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Nonlinear Observer-based Chaos Control for SPMSM with Both Uncertain Parameters and External Disturbances

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Abstract

This paper proposed a robust chaos control scheme for surface permanent magnet synchronous motor (SPMSM) drive system considering the uncertain parameters and external disturbances. A nonlinear time delay estimator is utilized to estimate the nonlinearities, uncertain parameters and disturbances of the system on line, so it is not necessarily required the exact model of the system. Then, based on the time delay estimator, a simple feedback controller, which is only related to the system errors and control input, is simple and easy to be constructed. The control gains can be obtained easily using pole placement method. The stability of the proposed control scheme is analyzed according to Lyapunov stability theory. Law-filter is also used to improve the performance of the system. Simulation results illustrate the effectiveness of the presented control method.

***Keywords**: surface permanent magnet synchronous motor, chaos control, nonlinear estimator, time delay estimation*

1. Introduction

Permanent magnet synchronous motor (PMSM) drive system has been widely used in industrial application, thanks to the advantages of high torque/inertia ratio, high torque/weight ratio, compact size and no rotor loss, such as in robotic system, CNC system, diskdrive systems, and so on. During the past years, the stability of the motor drive system, which is an essential requirement for industrial automation manufacturing, has received considerable attenuation. It has been found that chaos widely exists in all kinds of motor drive systems, such as induction motors, DC motors, and switched reluctance motors [1]. Chaotic behavior in permanent magnet DC motor with its parameters fall into a certain area is first addressed by Hemati [2]. Li has found that chaos was also existed in surface permanent magnet synchronous motor (SPMSM) [3]. Without considering power electric switching, SPMSM drive system can be transformed into a typical Lorenz system, which is well known exhibiting chaotic behavior. In most engineering applications, this undesirable chaotic oscillation, which will extremely destroy the stabilization of the system or even induce system collapse, should be suppressed or even eliminated.

Up to now, numerous control methods have been successfully used to control chaotic SPMSM system, such as decoupling control [4], feedback control [5, 6], dynamic surface control [5], Lyapunov exponents placement method [7, 8], sliding mode control [9], adaptive control [10, 11], and fuzzy control [12, 13]. However, decoupling control, feedback control, back-stepping control, and Lyapunov exponents placement method, all of which depend on the mathematical model of the system, cannot guarantee dynamic performance because of uncertain system parameters. It requires a parameter adaptive mechanism for the adaptive control technique, which increases the expense and complexity of the system and reduces its response capacity. Sliding mode control requires uncertain terms to meet specific match conditions and exhibits inherent chattering. Fuzzy control is usually based on Takagi–Sugeno fuzzy models of the system. Li et al. [12] proposed the fuzzy feedback control scheme, which exhibits poor responsiveness. Li et al. [14] then proposed optimal fuzzy guaranteed cost control, which improves its responsiveness, but has a structure that is too complicated for application.

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Moreover, the abovementioned methods are based on the assumption that there is no external disturbance in SPMSM. In fact, the uncertainties in a system usually are comprised of unpredictable plant parameter perturbation from the norm, external disturbance acting on the system, and unmodeled plant dynamics including undesirable nonlinear effects [15].Wu et at. [16] presented L2-gain passivity control method to restrain the disturbances. Although this control strategy can effectively inhibit the external disturbances in the system, but the uncertain parameters is not considered. To solve these problems, based on time delay estimation method, we propose a novel robust controller for SPMSM chaotic system with both parameter uncertainties and external disturbances. Time delay estimation, which has been successfully applied to control robot manipulators [17-19], is used to estimation of the nonlinearities, uncertainties and external disturbances. Low-pass filter are used to obtain the smooth delay estimation signals and control input signals. With this controller, the system exhibits not only rapid response, but also robustness under uncertain parameters and external disturbances.

