to predict the combined weights.

3.2.1. Sequence Transformation
Assuming the weight vector of AR(p) model at time before t is $^0 = \{ {}^0(1), \alpha^0(2), \dots \, t^0(t) \}$, the accumulative sequence will be $^1 \{ {}^1(1), {}^1(2), \dots {}^1(t) \}$. In which,

$$\alpha^1(k) = \sum_{j=1}^{k} \alpha^1(j) \tag{18}$$

3.2.2. Model Solution
Accumulative sequence ^{1} satisfies the following grey differential equation:

$$\begin{cases} \dfrac{d\alpha^1(t)}{dt} + a\alpha^1(t) = b \\ \alpha^1(1) = \alpha^0(1) \end{cases} \tag{19}$$

In which a and b are parameters to be estimated. Using the least squares estimate we can obtain discrete time response function of GM(1,1) model:

$$\alpha^1(k + 1) = \left(\alpha^0(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a} \tag{20}$$

After IAGO (Inverse Accumulated Generating Operation) of sequence $\alpha^1(k + 1)$, we can predict the values of weight vector.

$$\alpha^0(k+1) = \alpha^1(k+1) - \alpha^1(k) = \left(\alpha^0(1) - \frac{b}{a}\right)[1 - e^a]e^{-ak} \tag{21}$$

According to the model we get the weight vector $(t+1)$ at time $t+1$, and the weight vector of LS-SVM model is:

$$\beta(t+1) = 1 - \alpha(t+1) \tag{22}$$

4. Spindle Thermal Error Modeling of Five Axis Vertical Machining Center

Taking a five axis vertical machining center as experimental subject, spindle thermal error is measured. Measuring instrument installation is shown in Figure 1.
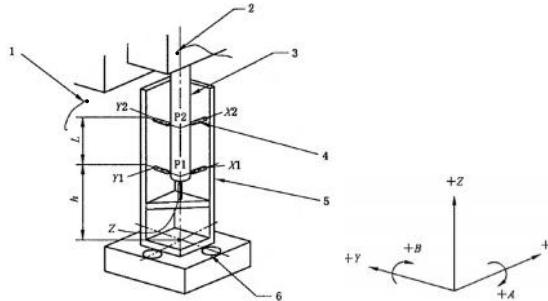
1 Environment temperature sensor T_1； 2 Spindle bearing temperature sensor T_2；
3 Inspection bar   4   Displacement sensor； 5   Fixture
6 Fixture fixed on platform with bolt

Figure 1. Installation Diagram

T_1 and T_2 are temperature sensors. T_1 is used to measure environment temperature, while T_2 is installed near the location of the spindle bearing to measure its temperature rise. X_1, X_2, Y_1, Y_2 and Z are displacement sensors planted along three perpendicular axis parallel to the machine direction to measure thermal drift errors along the X, Y, Z direction. Sensor placement point P_1 is 220 mm apart from the platform, and the distance between P1 and P2 is 150 mm.

The machine is started from cold state. When spindle runs to normal temperature (23), displacement sensors are used to measured spindle radial run-out. While spindle speed reaches 4500r/min, we drive the machine free run until the spindle temperature and deformation are stable. The data measured in this phase are used as spindle thermal error data. We get a total of 30 groups of samples by collecting data every 10 minutes. Taking spindle thermal error of Z direction for example, the measured results are shown in Figure 2.
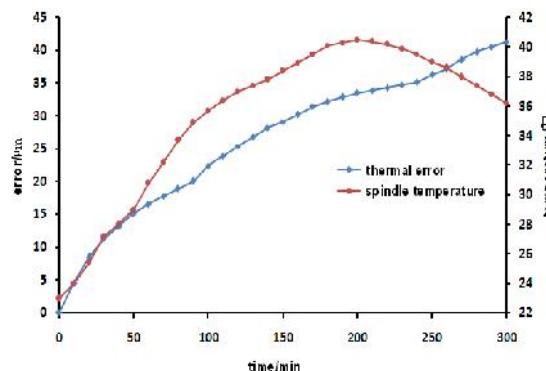
Figure 2. Measurement Result of Spindle Thermal Error in Z Direction

4.1. LS-SVM Modeling of Thermal Error

On the basis of the collected thermal error, we can calculate the parameter a and b of LS-SVM model. Using dynamic adaptive algorithm, model parameters are obtained.

$$=56.2, \quad \sigma^2=71.4$$

According to Equation (5), we get the values of parameter a and b of LS-SVM model.

$$a = (-132.52, 476.73, -563.28, 146.54, \cdots, 187.73), \quad b=14.52$$

4.2. AR(p) Modeling of Thermal Error

Due to the change trend between thermal error of machine tool and temperature is obvious, the nonstationary sequence should be smoothed. After that, time series analysis model is used to fit it. Fitting formula is shown in Equation (23).

$$\hat{x}_{\mathrm{AR}(p)}(t) = C_0 + C_s T + x_t \tag{23}$$

Where C_0 is constant, C_s is the temperature coefficient, T is temperature and x_t is remnant.

