

Control Strategy Analysis on Preventive Maintenance

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Abstract

In view of the limitations that the optimal examining item do not have dynamic real-time speciality and can't reflect the actual state of devices in the traditional preventive maintenance (PM) model of repairable devices, in this paper a real-time control project on the checking rate of PM is proposed based on the state of devices. The differential equations used to describe the dynamic behavior of the system are established, and some performance indexes of maintenance systems including the steady-state availability, and the mean time to failure (MTTF), and as well as the average time of staying in each state are calculated. The control strategy on the checking rate is then proposed and the adaptability and the stability of the corresponding control system are analyzed. The essence of the method is to achieve the expected steady-state behavior by controlling the dynamic behavior of the system, which will ensure reliable completion of the task and reduce the maintenance cost meantime. Researches indicate that the proposed method is very effective to improve the utilization of devices and provide theoretical support for the practical applications.

Keywords: repairable device, reliability, checking rate, dynamic control

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1. Introduction

The equipment maintenance is the important safeguard to keep it in a good state. To improve its utilization and prolong its service life, the maintenance transformation to aim at precise guarantee objectively requires that relevant system in operation must be changed from fixed periodic preventive maintenance (PM) to dynamic maintenance strategy based on equipment actual state [1].

At present, preventive maintenance strategy has regular maintenance and maintenance based on state (CBM) [2-6]. The shortcoming of regular maintenance is that the optimal examining times or frequency will not change once determined in advance. It isn't dynamic and real-time and can't reflect the actual state of equipment. And among the CBM models, the information data is difficult to collect, and the actual state of equipment is difficult to estimate and the model to be resolved is more complicated, so that they are difficult to be practically applied.

In view of the above issues, the paper starts from analyzing the state of maintenance system, and then establishes the dynamic equations used to describe the operation process of the system, and analyzes the behaviour of the system based on state transition matrix. The performance indexes of maintenance system that include the steady-state availability, the mean time to failure (MTTF) and the average time of staying in each state are given out. The checking rates are selected as control variable and control the dynamic behaviour of the system to achieve the expected steady-state aims. In the end, the adaptability and the stability of the corresponding control system are analyzed.

2. Model Description

In order to establish the lifecycle model of repairable devices, we do the following assumptions [7-8].

Hypothesis 1: Whether device is in working state or storage state, it is not existed for the failure that can't be detected out.

Hypothesis 2: The probability of device from state S_i at time t to state S_j at time $t + \Delta t$

is only proportional to the time interval Δt , the transfer rate is a constant which does not depend on the time t and Δt .

Hypothesis 3: Maintenance will not change failure rate of the devices.

Hypothesis 4: PM will not cause the failure of system—during PM system can still work normally.

Based on above assumptions, the lifecycle model of repairable device can be denoted using the state transition diagram as shown in Figure 1. Its symbolic meaning is below.

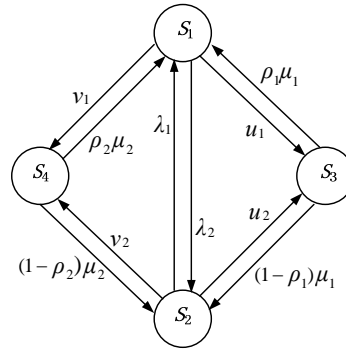


Figure 1. State Transition Diagram of Repairable Devices

In Figure 1, S_1 denotes that device is in good state in the warehouse; S_2 denotes that device is in working state; S_3 denotes that device is in preventive maintenance state; S_4 denotes that device is in corrective maintenance state. μ_i is the maintenance rate of device. Among them, μ_1 is the maintenance rate of device in preventive maintenance state; μ_2 is the maintenance rate of device in corrective maintenance state; $\rho_1\mu_1$ is the transfer rate from preventive maintenance state to storage state; $(1-\rho_1)\mu_1$ is the transfer rate from preventive maintenance state to working state; $\rho_2\mu_2$ is the transfer rate from corrective maintenance state to storage state; $(1-\rho_2)\mu_2$ is the transfer rate from corrective maintenance state to working state. λ_i is the state transition probability during the time interval $[t, t + \Delta t]$. Among them, λ_1 is the probability from working state to storage state; λ_2 is the probability from storage state to working state. Let $\lambda_3 = v_1 + u_1$, $\lambda_4 = v_2 + u_2$, v_1 and u_1 denote respectively the failure rate and the checking rate of the stored device, v_2 and u_2 denote respectively the failure rate and the checking rate of the working device.

