

Spread of harmful substances in the atmosphere of industrial cities of Kazakhstan: modeling and data refinement

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ABSTRACT

In Kazakhstan, air pollution in industrial cities poses a significant challenge that requires urgent attention. This study investigates the dispersion of harmful pollutants in the air across nine prominent industrial cities in Kazakhstan. The research involves modeling the emissions from major pollution sources for each city, which provides a comprehensive view of how these substances spread through the atmosphere. The study also examines the distribution patterns of these pollutants to gauge their concentration levels in each urban area. Additionally, it addresses the inverse problem of data assimilation from automated monitoring stations (AMS), aiming to refine the information on pollution sources. By utilizing the conjugate equations method, the study successfully converged to an accurate solution. Detailed visualizations for Almaty, Ust-Kamenogorsk, and Pavlodar illustrate the pollution dynamics and pinpoint the most affected regions. These findings are crucial for formulating strategies to mitigate the adverse effects of industrial emissions on both the environment and public health.

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1. INTRODUCTION

Currently, the quality of the air is a significant concern for many, particularly in large industrial cities. The scarcity of detailed data and research on air pollution in Central Asia, including Kazakhstan, has drawn considerable attention from environmental specialists. According to recent data [1], Kazakhstan is ranked 35th globally for air pollution. The primary contributors to atmospheric contamination include emissions from factories, power plants, and vehicles. Industrial cities in Kazakhstan, which are pivotal for economic growth, are grappling with escalating levels of harmful emissions that pose risks to public health and environmental quality.

Kazakhstan hosts several cities with robust industrial sectors. For instance, Pavlodar is home to major facilities such as JSC Transnational Company Kazchrome, LLP KSP Steel, JSC Pavlodarenergo (including TPP-3), and JSC Aluminum of Kazakhstan, among others. Similarly, Karaganda houses significant enterprises like Karaganda TPP-1, TPP-2, and TPP-3, along with other key industries such as the Karaganda Foundry and Machine-building Plant and the Karaganda Metallurgical Plant. Ust-Kamenogorsk features prominent facilities such as Ust-Kamenogorsk TPP, Kazzinc LLP, and the Ulba Metallurgical Plant. In Almaty, JSC TPP-1, TPP-2, and TPP-3 are crucial for regional energy needs. The effect of harmful

impurities on the environment has a direct influence on the health of the population. Toxic substances released into the atmosphere not only threaten flora and fauna, but can also lead to serious consequences for the human body. There are a number of researches that consider harmful substances' effects on public health and the environment.

In [2], [3], the health risks linked to atmospheric air pollution levels in twenty-six cities across Kazakhstan were examined. Ust-Kamenogorsk, Almaty, and Balkhash were found to have a particularly high risk of chronic health effects from heavy metal exposure. Elevated concentrations of heavy metals such as Ba, Mn, Pb, V, and Zn were detected in the blood of residents in Aksu and Ust-Kamenogorsk, likely attributable to the operations of metallurgical industries.

Bekbossynova *et al.* [4] explores public attitudes toward air pollution in Almaty, Kazakhstan, by employing a healthy lifestyle model (HLM) to gauge citizen reactions. The initial survey revealed that, although people are aware of the severe air pollution issues, they often underestimate the associated health risks. This underestimation leads to behaviors that worsen the impact of pollution in their daily lives. The research underscores the need for targeted interventions to align awareness with perception, which could inform policies, public health initiatives, and urban planning efforts aimed at mitigating the detrimental impacts of air pollution on health.

Temirbekov *et al.* [5] details a study conducted in Almaty to assess the city's morbidity rates and their correlation with air pollution levels. Machine learning algorithms were utilized to analyze the connection between air pollution and public health, with a specific focus on respiratory diseases. The research covered the period from 2017 to 2022, examining respiratory conditions and pollution levels of particulate matter. It offers recommendations for reducing harmful emissions through machine learning techniques, highlighting the significant impact of air pollution on respiratory health.

Kerimray *et al.* [6] explores the patterns and health impacts associated with significant urban air pollutants in Kazakhstan, utilizing data from national air quality monitoring systems. Using the global exposure mortality model (GEMM), the research estimates the increase in mortality rates connected to PM_{2.5} exposure and the rates of illness related to PM₁₀ exposure. The results indicate elevated pollutant levels in cities across Kazakhstan, with an estimated 8,134 adult deaths per year attributed to PM_{2.5} exposure, highlighting the critical health risks posed by air pollution in the nation.

Globally, issue of harmful substance dispersion is actively researched. International studies focus on assessing air, water, and soil pollution levels and identifying major emission sources. These studies aim to develop effective pollution management and reduction strategies, thereby enhancing life quality and public health. Several methods are available for determining air pollution levels in urban areas, including atmospheric monitoring systems, satellite observations, mathematical modeling, and artificial intelligence techniques. Mathematical modeling and AI offer advantages in economic efficiency and financial feasibility, significantly reducing the costs of air quality monitoring while ensuring high accuracy and predictive capabilities.

Dutta *et al.* [7] introduces the RSA algorithm within a hybrid deep learning model for air pollution monitoring (OSSO HDLAPM). Angelena *et al.* [8] discusses the evaluation and prediction of PM₁₀ air quality in the Chennai District using soft computing methods, specifically neural networks (NN), to address nonlinear problems. The study employs a wavelet approach with energy spectrograms to estimate PM₁₀ levels in specific areas and utilizes a multilayer NN direct propagation algorithm for air quality prediction. The results show the effectiveness of NN in forecasting short-term nonlinear parameters related to air pollution, offering insights into existing issues and suggesting necessary control measures.