This paper is organized as follows. Section 2 presents the chaos model of SPMSM. The proposed time delay controller is designed in detail in Section 3, and the stability is verified according to Lyapunov stability. Section 4 presents the simulation results to illustrate the effectiveness of the method. Finally, Section 5 concludes.

2. Chaos in SPMSM

The transformed model of SPMSM can be expressed as follows [3]:

$$\begin{cases} \dot{i}_d = -i_d + wi_q + v_d \\ \dot{i}_q = -i_q - wi_d + \text{¥} w + v_q \\ \dot{w} = \pm (i_q - w) - T_L \end{cases}, \quad (1)$$

Where v_d, v_q, i_d, and i_q are the transformed stator voltage components and current components in the d-q frame, w and T_L are the transformed angle speed and external load torque respectively, and *γ* and *σ* are the motor parameters.

Considering the case that, after an operation of the system, the external inputs are set to zero, namely, $v_d = v_q = T_L = 0$, system (1) becomes an autonomous system:

$$\begin{cases} \dot{i}_d = -i_d + wi_q \\ \dot{i}_q = -i_q - wi_d + \text{¥} w \\ \dot{w} = \pm (i_q - w) \end{cases}. \quad (2)$$

The modern nonlinear theory such as bifurcation and chaos has been used to study the nonlinear characteristics of SPMSM drive system in [3]. It has found that, with the operating parameters*γ* and *σ* falling into a certain area, SPMSM will exhibit complex dynamic behavior, such as periodic, quasi periodic and chaotic behaviors. In order to make an overall inspection of dynamic behavior of the SPMSM, the bifurcation diagram of the angle speed w with increasing of the parameter ¥ is illustrated in Figure 1(a).We can see that the system shows abundant and complex dynamical behaviors with increasing parameter ¥ . The typical chaotic attractor is shown in Figure 1(b) with $v_d = v_q = T_L = 0$, $\text{¥} = 26$, and $\pm = 5.46$.

According to chaos theory, the Lyapunov exponents and power spectrum are two effective methods to determine whether a continuous dynamic system is chaotic. In general, a three-dimensional nonlinear system has one positive Lyapunov exponents, implying that it is chaotic. Figure 1(c) and (d) show the Lyapunov exponents and power spectrum of SPMSM chaotic system (6) with $\text{¥} = 26$, and $\pm = 5.46$. When the parameters are set as obove, calculated Lyapunov exponents are: $L_{E1} = 0.483474$, $L_{E2} = -0.000926$, $L_{E3} = -7.942549$,and the Lyapunov dimension is $D_L = 2.060755$, which means the system is chaotic.