3. Reliability Analysis

According to Figure 1 and reliability theory [9-10], we have:

$$\left. \begin{aligned} \frac{dx_1(t)}{dt} &= -(\lambda_2 + \lambda_3)x_1(t) + \lambda_1x_2(t) + \rho_1\mu_1x_3(t) + \rho_2\mu_2x_4(t) \\ \frac{dx_2(t)}{dt} &= \lambda_2x_1(t) - (\lambda_1 + \lambda_4)x_2(t) + (1-\rho_1)\mu_1x_3(t) + (1-\rho_2)\mu_2x_4(t) \\ \frac{dx_3(t)}{dt} &= u_1x_1(t) + u_2x_2(t) - \mu_1x_3(t) \\ \frac{dx_4(t)}{dt} &= v_1x_1(t) + v_2x_2(t) - \mu_2x_4(t) \end{aligned} \right\} \quad (1)$$

$$x_1(t) + x_2(t) + x_3(t) + x_4(t) = 1; \quad t \geq 0 \quad (2)$$

where $x_1(t)$ denotes the probability of being in the state S_1 at time t , and $x_2(t)$ denotes the probability in S_2 , and $x_3(t)$ is the probability in S_3 , and $x_4(t)$ is the probability in S_4 .

Conducting the Laplace transformation on (1), then we have:

$$\left. \begin{aligned} s x_1(s) &= -(\lambda_2 + \lambda_3)x_1(s) + \lambda_1 x_2(s) + \rho_1 \mu_1 x_3(s) + \rho_2 \mu_2 x_4(s) + x_1(0) \\ s x_2(s) &= \lambda_2 x_1(s) - (\lambda_1 + \lambda_4)x_2(s) + (1 - \rho_1)\mu_1 x_3(s) + (1 - \rho_2)\mu_2 x_4(s) + x_2(0) \\ s x_3(s) &= u_1 x_1(s) + u_2 x_2(s) - \mu_1 x_3(s) \\ s x_4(s) &= v_1 x_1(s) + v_2 x_2(s) - \mu_2 x_4(s) \end{aligned} \right\} \quad (3)$$

Theorem 1: As the checking rate is constant, the steady-state availability of the system is:

$$A_0 = \lim_{t \rightarrow \infty} A(t) = x_1(\infty) + x_2(\infty) = \frac{\mu_1 \mu_2 d_1}{\mu_2 d_2 + \mu_1 d_3 + \mu_1 \mu_2 d_4} \quad (4)$$

Where $d_1 = \lambda_1 + \lambda_2 + \lambda_3 + \rho_1 \mu_2 + \rho_2 v_2 - \rho_1 \mu_1 - \rho_2 v_1$, $d_2 = u_1 \lambda_1 + u_1 u_2 + u_2 \lambda_2 + u_2 v_1 + u_1 v_2 \rho_2 - u_2 v_1 \rho_2$, $d_3 = v_1 \lambda_1 + v_2 \lambda_2 + v_2 u_1 + v_1 v_2 + u_2 v_1 \rho_1 - u_1 v_2 \rho_1$, $d_4 = \lambda_1 + \lambda_2 + u_1 \rho'_1 + u_2 \rho_1 + v_1 \rho'_2 + v_2 \rho_2$.