Haq [9] involves the development of five machine learning models for air pollution classification, with a focus on efficient data preprocessing and precise hyperparameter optimization. Innovative approaches, such as machine learning, artificial intelligence, the internet of things, and mathematical modeling, are increasingly used to analyze large datasets, predict pollutant dispersion, and assess the impact of pollutants on public health and ecosystems.

Mathematical modeling is essential for assessing the effects of various factors on air quality and predicting how it might change under different conditions. It enables the development of intricate models that account for multiple variables, such as geographic features, traffic patterns, industrial activities, and weather conditions [10], [11]. These models help researchers evaluate the impact of different factors on air quality in specific regions or cities and forecast future air quality changes under various scenarios [12], [13]. This capability is crucial for making informed decisions about environmental protection.

Oralbekova *et al.* [14] explores the use of a data assimilation algorithm to address air pollution issues in an industrial region, focusing on Karaganda, Kazakhstan, known for its high levels of atmospheric pollution. The study details the creation and application of an algorithm that effectively monitors air quality in the city, offering practical insights for real-time assessment and identification of environmental risk areas in industrial settings.

Aidosov *et al.* [15] applies mathematical models to examine the impact of oil and gas production, particularly from fires, on air quality. It emphasizes emissions from burning oil and gas, including compounds such as carbon, hydrogen, sulfur, nitrogen, and oxygen. Despite the challenges, pollution from well fires typically remains within acceptable limits for Class 1 hazard zones.

Assanov *et al.* [16] investigates air quality issues in the industrial city of Ust-Kamenogorsk, Kazakhstan, focusing on identifying pollution sources and their spatial and temporal distribution. By analyzing retrospective data from five monitoring stations between 2017 and 2021, this study employs multidimensional statistical techniques and hierarchical cluster analysis. The findings reveal distinct pollution patterns based on seasonal variations, with nitrogen dioxide (NO₂) and sulfur dioxide (SO₂) levels frequently exceeding safe limits. This indicates a significant impact of industrial activities, such as coal burning and heavy metallurgy, on the city's air quality. Smirnova *et al.* [17] presents a mathematical model designed to assess the effects of road traffic and airflow dynamics within city tunnels. The model effectively simulates traffic behavior and exhaust emissions, providing valuable insights for urban transportation planning. It suggests strategies for optimizing traffic flow to reduce stop-and-go conditions in tunnels and recommends aligning ventilation systems with traffic patterns to reduce pollution effects.

Madiyarov *et al.* [18] introduces a novel approach for predicting the spatial distribution of airborne pollutants using a combination of machine learning algorithms and parameter estimation methods. The study evaluates three machine learning techniques and compares them with measurement data, demonstrating their potential to solve problems associated with limited data on pollution sources. Testing of the method in two Kazakh cities confirms its effectiveness. Temirbekov *et al.* [19] explores the impact of atmospheric pollution on industrial cities in Kazakhstan, with a primary focus on Almaty. The study analyzes data from 20 automated monitoring stations (AMS) and chemical tests of meltwater samples, revealing the negative environmental impacts of pollution. A machine learning model was developed to correlate bioexperiment results with chemical analytical data, showing a strong relationship between them. Assanov *et al.* [20] investigates the patterns of industrial emissions in Kazakhstan and their effects on air pollution levels in major industrial cities. By reviewing emission limit data from permits, the research reveals high pollution levels in these urban areas and notes an increase in emission limits at several facilities in 2019.

Currently, mathematical models are available for analyzing atmospheric processes, typically falling into two main categories of tasks. The first category involves solving “direct” problems, where the goal is to determine the concentration distribution of pollutants based on known source characteristics and surface air layer parameters. The second category addresses “inverse” problems, where the aim is to deduce the type, location, and intensity of pollution sources based on measured pollutant concentrations at various observation points and the prevailing meteorological conditions [21], [22].

Addressing environmental issues presents challenges that extend beyond data analysis to include identifying the impact zones of pollution sources across different regions. The theory of inverse problems, particularly through the application of conjugate equations, plays a vital role in solving these applied problems [23], [24]. Initially used to estimate particle values in nuclear reactor simulations, conjugate equations were later adapted by Marchuk [25] to tackle specific atmospheric dynamics challenges. This method helps pinpoint critical integration areas that significantly influence the studied regions [26].

Kochergin and Kochergin [27] explores the inverse problem of reconstructing pollution sources for the convective diffusion equation with constant coefficients within a rectangular domain. The research presents algorithms based on Tikhonov regularization to address this issue, accompanied by comprehensive numerical analysis. Aloyan and Arutyunyan [28] addresses two key environmental issues: the application of control theory and optimization techniques to minimize environmental damage, and the numerical simulation of atmospheric gases and aerosols. The study uses a 3D mesoscale hydrodynamic model to compute wind fields and turbulence parameters, which are essential for understanding the behavior and transformation of pollutants in the atmosphere. Penenko *et al.* [29] evaluates two approaches to solving the inverse problem of atmospheric chemical composition modeling: one based on pre-existing measurements and another utilizing data produced through the modeling process. The computational experiments reveal that simulation-generated data can offer better results, even in cases where initial measurements are sparse.

This study investigates the distribution of pollutants in the air across nine industrial cities in Kazakhstan: Almaty, Astana, Pavlodar, Ust-Kamenogorsk, Ekibastuz, Karaganda, Zhezkazgan, Temirtau, and Petropavlovsk. It identifies the primary pollution sources for each city and models the distribution of carbon monoxide (CO), SO₂, and NO₂. The analysis assesses the spread of these harmful substances in each city. Additionally, the paper explores the inverse problem of data assimilation from AMS to refine data from sources. The method of conjugate equations is applied to solve this problem, with results demonstrating convergence to an exact solution. Visualizations for Almaty, Ust-Kamenogorsk, and Pavlodar are included.

