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Forward Position Solution of 3-RPS in-Parallel Manipulator Based on Particle Swarm Optimization

Zhang Hongli*, Ren Tiantian
College of Electrical Engineering, Xinjiang University, Urumqi 830047, China
*Corresponding author, e-mail: 1606829274@qq.com

Abstract

Particle swarm optimization is introduced to solve the problem in this paper. Instead of solving a group of non-linear equations, forward kinematics is solved by computing the extrenum of a function. And accurate solutions can be obtained by the global and local searching abilities of advanced particle swarm optimization. It overcomes the shortage that precision is greatly influenced by initial values with conventional numerical methods. Calculation results show that this new method is simple, convenient, and with generality for solving the parallel manipulator forward kinematics problems.

Keywords: *particle swarm optimization, parallel manipulator, forward kinematics*

1. Introduction

Compared with serial mechanism, the parallel manipulator is widely used in application fields, which has several advantages such as high stiffness, strong load-carrying capacity, small self-weight/load ratio, fast response speed, nice dynamic performance and so on. Forward kinematics is to determine the position and orientation of the platform with given limbs lengths, while inverse kinematics is to determine the limbs lengths for given position and orientation of the platform. Contrary to serial mechanism, inverse kinematics of parallel mechanism is relatively easy to achieve but the forward kinematics is more complicated. The analysis of forward kinematics is not only one of the basic problems of the theory of parallel mechanisms, but also the foundation for analysis and synthesis of mechanism, solution of velocity and acceleration, dynamics analysis and error analysis. Researchers have carried out many studies on numerical solution and analytical solution, and have made a series of progresses [1-3].

The essence of forward kinematics is to sovle highly nonlinear equations, and the main methods include analytic method and homotopy method. Analytic method is to reduce the unknown numbers of mechanism constrained equations by elimination, thus the input-output equation turns into a higher equation containing only one unknown number. The advantages of this method are no need of initial value, getting all the possible solutions and having no limitations of some special mechanism configurations. However, the elimination process is usually diverse and complicated, and solving speed is lower. Meanwhile, homotopy method has its advantages such as no need of elimination and initial value, getting all possible solutions, but its solving speed is low. Meanwhile some scholars do studies on neural network method for forward kinematics [4]. The nonlinear mapping from the joint-variable-space to the operation-variable-space for the platform is accomplished with neural network after training and learning so as to get solutions easier by avoiding complicated formula derivation and programming calculation. But the method needs further study to solve multiple solutions problem.

PSO algorithm is a non-numeric parallel algorithm based on population. Relative to traditional evolutionary algorithms, PSO has fewer adjustment parameters and no complicated operations of auto regulating, and it has better global searching optimization capacity. In this paper, PSO algorithm is used for forward kinematics of 3RPS parallel manipulator, and the result is satisfactory.

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2. Forward Kinematics of 3RPS Parallel Mechanisms

3RPS parallel mechanism, which was proposed by Hunt in 1983, is a typical parallel mechanism with few degrees of freedom. As Figure 1 shows that the lower platform ABC (the fixed platform) and the upper platform abc (the moving platform) are both equilateral triangles with their circumradius of R and r respectively. The upper platform is connected with cylinder linkages by spherical joints, and the lower one is connected with the bottom by revolute joints of which the axes are perpendicular to the axis of cylinder linkages. The upper platform is promoted by cylinder linkages (revolute joints) moving.

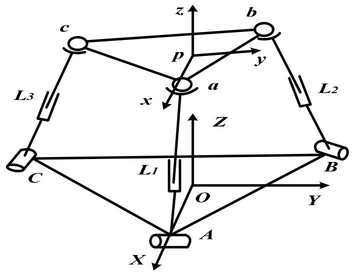


Figure 1. 3RPS Parallel Manipulator

O-XYZ is the reference coordinate system on the fixed platform, while p-xyz is the moving coordinate system on the moving platform. The coordinates of the upper acmes are represented as: A$(R,0,0)$, B$(-R/2,\sqrt{3}R/2,0)$, C$(-R/2,-\sqrt{3}R/2,0)$; the coordinates of the lower acmes are represented as: a$(r,0,0)$, b$(-r/2,\sqrt{3}r/2,0)$, c$(-r/2,-\sqrt{3}r/2,0)$.

r is a vector of the moving coordinate system, and it can be transformed as R by coordinate transformation matrix T to the reference coordinate system.

