Finite-Time Stabilization of Networked Control Systems with Packet Dropout

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Abstract

The problem of finite-time stabilization for networked control systems with both sensor-to-controller and controller-to-actuator packet dropouts is investigated in this paper. By using the iterative approach, the NCSs with bounded packet dropout is modeled as switched linear systems. Sufficient conditions for finite-time stabilization of the underlying systems are derived via linear matrix inequalities (LMIs). Lastly, an illustrative example is given to demonstrate the effectiveness of the proposed results.

Keywords: networked control systems, packet dropout, finite-time stability, LMIs

1. Introduction

Networked control systems (NCSs) are feedback control systems with network channels used for the communications. Compared with the traditional point-to-point wiring, the use of the communication channels can reduce the costs of cables and power, simplify the installation and maintenance of the whole system, and increase the reliability. The NCSs have many industrial applications in automobiles, manufacturing plants, aircrafts, and HVAC systems [1]. However, the insertion of the communication network in feedback control loop makes the analysis and design of an NCS complicated because it introduces some problems existing in the network into control systems such as limited communication bandwidth, network-induced delay, packets disorder and packets loss which often happen inevitably during information transmission see the references [2-8]and the references cited therein.

Among a number of issues arising from such a framework, packet loss of NCSs is an important issue to be addressed and has been receiving great attentions. For instance, Xiong and Lam [9] studied the problem of stability and stabilization of linear systems over networks with bounded packet loss. Bakule and De La Sen [10] tackled the problem of decentralized stabilization of networked complex composite systems with nonlinear perturbations. Wang and Yang [11] investigated the problem of state-feedback control synthesis for networked control systems with packet dropout. Sun and Qin [12] studied NCSs with both sensor-to-controller and controller-to-actuator packet dropouts via switched system approach. For more details of the literature related to networked problems with packet dropout, the reader is referred to [13-18] and the references therein.

It is worth pointing out most of existing literature relate to stability and performance criteria defined over an infinite-time interval. However, the main attention in many practical applications is the behavior of the dynamical systems over a fixed finite-time; for example, large values of the state are not acceptable in the presence of saturations [19, 20]. In this sense it appears reasonable to define as stable a system whose state, given some initial conditions, remains within prescribed bounds in the fixed finite-time interval. For this purposes finite-time stable (FTS) could be used [21, 22]. Recently, Amato *et al.* extends this concept to finite-time boundedness in [23]. To date, with the aid of linear matrix inequalities (LMIs) formulation, more results of finite-time stability and stabilization of various systems. For more details of the literature related to finite-time stability, the reader is referred to [24-28], and the references therein.

However, to the best of our knowledge, the finite-time stability and stabilization problems for NCSs with packet dropout have not been fully investigated to date. Especially, for the case where both sensor to-controller and controller-to-actuator packet dropouts are considered simultaneously, very few results related to NCSs are available in the existing literature, which motivates the study of this paper.

In this paper, the finite-time stabilization problems of a class of NCSs with bounded packet dropout is studied. Firstly, we model the NCSs with bounded packet dropout as switched linear systems. Then, the concepts of the finite-time stability (FTS) and problem formulation are given. The main contribution of this paper is to design a state-feedback controller which guarantees the resulting the resulting closed-loop discrete-time system uniform finite-time stable.

In the sequel, the following notation will be used: The symbols R^n and $R^{n \times m}$ stand for an n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively, A^T and A^{-1} denote the matrix transpose and matrix inverse, $\mathrm{diag}\{A, B\}$ represents the block-diagonal matrix of A and B, $P > 0$ stands for a positive-definite matrix, I is the unit matrix with appropriate dimensions, and $Z_+ = \{1, 2, \cdots\}$.