2. METHODS

2.1. Problem formulation

To model the distribution of harmful substances in industrial cities, the transport equation is examined by reducing the initial city area to a dimensionless region. In computational domain with dimensionless sizes $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq t \leq T$ we will consider following problem:

$$\frac{\partial \varphi_q}{\partial t} + u \frac{\partial \varphi_q}{\partial x} + v \frac{\partial \varphi_q}{\partial y} = \frac{\partial}{\partial x} \left(\frac{\partial \varphi_q}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\partial \varphi_q}{\partial y} \right) + f_q \tag{1}$$

$$\varphi_q(x, y, 0) = \varphi_0(x, y) \tag{2}$$

$$\varphi_q(0, y, t) = 0, \quad \varphi_q(1, y, t) = 0, \tag{3}$$

$$\varphi_q(x, 0, t) = 0, \quad \varphi_q(x, 1, t) = 0, \tag{4}$$

where u, v are the wind velocity components, φ_q is the amounts of pollutants and the output rates of contamination sources are specified in the following way:

$$f_q = \sum_{j=1}^m Q_j \delta(\vec{r} - \vec{r}_j), \tag{5}$$

where r_j is the radius vector representing locations of pollution point sources, $\delta(x)$ is the Dirac delta function, Q_j is power of point sources, m is the number of point sources. The boundary conditions are set to zero, reflecting the assumption that pollution is confined within the city limits and does not extend beyond them.

To tackle the problem, the initial approximation is represented by a Gaussian distribution.

$$\varphi_0(x, y) = \frac{Q_j}{2\sqrt{\mu\sigma}} e^{-\sqrt{\frac{\sigma}{\mu}}((x-x_j)^2+(y-y_j)^2)} \tag{6}$$

Let's examine the direct problem (1)-(4) in its discretized form. We will create a grid $\omega_{h,\tau}$ within the specified domain using a step size $h = 1/N, \tau = T/N_t$, where N, N_t are positive integers.

Then we will write the corresponding direct difference problem in the following area $\omega_{h,\tau} = \{x_i = ih, y_j = jh, t_n = n\tau; i, j = \overline{0, N}, n = \overline{0, N_t}\}$. Thus, problem (1)-(4) has following form:

$$\begin{aligned} & \frac{1}{h_0} \frac{\varphi_{ij}^{n+1} - \varphi_{ij}^n}{\tau} + \frac{1}{2} \left((u + |u|) \frac{\varphi_{ij}^n - \varphi_{i-1j}^n}{h_1} + (u - |u|) \frac{\varphi_{i+1j}^n - \varphi_{ij}^n}{h_1} \right) + \frac{1}{2} \left((v + |v|) \frac{\varphi_{ij}^n - \varphi_{ij-1}^n}{h_2} + (v - |v|) \frac{\varphi_{ij+1}^n - \varphi_{ij}^n}{h_2} \right) \\ & = \frac{L}{U^*} \alpha_q \varphi_{ij}^n + \frac{L}{U^* \varphi_q^*} (f_{ij}^n + \beta_q) + \frac{1}{A} (\varphi_{xx,ij}^n + \varphi_{yy,ij}^n) \end{aligned} \tag{7}$$

with homogeneous first kind boundary conditions,

$$\varphi_{0,j}^n = \varphi_{i,0}^n = \varphi_{0,N_x}^n = \varphi_{0,N_y}^n = 0,$$

and the initial condition,

$$\varphi_{ij}^0 = \varphi_{0i,j}$$

2.2. The inverse problem of data assimilation from AMS

When modeling the dispersion of harmful substances from point sources, inaccuracies or a lack of data on total emissions from industrial sources can pose significant challenges. Additionally, the data may be affected by changes occurring during photochemical reactions or the transformation of pollutants. In relation to this issue, the study addresses the inverse problem associated with the model of pollutant transport in the atmospheric air of an industrial city. To achieve a more accurate depiction of pollution, an algorithm was employed that integrates the pollutant transport model with data from AMS. The transport equation is analyzed, incorporating the processes of photochemical transformation and the formation of harmful pollutants.

$$\frac{\partial \varphi_q}{\partial t} + u \frac{\partial \varphi_q}{\partial x} + v \frac{\partial \varphi_q}{\partial y} = \Delta \varphi_q + \alpha_q \varphi_q + \beta_q + f_q \quad (8)$$

with initial boundary conditions (2)-(4).

To solve this problem, supplementary data on pollutant levels from AMS are used, and this data is formatted in the following form:

$$z_q = \sum_{i=1}^n g_i \delta(\vec{r} - \vec{r}_i),$$

where \vec{r}_i is the vector representing the position of the AMS, g_i is the pollutant levels recorded by the AMS, with n representing the total number of AMS.

The provided mathematical model integrates the dynamics of atmospheric processes and the transport of multi-component gas pollutants, including their transformations. As a result, (1) account for photochemical transformations and the development of harmful pollutants, as outlined in [12]. The behavior of chemical reactions is outlined by kinetic equations that adhere to the principles of mass conservation and particle count, with α_q , β_q serving as parameters for each chemical component q . To solve the inverse problem, the conjugate equations method is applied [29], [30]. This approach results in a gradient-driven iterative process aimed at improving emission source estimates [31].

Let's investigate the inverse problem of determining amount of pollution from source based on data from the monitoring system. The fundamental task is to minimize the Lagrange functional associated with this problem:

$$L(f_q) = \int_0^T dt \int_{\Omega} \left[\frac{\partial \varphi_q}{\partial t} + u \frac{\partial \varphi_q}{\partial x} + v \frac{\partial \varphi_q}{\partial y} - \Delta \varphi_q - \alpha_q \varphi_q - \beta_q - f_q \right] \varphi^* d\Omega + \sum_{i=1}^n \lambda_i \int_0^T dt \int_{\Omega} (z_q - \varphi_q)^2 \delta(\vec{r} - \vec{r}_i) d\Omega, \quad (9)$$

where λ_i is the preference coefficient.