Proof: From (3), we can have:

$$\left. \begin{aligned} x_3(s) &= \frac{1}{s + \mu_1} (u_1 x_1(s) + u_2 x_2(s)) \\ x_4(s) &= \frac{1}{s + \mu_2} (v_1 x_1(s) + v_2 x_2(s)) \end{aligned} \right\} \quad (5)$$

From (3) and (5), then:

$$s \mathbf{x}(s) = \mathbf{A} \mathbf{x}(s) + \mathbf{x}(0) \quad (6)$$

Where,

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}, \quad \mathbf{x}(s) = \begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix}, \quad \mathbf{x}(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}, \quad \rho'_1 = 1 - \rho_1, \quad \rho'_2 = 1 - \rho_2, \quad a_1 = -\lambda_2 - \lambda_3 + \frac{\rho_1 \mu_1 u_1}{s + \mu_1} + \frac{\rho_2 \mu_2 v_1}{s + \mu_2},$$

$$, \quad a_2 = \lambda_1 + \frac{\rho_1 \mu_1 \mu_2}{s + \mu_1} + \frac{\rho_2 \mu_2 v_2}{s + \mu_2}, \quad a_3 = \lambda_2 + \frac{\rho'_1 \mu_1 u_1}{s + \mu_1} + \frac{\rho'_2 \mu_2 v_1}{s + \mu_2}, \quad a_4 = -\lambda_1 - \lambda_4 + \frac{\rho'_1 \mu_1 u_2}{s + \mu_1} + \frac{\rho'_2 \mu_2 v_2}{s + \mu_2}.$$

Then,

$$\mathbf{x}(s) = \begin{bmatrix} \frac{(s - a_4)x_1(0) + a_2 x_2(0)}{\det |s\mathbf{I} - \mathbf{A}|} \\ \frac{a_3 x_1(0) + (s - a_1)x_2(0)}{\det |s\mathbf{I} - \mathbf{A}|} \end{bmatrix} \quad (7)$$

$$\det |s\mathbf{I} - \mathbf{A}| = s^2 - (a_1 + a_4)s + a_1 a_4 - a_2 a_3 \quad (8)$$

According to Laplace transformation, and we have:

$$\mathbf{x}(\infty) = \lim_{s \rightarrow 0} s \mathbf{x}(s). \quad (9)$$

Combining (7), (8) and (9), we can obtain:

$$x_1(\infty) = \frac{\mu_1 \mu_2 (\lambda_1 + \rho \mu_2 + \rho_2 \nu_2)}{\mu_2 d_2 + \mu_1 d_3 + \mu_1 \mu_2 d_4}, \quad x_2(\infty) = \frac{\mu_1 \mu_2 (\lambda_2 + \lambda_3 - \rho \mu_1 - \rho_2 \nu_1)}{\mu_2 d_2 + \mu_1 d_3 + \mu_1 \mu_2 d_4}$$

According to (4), the system steady-state availability can be proven, immediately.

Theorem 2: As the checking rate is constant, the mean time to failure (MTTF) of the system is:

$$\text{MTTF} = \frac{[\mu_1 h_1 + u_1 (\lambda_1 + \lambda_4) + u_2 \lambda_2] x_1(0) + [\mu_1 h_2 + u_2 (\lambda_2 + \lambda_3) + u_1 \lambda_1] x_2(0)}{\mu_1 [(\lambda_2 + \lambda_3 - \rho \mu_1)(\lambda_1 + \lambda_4 - \rho' \mu_2) - (\lambda_1 + \rho \mu_2)(\lambda_2 + \rho' \mu_1)]} \quad (10)$$

Where $h_1 = \lambda_1 + \lambda_2 + \lambda_4 + \rho' \mu_1 - \rho' \mu_2$, $h_2 = \lambda_1 + \lambda_2 + \lambda_3 + \rho \mu_2 - \rho \mu_1$.

Proof: Let S_4 be the absorbing state, and $R(t) = x_1(t) + x_2(t) + x_3(t)$, $\mu_2 = 0$, and then,

$$\text{MTTF} = \int_0^{\infty} R(t) dt = R(s)|_{s=0} = \left(x_1(s) + x_2(s) + x_3(s) \right) \Big|_{s=0}$$

Then the formula (10) can be obtained, immediately.