$$T=\begin{bmatrix} \cos\psi.\cos\varphi\cos\theta-\sin\psi.\sin\theta & -\cos\psi.\cos\varphi\sin\theta-\sin\psi.\cos\theta & \cos\psi.\sin\varphi \\ \sin\psi.\cos\varphi\cos\theta+\cos\psi.\sin\theta & -\sin\psi.\cos\varphi\sin\theta+\cos\psi.\cos\theta & \sin\psi.\sin\varphi \\ -\sin\varphi\cos\theta & \sin\varphi\sin\theta & \cos\varphi \end{bmatrix}$$

***R*=*T*r+p**

Where p(X_p,Y_p,Z_p) is the position vector of the moving coordinates origin in reference coordinate system; Euler angles ψ,φ,θ represent moving platform attitude. Coordinates of the upper platform acmes in the reference coordinate system are functions of $X_p,Y_p,Z_p\ \psi,\varphi,\theta$, only three of which are independent parameters for 3RPS parallel mechanisms [5]. Select Z_p,ψ,θ as the independent output posture parameters, and the other three will be represented by them. The 3RPS parallel mechanisms characters show that the moving platform cannot rotate about z-axis of the moving platform, so we can get $\psi+\theta=0$, that is $\psi=-\theta$. As a result, the formulas for calculating limbs lengths are represented as follows:

$$L_1^2=[r\cos 2\psi(\cos\phi-1)+\frac{r(\cos\phi+1)}{2} - R]^2+(Z_p-r\sin\phi\cos\psi)^2 \quad (1)$$

$$L_2^2=[\frac{r(\cos\phi-1)(\sqrt{3}\sin 2\psi+\cos 2\psi)}{4}-\frac{r(1+\cos\phi)}{4} + \frac{R}{2}]^2+[\frac{r\sin\phi(\cos\psi-\sqrt{3}\sin\psi)}{2}+Z_p]^2 + [\frac{r(1-\cos\phi)(3\sin 2\psi+\sqrt{3}\cos 2\psi)}{4} + \frac{\sqrt{3}r(1+\cos\phi)}{4}+\frac{R}{2}]^2 \quad (2)$$

$$L_3^2=[\frac{r(1-\cos\phi)(\sqrt{3}\sin 2\psi-\cos 2\psi)}{4}-\frac{r(1+\cos\phi)}{4}+\frac{R}{2}]^2+[\frac{r\sin\phi(\cos\psi+\sqrt{3}\sin\psi)}{2}+Z_p]^2+[\frac{r(1-\cos\phi)(3\sin 2\psi-\sqrt{3}\cos 2\psi)}{4}+\frac{\sqrt{3}R}{2}]^2 \quad (3)$$

Forward kinematics is to determine Z_p,ψ,θ for given L_1,L_2,L_3, and its essence is to solve highly nonlinear equations mentioned above. Newton-Raphson method is a usual numerical solution. According to the structural characteristics of 3RPS and the identity conditions for mechanisms motions, Li Shujun from Northeastern University extends the successive approximation method to the forward kinematics solutions of the spatial parallel mechanisms, which is a new approach to the position analysis of parallel mechanism. Some other solution methods are proposed by scholars [7], but generally speaking, some problems are not completely solved such as initial value sensitivity, solution speed and convergence problem.

3. Particles Swarm Optimization

Particles swarm optimization algorithm, has become an important branch of Evolutionary Algorithm [8], which was originally proposed by American social psychologist James Kennedy and electrical engineer Russel Eberhart in 1995 and inspired by the social behaviors of animals such as bird flocking and fish schooling. PSO algorithm has been widely applied in many field such as function optimization, neural network design, IC design, power network planning and so on because of its simple concept, easily implement, high speed and better global searching ability for large scale mathematical optimization problems and higher convergence speed than genetic algorithm.

The basic principle of PSO is described that in the n-dimensional space, m particles with their coordinates as $X_i=(x_{i1},x_{i2},...x_{in})$ respectively and fitness correlated to optimization object function (usually objective function is used as fitness directly), have their respective flight speeds represented as $V_i=(v_{i1},v_{i2},...v_{in})$. For the i-th particle, its best position named as Pbest can be represented as $P_i=(p_{i1},p_{i2},...p_{in})$, and all particles' best position named as Gbest can be represented as $P_g=(g_1,g_2,...g_n)$. For the i-th particle of the t-th generation, its j-th dimensional speed and position of the (t+1)-th generation can be solved according to the following two equations:

$$v_{ij}(t+1)=v_{ij}+c_1r_{1j}(t)(p_{ij}(t)-x_{ij}(t))+c_2r_{2j}(t)(p_{gj}(t)-x_{ij}(t)) \quad (4)$$