If φ_q is the solution of the direct problem described by (1)-(4), then exists a unique solution φ^* that fulfills the conjugate (10) along with conditions (11)-(13).

$$\frac{\partial \varphi^*}{\partial t} + u \frac{\partial \varphi^*}{\partial x} + v \frac{\partial \varphi^*}{\partial y} = -\Delta \varphi^* - \alpha_q \varphi^* - \beta_q + 2 \sum_{i=1}^n \lambda_i (z_q - \varphi_q) \delta(\vec{r} - \vec{r}_i) \quad (10)$$

$$\varphi^*(x, y, T) = 0 \quad (11)$$

$$\varphi^*(0, y, t) = 0, \quad \varphi^*(1, y, t) = 0, \quad (12)$$

$$\varphi^*(x, 0, t) = 0, \quad \varphi^*(x, 1, t) = 0, \quad (13)$$

The algorithm for solving the inverse problem of assimilation of AMS data is described by following way:

1. Selecting the initial approximation f_q^0 .
2. Given f_q^n , numerically solving the direct problem (1)-(4) to obtain $\varphi_q(x, y, t; f_q^n)$.
3. Minimization of the functional $L(f_q^n)$ using formula (9).
4. Solving the conjugate problem (10)-(13), if the value of the functional is not sufficiently small.
5. Computing the gradient of the functional as $L'f_q^n = \varphi^*(x, y, t; f_q^n)$.
6. Updating the approximation using the formula $f_q^{n+1} = f_q^n - \xi \cdot L'f_q^n$.

3. NUMERICAL RESULTS AND DISCUSSION

Data on gross emissions of industrial enterprises in cities were taken from the article [20]. The numerical computations were performed on the interactive Jupiter Notebook platform. Key system parameters and values used for generating the numerical outcomes are listed in Table 1. Figures 1-9 provide visual representations of CO, SO₂, and NO₂ distributions across the cities of Almaty, Astana, Pavlodar, Ust-Kamenogorsk, Ekibastuz, Karaganda, Zhezkazgan, Temirtau, and Petropavlovsk.

Table 1. Numerical data of the parameters used in the calculations

Parameter	Value
u - component of wind speed	$u = 5 \text{ m/s}$
v - component of wind velocity	$v = 5 \text{ m/s}$
μ - shear parameter	30
σ - scale parameter	2000
L - length scale	$L = 35000 \text{ m}$
T - time scale	$T = 3600 \text{ s}$
U^* - speed scale	$U^* = 10 \text{ m/s}$

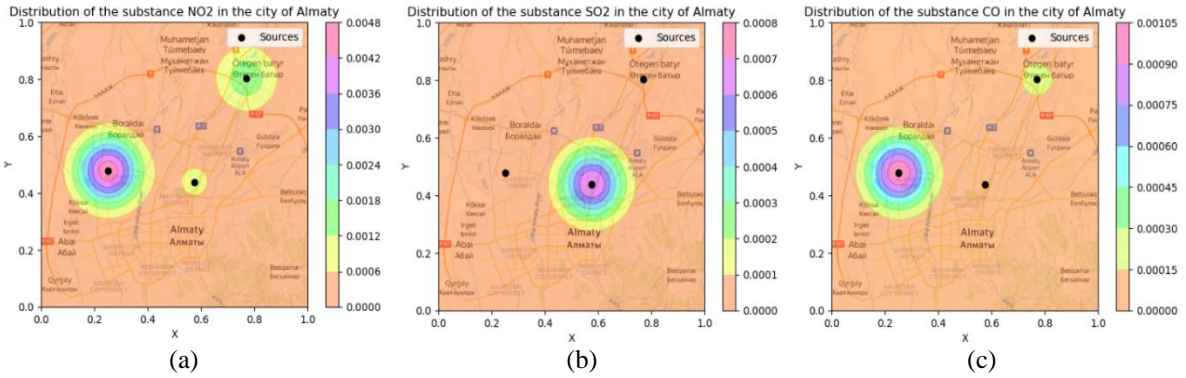


Figure 1. Distribution of harmful substances for the Almaty city: (a) NO₂, (b) SO₂, and (c) CO

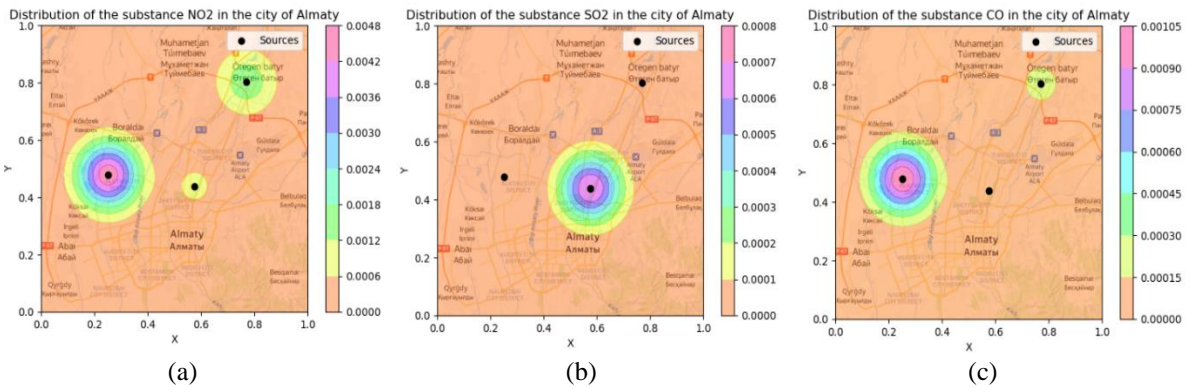


Figure 2. Distribution of harmful substances for Pavlodar city: (a) NO₂, (b) SO₂, and (c) CO