Taking the 30 samples of thermal error experimental data in Equation (23), we can work out the least squares estimate of C_0 and C_s based on the linear regression equation.

$$\hat{C}_0 = 194.83, \hat{C}_s = -1.762, R_{ss}=67.21$$

Where R_{ss} is residual sum of squares.

Upon inspection, the residual sequence is not white noise, so we fit it with time series analysis model after zero mean smooth processing and stationary test. By the minimum and final prediction error criterion of model order, remnants $\{x_t\}$ obey to $AR(1)$ model.

$$x_t{}' = \varphi_1 x_{t-1}{}' + a_t \tag{24}$$

Based on the sample estimates of auto-covariance function and autocorrelation function, we can obtain the moment estimation $\hat{\varphi}_1$ and $\hat{\sigma}^2{}_a$ of autoregressive parameter φ_1 and variance of a_t in the $AR(1)$ model by solving Yule-Walker equation.

$$\hat{\varphi}_1=0.213, \hat{\sigma}^2{}_a=1.379, R_{ss}=61.56$$

So the time series analysis model of thermal error is as follows:

$$\hat{x}_{\mathrm{AR}(p)}(t) = 153.33 + 0.213\hat{x}_{\mathrm{AR}(p)}(t-1) - 1.762T_t + 0.375T_{t-1} \tag{25}$$

4.3. Determination of Variable Weights

Using the method of determining variable weight coefficient in chapter 3.2, we obtain the weight vector of   and .

$$=\{0.67, 0.72, 0.65, \ldots, 0.58\}$$
$$=\{0.33, 0.28, 0.35, \ldots, 0.42\}$$

Therefore the weights (t+1) and (t+1) in time t+1 can be worked out by predicting variable weights using GM(1,1).

5. Result and Analysis of Variable-weight Combination Prediction

The variable-weight combination prediction model is applied to a five axis vertical machining center. It's used in the prediction of spindle thermal error of Z direction, which is shown in Figure 3.
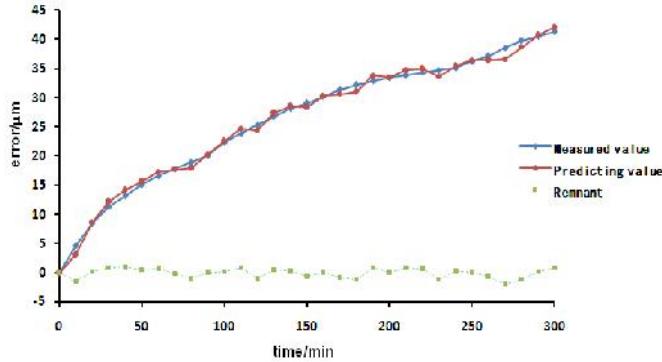
Figure 3. Modeling Result of Spindle Thermal Error in Z Direction

From Figure 3, we know that the model has high accuracy so it can predict the variation trend of thermal error effectively.

To analyze the prediction accuracy of variable-weight combination prediction model and single prediction model with LS-SVM and time series analysis model, we work out average residual value $\bar{e}$ and residual variance $\bar{\delta}$ of them respectively. Comparative analysis result is shown in Table 1.

Table 1. Accuracy Ccomparison of Different Models

Parameter	Modeling method		
	LS-SVM	AR(p)	variable-weight combination prediction
$\bar{e}/\mu m$	1.15	-0.43	-0.04
$\bar{\delta}/\mu m$	2.71	3.18	0.88

From Table 1 we know that variable-weight combined prediction model can get higher prediction accuracy than single modeling method with LS-SVM and time series analysis.

5. Conclusion

Due to the cause and change law of thermal deformation error in machining process are very complex, single prediction model will result in lower prediction accuracy. The work proposed the variable-weight combined prediction model by integrating LS-SVM and time series analysis. GM(1,1) model was used to predict the variable weights. Application of the grey variable-weight combined model on a five axis vertical machining center indicated that it can get higher prediction accuracy than single modeling method. So it can be used in the thermal error online compensation of CNC machine tool effectively.

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