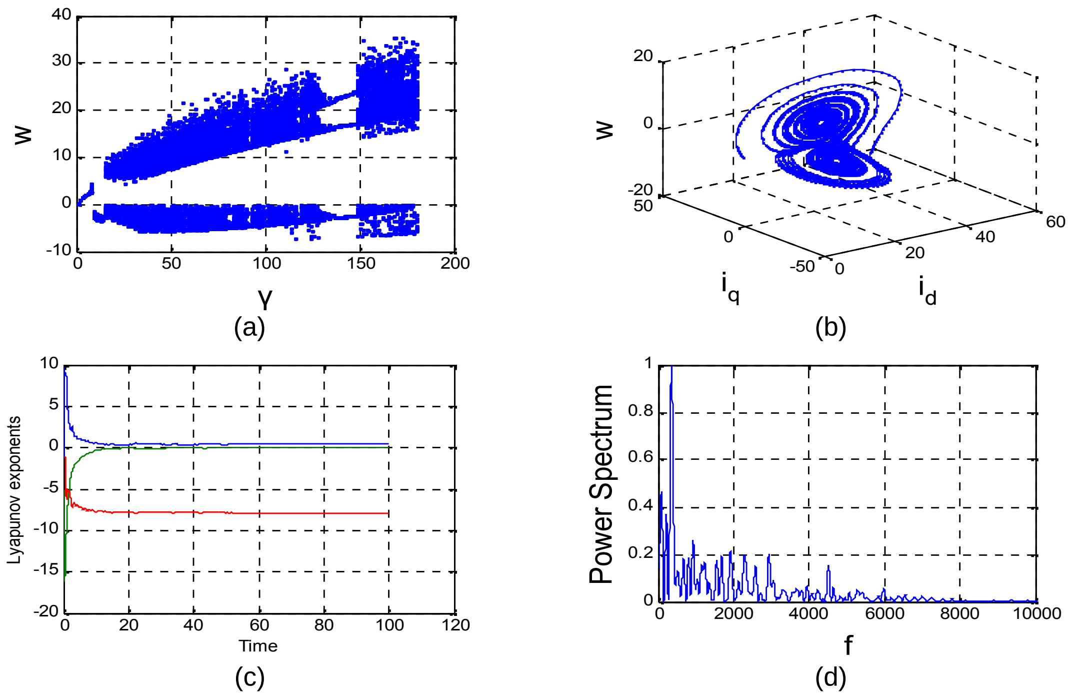

Figure 1. Bifurcation Deagram and the Characterizations of chaos in SPMSM (a) Bifurcation diagram of state variable w with the parameter ¥ (b) typical chaotic attractor (c) Lyapunov exponents (d) power spectrum of state variable w

Considering the uncertain parameters and external distanbances, the dynamic model of the system can be described as follows:

$$\begin{cases} \dot{i}_d = -i_d + w i_q + d_1 \\ \dot{i}_q = -i_q - w i_d + (¥ + \Delta ¥) w + d_2 \\ \dot{w} = (± + \Delta ±)(i_q - w) \end{cases}, \quad (3)$$

Where $\Delta ¥$ and $\Delta ±$ represent the uncertainty of ¥ and ± respectively d_1 and d_2 represent the total disturbance including the unmodeled dynamics and external disturbances, which are assumed to be continuous and bounded ($\|d_1\| \le £_1$ and $\|d_2\| \le £_2$).

Following an actual operation, this article assumes that the fluctuation range of system parameters is 30%, that is, $\|\Delta ¥\| \le ¢_1 \le 0.3¥$, $\|\Delta ±\| \le ¢_2 \le 0.3±$. The chaotic attractor of SPMSM with parameter uncertainties is shown in Figure 2.

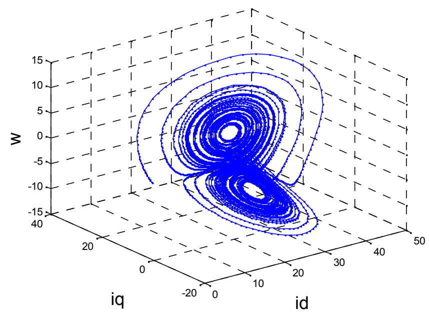

Figure 2. Chaotic Attractor of SPMSM with Parameter Uncertainties

3. Controller Design for SPMSM Chaotic System

System (1) indicates three equilibrium points: $S_0(0,0,0)$, $S_1(¥-1,\sqrt{¥-1},\sqrt{¥-1})$, and $S_2(¥-1,-\sqrt{¥-1},-\sqrt{¥-1})$. Given that *γ*=26, $S_0(0,0,0)$ is locally stable, and $S_1(25,5,5)$ and $S_2(25,-5,-5)$ are both locally unstable [3]. Assuming that one equilibrium point of system (1) is $S_d(i_{dd},i_{qd},w_d)$, then:

$$\begin{cases} \dot{i}_d = -i_{dd} + w_d i_{qd} = 0 \\ \dot{i}_{qd} = -i_{qd} - w_d i_{dd} + ¥ w_d = 0 \\ \dot{w}_d = ±(i_{qd} - w_d) = 0 \end{cases} . \tag{4}$$