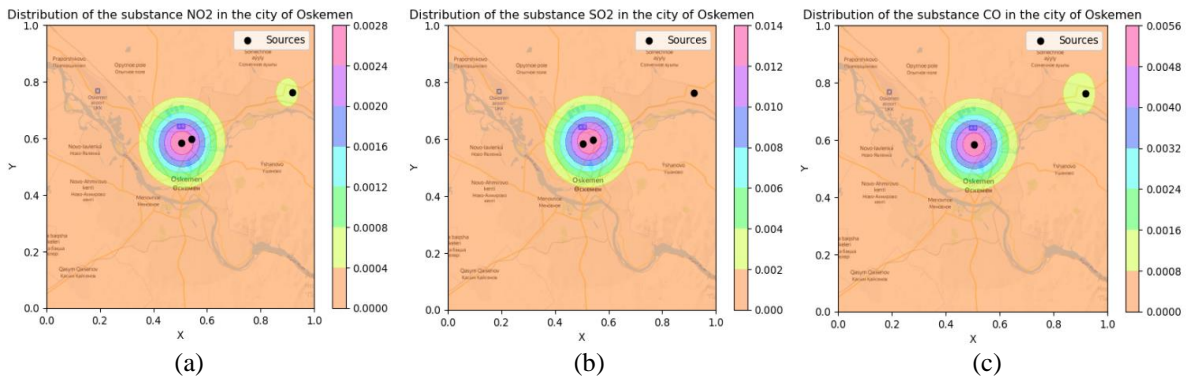


Figure 3. Distribution of harmful substances for Ust-Kamenogorsk city: (a) NO₂, (b) SO₂, and (c) CO

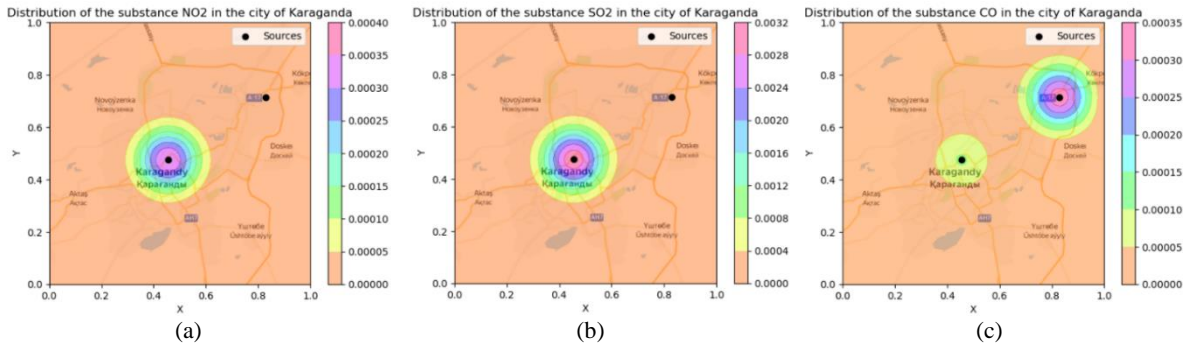


Figure 4. Distribution of harmful substances for Karagandy city: (a) NO₂, (b) SO₂, and (c) CO

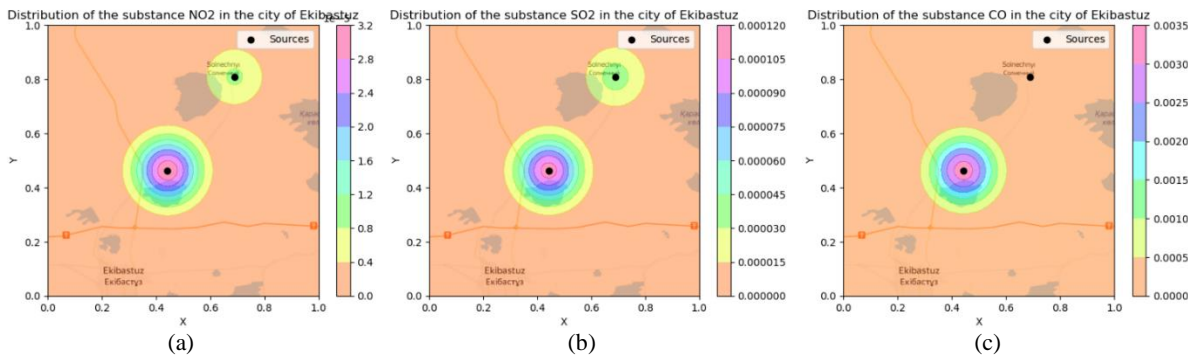


Figure 5. Distribution of harmful substances for Ekibastuz city: (a) NO₂, (b) SO₂, and (c) CO

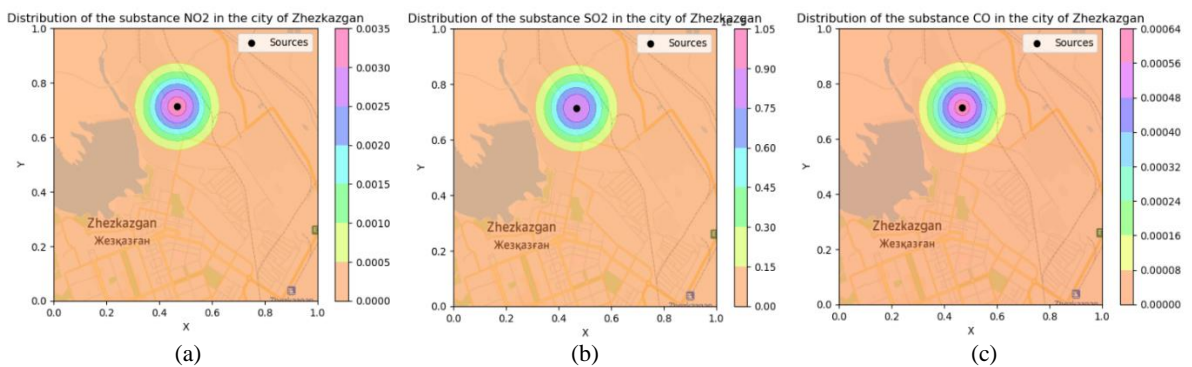


Figure 6. Distribution of harmful substances for Zhezkazgan city: (a) NO₂, (b) SO₂, and (c) CO