$$x_{ij}(t+1)=x_{ij}(t)+v_{ij}(t+1)) \quad (5)$$

Where i=1,2, … , m. m is the number of particles. r_1、r_2 are random numbers in the range (0,1). c_1、c_2 are the acceleration weights. In addition, the particle speed is limited by the maximum speed. The first part of Equation (4), representing the current speed, provides necessary momentum for particles' flying in the searching space. The second part, the cognitive portion, represents particles' thinking and impels particles to fly to the personal best position Pbest. Then the third part is the social portion, represents the mutual cooperation and influence between particles and impels particles fly to the best position Lbest in the neighborhood initially to find the global optimal solution as much as possible and finally fly to the global best position Gbest. The 1st portion of the speed Evolution Equation guarantees the global searching capacity and the other two portions guarantee the local searching capacity. A modified PSO is usually applied [9], and the corresponding speed evolution equation is represented as:

$$v_{ij}(t+1)=w(t)v_{ij}+c_1r_{1j}(t)(p_{ij}(t)-x_{ij}(t))+c_2r_{2j}(t)(p_{gj}(t)-x_{ij}(t)) \quad (6)$$

Where *w* is inertia weight. Similar to the temperature parameter in simulated annealing algorithm, larger *w* means better global searching ability and smaller *w* means better local searching ability. As the iteration times increases, *w* decreases gradually and the algorithm gets better global convergence ability initially and better local convergence ability later. Select $w = 0.9 - 0.5k/k_{\max}$, where *k* is iteration times, and $k_{\max}$ is cut-off iteration times, so the actual variation range of *w* is 0.9~0.4.

In usual applications, a constrained condition must be taken into account that the center point of the upper platform is required to run according to the preconceived track. Based on 3RPS parallel mechanisms structural characteristics, points a , b, c of the upper platform are limited in three planes: $Y = 0, Y = -\sqrt{3}X$, $Y = \sqrt{3}X$, and as a result the upper platform center point is constrained by the following equations:

$$X_P = \frac{r\cos 2\psi(\cos\varphi - 1)}{2}, Y_P = \frac{r\sin 2\psi(1 - \cos\varphi)}{2}$$

If the center point is required to follow circle locus with radius r_0 , then:

$$X_p{}^2 + Y_P{}^2 - r_0{}^2 = 0$$

That is:

$$\left(\frac{r\cos 2\psi(\cos\varphi - 1)}{2}\right)^2 + \left(\frac{r\sin 2\psi(1 - \cos\varphi)}{2}\right)^2 - r_0{}^2 = 0 \tag{7}$$

The nonlinear equations made up of Equation (1), (2), (3) and (7) are modified to homogeneous equations as follows:

$$f_1(\psi, \varphi, Z_p) - L_1^2 = 0 \tag{8}$$

$$f_2(\psi, \varphi, Z_p) - L_2^2 = 0 \tag{9}$$

$$f_3(\psi, \varphi, Z_p) - L_3^2 = 0 \tag{10}$$

$$f_4(\psi, \varphi, Z_p) = 0 \tag{11}$$

Then a new function can be constructed by the four equations above:

$$\begin{aligned} f(\psi, \phi, Z_p) &= (f_1 - L_1^2)^2 + (f_2 - L_2^2)^2 \\ &+ (f_3 - L_3^2)^2 + f_4{}^2 \end{aligned} \tag{12}$$

So the minimum value of the unconstrained function $f(\psi, \varphi, Z_p)$ is the solution of trajectory-bounded nonlinear equations. In PSO, $f(\psi, \varphi, Z_p)$ is used as the fitness function to evaluate the position of particles, and it is the fitness value that guides the evolution process.

The calculation flow of PSO is as follows:

1) Initialize the particle swarm (including random position and speed);
2) Calculate fitness value of every particle, assume the initial position as the historical best position $P_i, i = 1,2,...n$, and obtain the global best position P_g ;
3) Compare the fitness value of every particle with its historical best position P_i, and let it be the current best position if the fitness value is better;
4) Compare the fitness value of every particle with the global best position P_g , and let it be the historical best position if the fitness value is better;