To quickly stabilize to equilibrium point $S_d(i_{dd},i_{qd},w_d)$, u_1 and u_2 are used to control the system. Under the control of u_1 and u_2, the system model can be represented as:

$$\begin{cases} \dot{i}_d = -i_d + w i_q + d_1 + u_1 \\ \dot{i}_q = -i_q - w i_d + (¥ + \Delta_¥)w + d_2 + u_2 \\ \dot{w} = (± + \Delta_±)(i_q - w) \end{cases} . \tag{5}$$

Let $e_1 = i_d - i_{dd}$, $e_2 = i_q - i_{qd}$, $e_3 = w - w_d$, we can obtain the dynamic error equations of the system:

$$\begin{cases} \dot{e}_1 = -e_1 + e_2 e_3 + e_2 w_d + e_3 i_{qd} + d_1 + u_1 \\ \dot{e}_2 = -e_2 - e_1 e_3 - e_1 w_d - e_3 i_{dd} + ¥ e_3 + \Delta_¥ w + d_2 + u_2 \\ \dot{e}_3 = (± + \Delta_±)(e_2 - e_3) \end{cases} . \tag{6}$$

System (6) indicates that error state e_3 is internally stable when e_1 and e_2 converge to zero (because $± + \Delta_± > 0$). So the focus of the controller design is to make e_1 and e_2 converge to zero. The first and second equation of system (6) can be rewritten as:

$$\begin{cases} \dot{e}_1 = f_1(e_1,e_2,e_3) + u_1 \\ \dot{e}_2 = f_2(e_1,e_2,e_3) + u_2 \end{cases} . \tag{7}$$

Where,

$$\begin{cases} f_1(e_1,e_2,e_3) = -e_1 + e_2 e_3 + e_2 w_d + e_3 i_{qd} + d_1 \\ f_1(e_1,e_2,e_3) = -e_2 - e_1 e_3 - e_1 w_d - e_3 i_{dd} + ¥ e_3 + \Delta_¥ w + d_2 \end{cases} . \tag{8}$$

System analysis is required prior to designing the controller. Since $f_1(e_1,e_2,e_3)$ and $f_2(e_1,e_2,e_3)$ are both continuous, if the time delay *τ* is sufficiently small, according to time delay estimation method [19], the following approximation holds:

$$\begin{cases} f_1(e_1,e_2,e_3)_t \cong f_1(e_1,e_2,e_3)_{t-²} \\ f_2(e_1,e_2,e_3)_t \cong f_2(e_1,e_2,e_3)_{t-²} \end{cases} \tag{9}$$

That is, $f_1(e_1,e_2,e_3)_t$ and $f_2(e_1,e_2,e_3)_t$ can be estimated by $f_1(e_1,e_2,e_3)_{t-²}$ and $f_2(e_1,e_2,e_3)_{t-²}$ respectively. So, according to this method, it can be formally defined as :

$$\begin{cases} \hat{f}_1(e_1,e_2,e_3)_t = f_1(e_1,e_2,e_3)_{t-\tau} \\ \hat{f}_2(e_1,e_2,e_3)_t = f_2(e_1,e_2,e_3)_{t-\tau} \end{cases}. \quad (11)$$

Where $\hat{\bullet}$ means the estimated value of $\bullet$, and $\bullet_{t-\tau}$ means time delayed value of $\bullet$. Note (7), (11) can be rewritten as:

$$\begin{cases} \hat{f}_1(e_1,e_2,e_3)_t = (\dot{e}_1)_{t-\tau} - (u_1)_{t-\tau} \\ \hat{f}_2(e_1,e_2,e_3)_t = (\dot{e}_2)_{t-\tau} - (u_2)_{t-\tau} \end{cases}. \quad (12)$$

Theorem. Consider dynamic error system (6). If the controller is designed as:

$$\begin{cases} u_1 = -(\dot{e}_1)_{t-\tau} + (u_1)_{t-\tau} - k_1 e_1 \\ u_2 = -(\dot{e}_2)_{t-\tau} + (u_2)_{t-\tau} - k_2 e_2 \end{cases}, \quad (13)$$

Then system (6) globally asymptotically stable at the equilibrium point O (0, 0, 0), that is, system state (i_d, i_q, w) converge to (i_{dd}, i_{qd}, w_d), where k_1 and k_2 are the controller parameters that are positive real numbers.