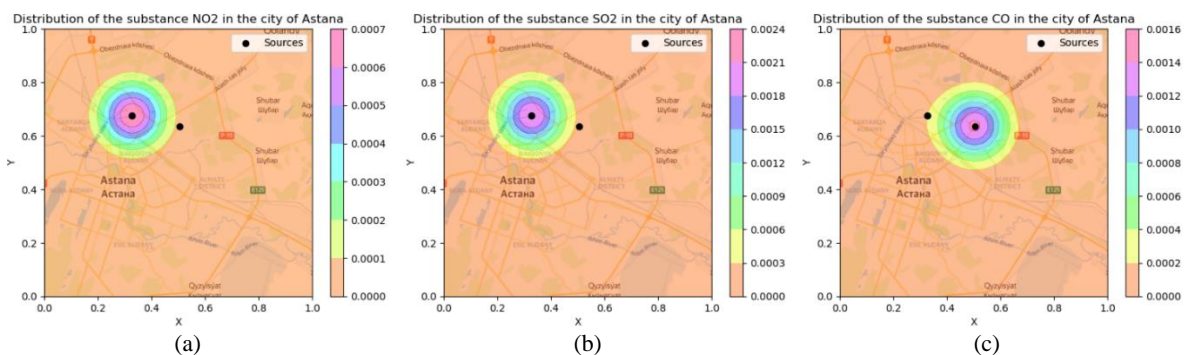


Figure 7. Distribution of harmful substances for Astana city: (a) NO₂, (b) SO₂, and (c) CO

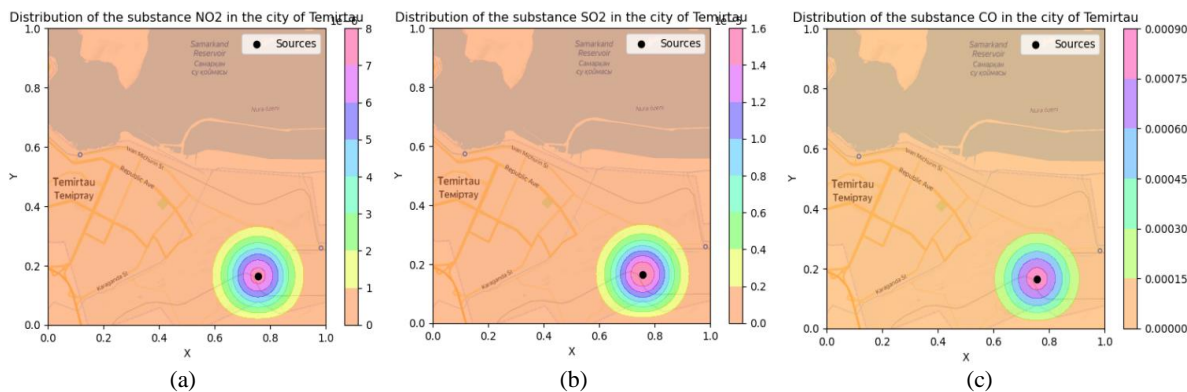


Figure 8. Distribution of harmful substances for Temirtau city: (a) NO₂, (b) SO₂, and (c) CO

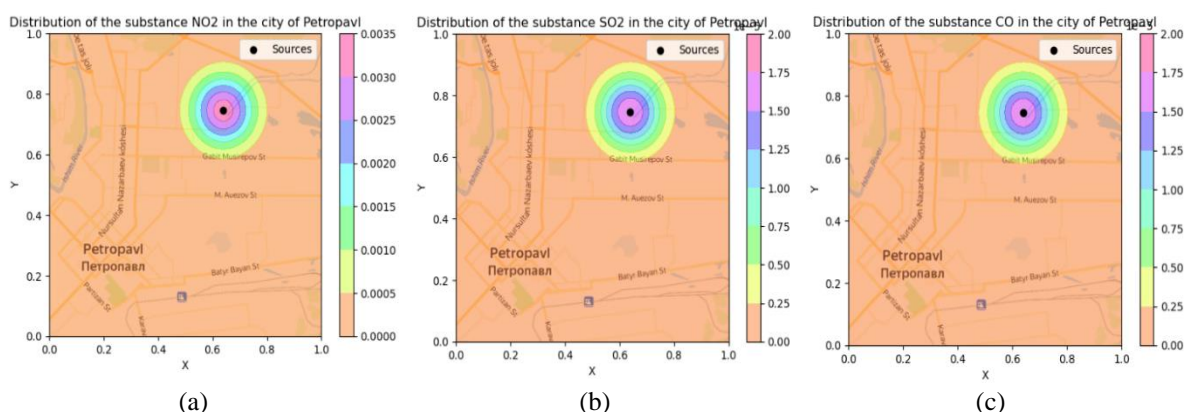


Figure 9. Distribution of harmful substances for Petropavlovsk city: (a) NO₂, (b) SO₂, and (c) CO

There is no information on the gross emissions of some substances from point sources, so there was conducted a simulation of the distribution of known substances from these sources. Figure 1 provides a detailed depiction of the dispersion of pollutants in Almaty, stemming from three primary pollution sources: JSC TPP-1, JSC TPP-2, and JSC TPP-3. The graph shows NO₂ emissions from all three sources, with JSC TPP-1 being the largest emitter of SO₂. This suggests that JSC TPP-1 is the major contributor to hydrogen sulfide pollution in Almaty. JSC TPP-2, on the other hand, is responsible for releasing significant quantities of both NO₂ and CO, likely due to increased reliance on hydrocarbon fuels at this plant.