2. Problem Formulation and Preliminaries

The framework of NCSs considered in the paper is depicted in Figure 1. The process to be controlled is modeled by a linear discrete-time system.

$$x(k+1) = Ax(k) + Bu(k) \qquad (1)$$

Where $k \in Z_+$ is the time step, $x(k) \in R^n$ and $u(k) \in R^m$ are is system state and control input, respectively. A and B are known real constant matrices with appropriate dimensions.
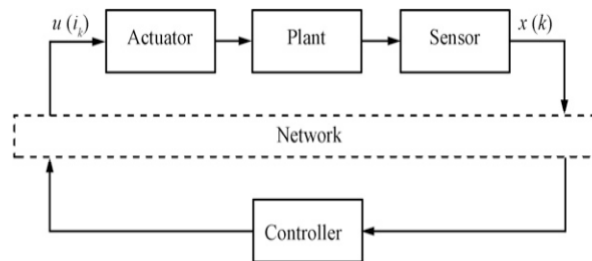

Figure 1. Illustration of NCSs over Communication Network

We make the following assumptions about the NCS:
1) Networks exist between sensor and controller, and between controller and actuator;
2) The sensor is clock driven; the controller and the actuator are event driven;
3) The data are transmitted in a single packet at each time step.

Let $\ell = \{i_1, i_2, \cdots\}$, which a subsequence is of $Z_+ = \{1, 2, \cdots\}$, denote the sequence of time points of successful data transmission from the sampler to the zero-order hold, and $s = \max_{i_k \in \ell}(i_{k+1} - i_k)$ be the maximum packet-loss upper bound. Then the following concept and mathematical models are introduced to capture the nature of packet losses.

The state feedback controller law is:

$$u(k) = Kx(k) \qquad (2)$$

Where $K \in R^{m \times n}$ is to be designed. From the viewpoint of the zero-order hold, the control input is:

$$u(l) = u(i_k) = Kx(i_k)$$

For $i_k \leq l \leq i_{k+1} - 1$. The initial inputs are set to zeros: $u(l) = 0, \ 0 \leq l \leq i_1 - 1$. Hence the closed-loop system becomes:

$$x(l+1) = Ax(l) + BKx(i_k), \quad i_k \leq l \leq i_{k+1} - 1 \tag{3}$$

From the closed-loop system (3), we can obtain:

$$x(i_{k+1}) = \left(A^{i_{k+1} - i_k} + \sum_{r=0}^{i_{k+1} - i_k - 1} A^r BK \right) x(i_k), \ i_k \in \ell \tag{4}$$

Define the packet dropout process as follows:

$$\eta(i_k) = i_{k+1} - i_k \tag{5}$$

Which takes values in the finite state space $\ell = \{1, 2, \cdots, s\}$.

Let,

$$\begin{cases} z(k) = x(i_k) \\ z(k+1) = x(i_{k+1}) \\ \overline{A}_{i(k)} = A^{i_{k+1} - i_k} \\ \overline{B}_{i(k)} = \sum_{r=0}^{i_{k+1} - i_k - 1} A^r B \end{cases} \tag{6}$$

It is easily seen that the closed-loop system (4) can be described by the follow in switched system.

$$z(k+1) = (\overline{A}_{i(k)} + \overline{B}_{i(k)} K) z(k) \tag{7}$$

Where $i(k) = i_{k+1} - i_k$ is arbitrarily switching signal.

For simplicity, at any arbitrary discrete time $k \in Z_+$, the switching signal $i(k)$ is denoted by i. Then, the closed-loop system (7) can be rewritten as:

$$z(k+1) = (\overline{A}_i + \overline{B}_i K) z(k) \tag{8}$$

The general idea of finite-time stability concerns the boundedness of the state of a system over a finite-time interval for the given initial conditions; this concept can be formalized through the following definition, which is an extension to discrete-time systems of the one given in [14].

Definition 1: (Finite-time stability (FTS)). The discrete-time switched system

$$x(k+1) = A_i x(k), \ k \in Z_+$$

Is said to be FTS with respect to (c_1, c_2, N, R) where $0 < c_1 < c_2, R$ is a symmetric positive-

definite matrix and $N \in Z_{k \geq 0}$, if:

$$x^T(0)Rx(0) \leq c_1 \Rightarrow x^T(k)Rx(k) \leq c_2, \forall k = \{1, 2, \cdots, N\}$$

The following problem will be dressed in this paper.