Proof. Add control u_1 and u_2 to the first and second equations of Equation (6), we can obtain that:

$$\begin{cases} \dot{e}_1 = f_1(e_1,e_2,e_3) - f_1(e_1,e_2,e_3)_{t-\tau} - k_1 e_1 \\ \dot{e}_2 = f_2(e_1,e_2,e_3) - f_2(e_1,e_2,e_3)_{t-\tau} - k_2 e_2 \end{cases}. \quad (14)$$

If the candidate Lyapunov function is defined as $V = \frac{1}{2}(e_1^2 + e_2^2)$, then the time derivative of V along the trajectory of (14) is:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 \\ &= e_1(f_1(e_1,e_2,e_3) - f_1(e_1,e_2,e_3)_{t-\tau} - k_1 e_1) + e_2(f_2(e_1,e_2,e_3) - f_2(e_1,e_2,e_3)_{t-\tau} - k_2 e_2) \\ &= -k_1 e_1^2 - k_2 e_2^2 - e_1 \zeta_1 - e_2 \zeta_2 \\ &\le -k_1(e_1^2 - |\zeta_1 e_1|/k_1) - k_2(e_2^2 - |\zeta_2 e_2|/k_2) \end{aligned}$$

Where ζ_1 and ζ_2 denotes the estimated error, they are expressed by:

$$\begin{cases} \zeta_1 = f_1(e_1,e_2,e_3)_t - \hat{f}_1(e_1,e_2,e_3)_t \\ \zeta_2 = f_2(e_1,e_2,e_3)_t - \hat{f}_2(e_1,e_2,e_3)_t \end{cases}. \quad (15)$$

$\dot{V}$ is negative outside the set $\{|e_1| \le |s_1|/k_1\} \cup \{|e_2| \le |s_2|/k_2\}$, so the solutions are ultimately bounded. If e_1=0 (an ideal case as $\tau \to 0$) and e_2=0 (an ideal case as $\tau \to 0$), then the controlled subsystem (14) is globally asymptotically stable.

If we assumes that the stable time of e_1 and e_2 are t_{m1} and t_{m2},respectively, then after t_m ($t_m = \max\{t_{m1}, t_{m2}\}$), $e_1 \equiv 0$ and $e_2 \equiv 0$. Substituting $e_1 = 0$ and $e_2 = 0$ into the third equation of system (6) yields:

$$\dot{e}_3 = -(\alpha + \Delta_\alpha)e_3, \quad (16)$$

Obviously, subsystem (16) is globally asymptotically stable because of $\pm + \Delta_{\pm} > 0$.

Thus, system (6) is globally asymptotically stable. That is, system state (i_d, i_q, w) converge to $S_d(i_{dd}, i_{qd}, w_d)$.

4. Simulation Results

We use SIMULINK of MATLAB to verify the feasibility of the proposed control scheme for SPMSM chaotic system. In the simulation, the fourth-order Runge–Kutta method is used to solve the systems with time step size 0.001. The parameters of SPMSM are selected as $¥ = 26$, and $\pm = 5.46$. The uncertain parameters are selected as the same as in Section 3, and the external distances are $d_1 = 0.5\sin(5®t)$ and $d_2 = 0.5\sin(5®t)$. The control method takes effect after *t*=10 s.

Supposed we choose the control parameters to be $k_1 = 30$, $k_2 = 60$, and $^2 = 0.001$. The desired equilibrium point $S_d(i_{dd}, i_{qd}, w_d)$ is set as: if $0 \le t < 20$, $S_d(i_{dd}, i_{qd}, w_d) = S_1(25,5,5)$; if $t \ge 20$, $S_d(i_{dd}, i_{qd}, w_d) = S_1(25,5,5)$.