In Pavlodar, according to the analysis in Figure 2, emissions of NO₂ and CO primarily originate from TPP-1 and TPP-3, indicating their substantial impact on the city’s air quality. In contrast, TPP-2 stands out as the main emitter of SO₂. Figure 3 illustrates pollutant spread in Ust-Kamenogorsk, with Kazzinc LLP and Ust-Kamenogorsk CHP contributing to the distribution of NO₂, SO₂, and CO. Although the Sogrinskaya CHP plays a smaller role, it still contributes to NO₂ and CO emissions.

The atmospheric conditions in Karaganda are shown in Figure 4, where TPP-3 emerges as the dominant source of CO emissions. TPP-1 plays a significant role in releasing NO₂ and SO₂, although its CO emissions are notably lower than those of TPP-3. In Ekibastuz, as seen in Figure 5, JSC Ekibastuz GRES-1 is identified as the main polluting entity, with JSC Ekibastuz GRES-2 also contributing to NO₂ and SO₂ emissions, albeit in smaller amounts. Figure 6 focuses on Zhezkazgan, revealing that TPP-2 is a significant source of air pollutants in the region.

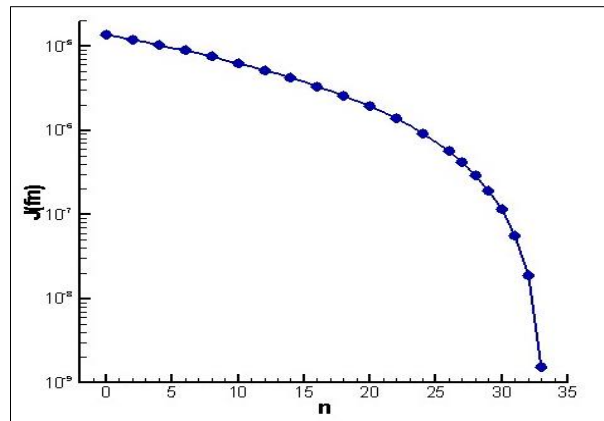
Lastly, in Astana, Figure 7 highlights TPP-2 as the principal emitter of CO, while NO₂ and SO₂ emissions are largely associated with TPP-1. In Temirtau and Petropavlovsk, the graphs reflect, respectively, the spread of harmful substances from TPP-2 for each city (Figures 8 and 9). Comparing the results of this study with previous ones [10], it can be noted that the previously identified main sources of pollution in these cities are confirmed, but this study provides a more detailed picture of the distribution of harmful substances. The strength of the study is the use of simulation, which compensates for the lack of data on gross emissions, although dependence on the accuracy of the model remains a weak point. Questions remain open about the long-term effects of pollution on public health and ecosystems, as well as the impact of seasonal and climatic

changes on the spread of harmful substances. Future research may focus on improving simulation models, collecting more detailed emission data, and examining the interaction of various factors affecting air pollution.

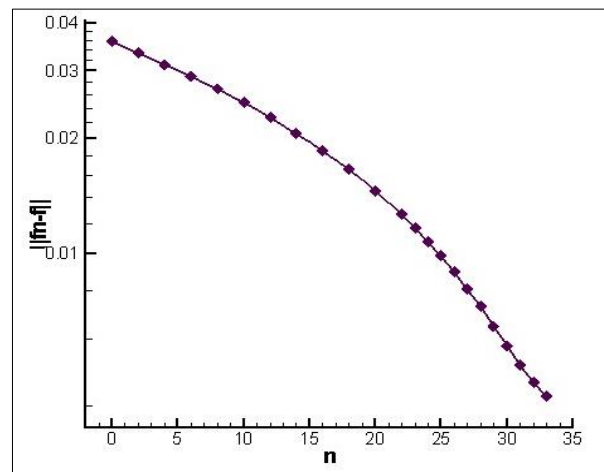
3.1. Numerical results and calculations for the task of data assimilation

For solving the inverse problem, a starting estimate based on a Gaussian distribution was utilized, as specified in (6). The numerical values used were obtained from Table 1. Emission data were taken as in [32]. Air quality data is provided by Ecoservice-S LLP, which updates every 20 minutes via an API connected to AMS. This information is processed in real-time and stored on a server at Akademset LLP's data processing center.

For the computational experiment on pollutant dispersion, chemical compounds CO, SO₂, and NO₂ were selected. These substances were selected because they have a significant environmental impact and their concentrations are continuously monitored by AMS in cities of Ust-Kamenogorsk, Almaty, and Pavlodar. The outcomes of computational analysis for the inverse problem of source reconstruction demonstrate effective convergence to an accurate solution. The value of the functional reduces to approximately 10⁻⁹ after 33 iterations (Figure 10). Figure 6 shows that the measure of deviation between the true and estimated functions drops below 10⁻², indicating that a minimum has been achieved. Figures 11 display the distribution of CO emissions from three different sources, with the metrics retrieved from AMS data for the cities of Almaty in Figure 11(a), Ust-Kamenogorsk in Figure 11(b), and Pavlodar in Figure 11(c).