Problem 1. For the discrete-time system (1), we find a networked state feedback controller (5) such that the closed-loop system is FTS with respect to (c_1, c_2, N, R).

We next provide a lemma which will play an important role in the late development.

Lemma 1. The LMI

$$\begin{bmatrix} Y(x) & W(x) \\ W^T(x) & R(x) \end{bmatrix} > 0$$

Is equivalent to:

$$R(x) > 0, \quad W(x)R^{-1}(x)W^T(x) > 0$$

Where $Y(x) = Y^T(x), \ R(x) = R^T(x)$ and $W(x)$ depend on x.

3. Main Results

In this section, we will develop the stabilization results for the closed-loop NCS (8). The following theorem presents a sufficient condition for the finite-time stability of the considered system with arbitrary packet-loss process.

Theorem 1. The closed-loop NCS (8) with arbitrary packet-loss process is FTS with respect to (c_1, c_2, N, R) if, there exist positive definite matrix $S \in R^{n \times n}$ and scalar $\gamma \geq 1$ such that the following matrix inequalities hold:

$$\begin{bmatrix} S & A_i S + B_i X \\ * & -\gamma S \end{bmatrix} < 0 \tag{9}$$

And,

$$\frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)} \gamma^N c_1 < c_2 \tag{10}$$

Where $Q = R^{-1/2} S^{-1} R^{-1/2}$ and $\lambda_{\max}(\Box)$ and $\lambda_{\min}(\Box)$ indicate the maximum and minimum eigenvalue of the augment, respectively. Then state feedback controller is given by $K = XS^{-1}$.

Proof. Choose a Lyapunov functional candidate for the system (8) as follows:

$$V(k) = x^T(k)S^{-1}x(k) \tag{11}$$

Then, along the solution of system (8) we have:

$$\begin{aligned} V(k+1) &= x^T(k+1)S^{-1}x(k+1) \\ &= x^T(k)(\overline{A}_i + \overline{B}_i K)^T S^{-1}(\overline{A}_i + \overline{B}_i K)x(k) \end{aligned} \tag{12}$$

Substituting $K = XS^{-1}$ into (11) and pre- and post-multiplying by $\mathrm{diag}\{S^{-1}, \ S^{-1}\}$, we can obtain the equivalent condition.

$$\begin{bmatrix} -S^{-1} & S^{-1}A_i + S^{-1}B_iXS^{-1} \\ * & -\gamma S^{-1} \end{bmatrix} < 0 \tag{13}$$

From Lemma 1, (12) and (13), it follows that:

$$V(k) < \gamma V(k) \tag{14}$$

Applying iteratively (14), we can obtain:

$$V(k) < \gamma^k V(0), \quad \forall k = \{1, 2, \cdots, N\}$$

Noting that $Q = R^{-1/2}S^{-1}R^{-1/2}$ and using the fact $\gamma \geq 1$ we have:

$$\begin{aligned} \gamma^k V(0) &= \gamma^k x^T(0)S^{-1}x(0) \\ &= \gamma^k x^T(0)R^{1/2}QR^{1/2}x(0) \\ &\leq \gamma^k \lambda_{\max}(Q)x^T(0)Rx(0) \\ &\leq \gamma^N \lambda_{\max}(Q)x^T(0)Rx(0) \end{aligned} \tag{15}$$

And,

$$\begin{aligned} V(k) &= x^T(k)S^{-1}x(k) \\ &= x^T(0)R^{1/2}QR^{1/2}x(0) \\ &\geq \lambda_{\min}(Q)x^T(k)Rx(k) \end{aligned} \tag{16}$$

Putting together (14)-(16), we obtain:

$$x^T(k)Rx(k) \leq \frac{\lambda_{\max}(\tilde{Q})\gamma^N}{\lambda_{\min}(\tilde{Q})}x^T(0)Rx(0) \tag{17}$$

From (17), it follows that (10) implies that, for all $k = 1, 2, \cdots, N$ $x^T(k)Rx(k) \leq c_2$. Therefore, the proof follows.