For the purpose of examining the robustness of the proposed method for uncertain parameters and the anti-disturbance capacity in SPMSM chaotic system, the simulation results are shown in Figure 3.

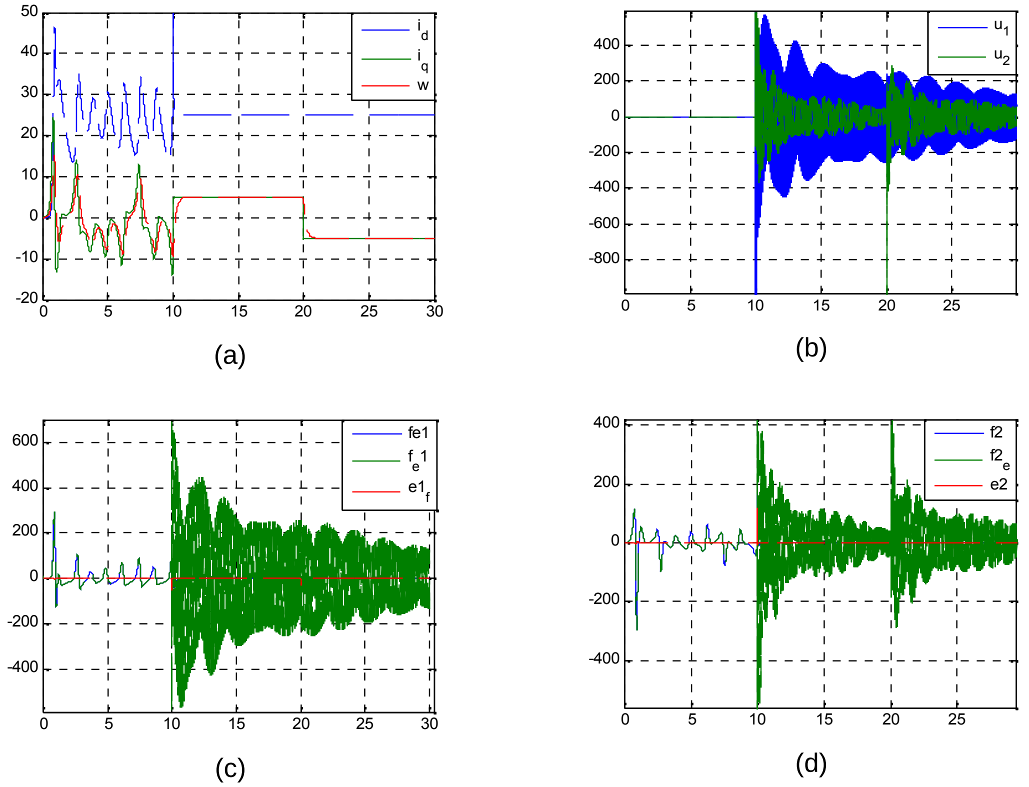

Figure 3. The Response of SPMSM Chaotic System under the Proposed Control Scheme with Uncertain Parameters and External Distanbances (a) state trajectories (b) control inputs (c) f_1, $\hat{f}_1$ and $e_1 = f_1 - \hat{f}_1$ (d) f_2, $\hat{f}_2$ and $e_1 = f_2 - \hat{f}_2$

As can be seen from Figure 3, the proposed controller can quickly stabilize the system state to a desired equilibrium point. However, the design of the controller has a big chattering phenomenon, which will affect the performance of the system.

In order to improve the performance of the designed controller, a low-pass filter is introduced to the controller in this paper. The cutoff frequency of the low-pass filter is set as $w_c = 100$ Hz. Moreover, the same low-pass filter is also introduced to estimate f_1 and f_2. the simulation results of the improved controller are shown in Figure 4.