(a)



(b)

Figure 10. Graphs (a) of the functional $L(f^n)$ and (b) of the measure of deviation between the initial function and the reconstructed function $\|f - f^n\|$

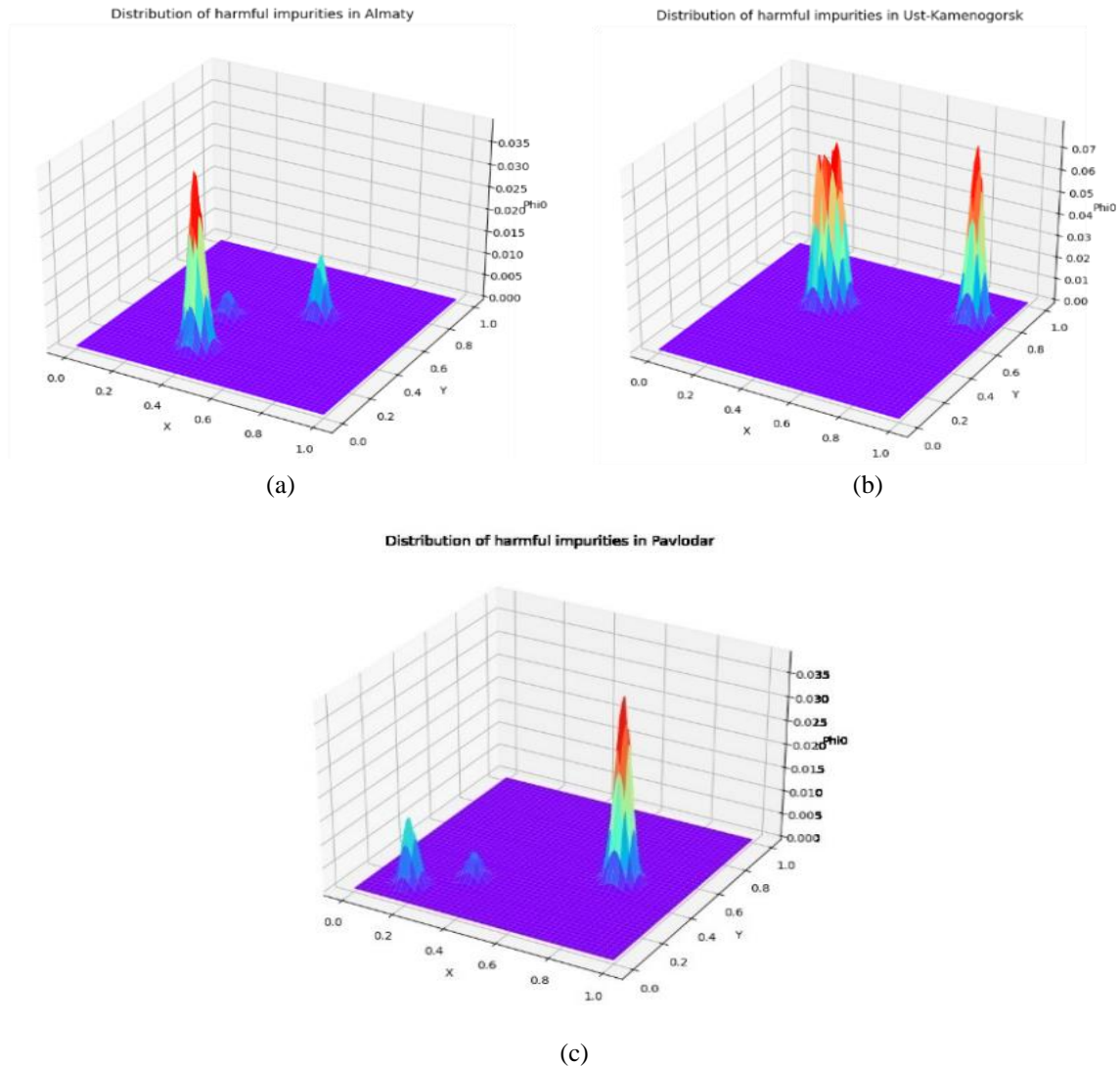


Figure 11. Distribution of CO from emission sources, taking into account AMS data in 3D format for the cities: (a) Almaty, (b) Ust-Kamenogorsk, and (c) Pavlodar

4. CONCLUSION

This study offers a thorough evaluation of the dispersion of harmful substances in the atmosphere across nine industrial cities in Kazakhstan: Almaty, Astana, Pavlodar, Ust-Kamenogorsk, Ekibastuz, Karaganda, Zhezkazgan, Temirtau, and Petropavlovsk. It identifies the main sources of air pollution in each city and focuses on key pollutants typical of industrial areas, such as CO, SO₂, and NO₂. The simulation results provide an understanding of how these pollutants spread in the environment of each city.

Additionally, the research tackles the inverse problem of refining pollution source information by assimilating data from AMS. The method of conjugate equations was employed to solve this problem, resulting in convergence to a precise solution. The paper details the implementation algorithm used for this process and presents visualizations for Almaty, Ust-Kamenogorsk, and Pavlodar, showing the dynamics of pollutant distribution. The integration of AMS data improved the accuracy of the modeling parameters, which is crucial for enhancing result precision. The findings are visually represented through graphs that depict pollution distribution patterns and highlight areas with elevated pollutant levels. This method contributes to a deeper understanding of atmospheric pollution and supports the development of strategies to reduce the impact of emissions on both the environment and public health.

In summary, further research is needed to explore the long-term effects of pollution on health and ecosystems, as well as the influence of seasonal and climatic changes on pollutant dispersion. Future studies should aim to refine simulation models, gather more comprehensive emission data, and investigate the interactions of various factors affecting air quality to better manage atmospheric conditions in urban environments.

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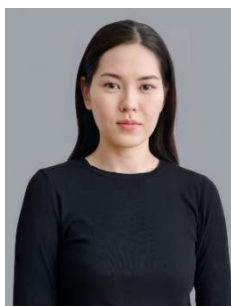
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