Remark 2. If conditions (9) and (10) in Theorem 1 is satisfied with $\gamma = 1$, then system (8) is finite-time stable with respect to (c_1, c_2, N, R) for all $N \in Z_{k \geq 0}$ and it is also asymptotically stable.

Theorem 2. The closed-loop NCS (8) with arbitrary packet-loss process is FTS with respect to (c_1, c_2, N, R) if, there exist positive definite matrix $S \in R^{n \times n}$ and scalars $\alpha \geq 0$, $\beta \geq 0$ and $\gamma \geq 1$ such that the following matrix inequalities hold:

$$\begin{bmatrix} S & A_iS + B_iX \\ * & -\gamma S \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} S & \alpha SR \\ * & -\alpha R \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} -\beta R & I \\ * & -S \end{bmatrix} < 0 \tag{20}$$

$$\beta\gamma^N c_1 - \alpha c_2 < 0 \qquad (21)$$

Where $Q = R^{-1/2}S^{-1}R^{-1/2}$ and $\lambda_{\max}(\square)$ and $\lambda_{\min}(\square)$ indicate the maximum and minimum eigenvalue of the augment, respectively. Then state feedback controller is given by $\mathrm{K} = \mathrm{X}\mathrm{S}^{-1}$.

  Proof. According to Theorem 1, it suffices to prove condition (10) is guaranteed by (19)-(21).
Using Lemma 1, it follows that:

$$\begin{bmatrix} S & \alpha SR \\ * & -\alpha R \end{bmatrix} < 0 \Rightarrow S\alpha RS - S < 0 \Rightarrow \alpha R < S^{-1} \text{ and } \begin{bmatrix} -\beta R & I \\ * & -S \end{bmatrix} < 0 \Rightarrow S^{-1} - \beta R < 0 \Rightarrow S^{-1} < \beta R$$

From the above two equations, we have:

$$\alpha R < S^{-1} < \beta R$$

Which means that:

$$\alpha I < R^{-1/2}S^{-1}R^{-1/2} < \beta I$$

  Noting that $Q = R^{-1/2}S^{-1}R^{-1/2}$, we can obtain the following relation:

$$\alpha I < Q < \beta I \qquad (22)$$

  On the other hand, from (21), we have:

$$\frac{\beta}{\alpha}\gamma^N c_1 < c_2 \qquad (23)$$

 Putting (22) and (23) together, we have:

$$\frac{\lambda_{\max}(\mathrm{Q})}{\lambda_{\min}(\mathrm{Q})}\gamma^N c_1 < \frac{\beta}{\alpha}\gamma^N c_1 < c_2 \qquad (24)$$

This completes the proof.
  Remark 4. We can see that the conditions in Theorem 2 are not LMIs. However, once we fix an γ, they can be turned into LMIs based feasibility problem which can be solved via existing software (for example the LMI Control Toolbox of MATLAB).

5. An Illustrative Example
  To illustrate the effectiveness of the proposed method, we present a numerical example. Consider the state-space plant model:

$$x(k+1) = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k)$$

  Since packet-loss process is arbitrary, we can obtain the packet-loss upper bound $s = 2$, for given $c_1 = 1$, $c_2 = 10$, $R = I_2$, $N = 100$, if let $\gamma = 1.11$, by solving LMIs (20)-(23) by Theorem 2, we can obtain state feedback controller:

$$u(k) = \begin{bmatrix} -3.5843 & -4.2463 \end{bmatrix} x(k)$$

Which ensure the closed-loop NCS is FTS with respect to (c_1, c_2, N, R).

5. Conclusion

In this paper, the finite-time stabilization problems of a class of NCSs with bounded packet dropout is investigated. The main contribution of this paper is that both sensor-to-controller and controller-to-actuator packet dropouts have been taken into account. The sufficient conditions for finite-time stabilization of the underlying systems are derived via LMIs formulation. Lastly, an illustrative example is given to demonstrate the effectiveness of the proposed results.

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