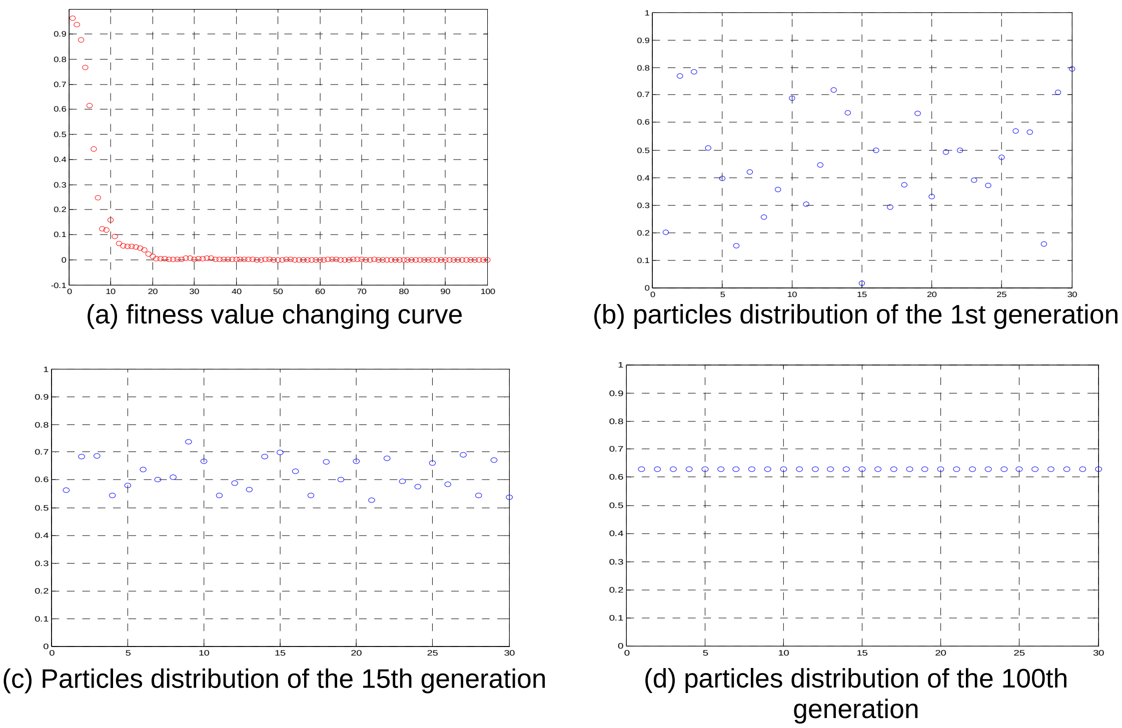
5) Evolve the speed and position of particles according to formulas (5) and (6), and obtain a new generation of particle swarm;
6) If the termination condition (usually good enough fitness value or presuppose maximum algebraic multiplicity) is not satisfied, return 2; otherwise finish the whole process.

4. Example of Forward Kinematics of Parallel Manipulator Based on PSO

As Figure 1 shows that the radii of the upper and lower platforms are 40cm and 30cm, and the limbs lengths are between 50cm and 100cm. Assume the size of particle swarm n=30, dimension d=3,and maximum iteration times is 100. The center point of upper platform runs following the circle with radius 4.4cm. Choose arbitrarily 5 groups of limbs lengths, and the calculation results of Table 1 shows that calculation precision already have reached 10^{-4}.

Table 1. Result of PSO

Number	Rod Length/m	Real Euler Angles/rad	PSO Caculated Angles/rad	Error×10^{-4}/rad
First	L_1 0.5494	0.5236	0.523789	1.89
	L_2 0.7022	0.7854	0.785446	0.46
	L_3 0.9035	0.7000	0.699997	0.03
Second	L_1 0.5554	0.6283	0.628370	0.07
	L_2 0.6803	0.7854	0.785381	0.19
	L_3 0.9165	0.7000	0.700002	0.02
Third	L_1 0.5685	0.7854	0.785203	1.97
	L_2 0.6487	0.7854	0.785470	0.70
	L_3 0.9313	0.700000	0.700015	0.15
Fouth	L_1 0.6023	1.047200	1.047198	0.02
	L_2 0.6023	0.785400	0.785342	0.58
	L_3 0.9411	0.700000	0.699997	0.03
Fifth	L_1 0.7022	1.570800	1.570606	1.94
	L_2 0.5494	0.785400	0.785446	0.46
	L_3 0.9035	0.700000	0.699997	0.03



(a) fitness value changing curve

(b) particles distribution of the 1st generation

(c) Particles distribution of the 15th generation

(d) particles distribution of the 100th generation

Figure 2. Evolution of PSO Algorithm

Taking the 2nd group data as an example, the changing curve of particle swarm fitness value is showed as Figure 2(a). Obviously the average fitness value of every generation particle swarm decreases constantly and finally converges to zero. Taking L_1 of the 2nd group data as an example and showing as Figure 2(b), 2(c), 2(d), particle swarm under the guidance of fitness value converge gradually from initial random distribution to actual value.

5. Conclusion

The global optimal solution can be obtained better by the modified PSO and its parallel searching ability. The method overcomes the problem that traditional solution of nonlinear equations is sensitive to the initial point, and avoids of formula derivation and complicated calculation for forward kinematics, and needs no special form of equations. Compared with other evolutionary algorithm, it is easy to understand and program needs fewer empirical parameters. The results show that PSO algorithm is a new effective method for forward kinematics.

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