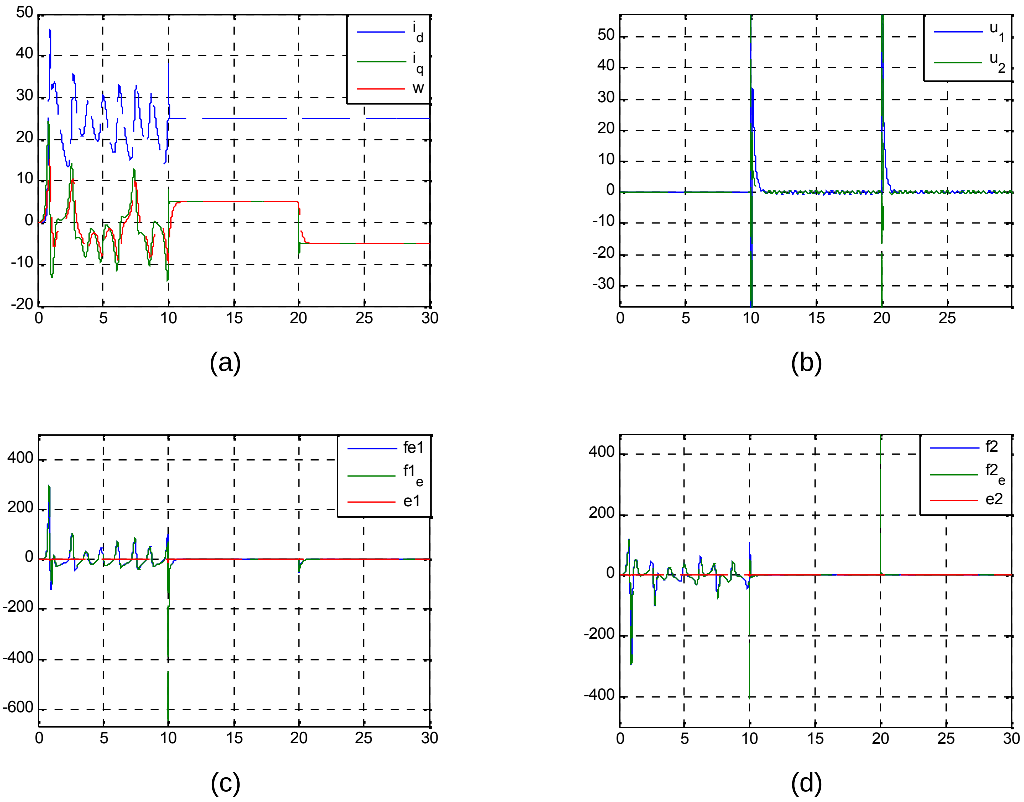

Figure 4. The Response of SPMSM Chaotic System under the Proposed Control Scheme with Uncertain Parameters and External Distanbances using Low-pass Filters (a) state trajectories (b) control inputs (c) f_1, $\hat{f}_1(f1_e)$ and $e_1 = f_1 - \hat{f}_1$ (d) f_2, $\hat{f}_2(f2_e)$ and $e_2 = f_2 - \hat{f}_2$.

We can see from Figure 4 that the control inputs and estimator outputs are both smooth. The states of the system stabilize the system state to a desired equilibrium point and the estimation errors converge to zero quickly.

5. Conclusion

We develop a novel robust control scheme for SPMSM chaotic system with both parameter uncertainties and external distances. This controller applies time delay estimator to estimate the nonlinearities, uncertain parameters and external disturbances. Law-filter has been used to improve the performance of the system. The structure of this controller is easy to design and implement. Simulation results verify that the proposed controller exhibits quick responsiveness and strong robustness. Future research should investigate the implementation of the proposed control scheme by using an experimental setup. The scheme can also be extended to synchronize SPMSM chaotic systems with uncertain parameters.

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