Theorem 3: As the checking rate is constant, the total average time that system stay in state S_1 and state S_2 before entering the absorbing state is:

$$T = \frac{(\lambda_1 + \lambda_2 + \lambda_4 + \rho' \mu_1 - \rho' \mu_2) x_1(0) + (\lambda_1 + \lambda_2 + \lambda_3 + \rho \mu_2 - \rho \mu_1) x_2(0)}{(\lambda_2 + \lambda_3 - \rho \mu_1)(\lambda_1 + \lambda_4 - \rho' \mu_2) - (\lambda_1 + \rho \mu_2)(\lambda_2 + \rho' \mu_1)} \quad (11)$$

Proof: Let X be the total time of staying in state S_1 and S_2 before entering the absorbing state. To get the distribution of X , we must deduct the time of staying in state S_3 . According to the physical meaning of derivative, let the derivative on t in the differential equation of state S_3 be zero [11], then:

$$\left. \begin{aligned} \frac{dx_1(t)}{dt} &= -(\lambda_2 + \lambda_3)x_1(t) + \lambda_1 x_2(t) + \rho_1 \mu_1 x_3(t) \\ \frac{dx_2(t)}{dt} &= \lambda_2 x_1(t) - (\lambda_1 + \lambda_4)x_2(t) + (1 - \rho_1) \mu_1 x_3(t) \\ 0 &= u_1 x_1(t) + u_2 x_2(t) - \mu_1 x_3(t) \end{aligned} \right\} \quad (12)$$

After Laplace transformation we have:

$$\left. \begin{aligned} s x_1(s) &= -(\lambda_2 + \lambda_3)x_1(s) + \lambda_1 x_2(s) + \rho_1 \mu_1 x_3(s) + x_1(0) \\ s x_2(s) &= \lambda_2 x_1(s) - (\lambda_1 + \lambda_4)x_2(s) + (1 - \rho_1) \mu_1 x_3(s) + x_2(0) \\ 0 &= u_1 x_1(s) + u_2 x_2(s) - \mu_1 x_3(s) \end{aligned} \right\} \quad (13)$$

Writing as matrix formula, and then:

$$s \mathbf{x}(s) = \mathbf{C} \mathbf{x}(s) + \mathbf{x}(0) \quad (14)$$

Where $\mathbf{C} = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix}$, $\mathbf{x}(s) = \begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix}$, $\mathbf{x}(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$, $c_1 = -(\lambda_2 + \lambda_3 - \rho_1 \mu_1)$, $c_2 = \lambda_1 + \rho_1 \mu_2$,

$c_3 = \lambda_2 + \rho' \mu_1$, $c_4 = -(\lambda_1 + \lambda_4 - \rho' \mu_2)$.

Then we can get:

$$\dot{\mathbf{x}}(s) = \begin{bmatrix} \frac{(s-c_4)x_1(0) + c_2x_2(0)}{\det|s\mathbf{I} - \mathbf{C}|} \\ \frac{c_3x_1(0) + (s-c_1)x_2(0)}{\det|s\mathbf{I} - \mathbf{C}|} \end{bmatrix} \quad (15)$$

$$\det|s\mathbf{I} - \mathbf{C}| = s^2 - (c_1 + c_4)s + c_1c_4 - c_2c_3 \quad (16)$$

Let $T(t) = P(X > t) = x_1(t) + x_2(t)$, then, the total average time of staying in state S_1 and state S_2 before entering the absorbing state is:

$$\Gamma = \int_0^{\infty} T(t)dt = T(s)|_{s=0} = \left(x_1(s) + x_2(s) \right) \Big|_{s=0} = \frac{(c_3 - c_4)x_1(0) + (c_2 - c_1)c_2x_2(0)}{c_1c_4 - c_2c_3}$$

After related parameters are substituted into the formula above, the formula (11) is proven.

4. Control Strategy Design

To make the dynamic behavior of the system achieve the expected steady-state aim, we may control the checking rates of maintenance system, which are the inverse of mean time between checks. This control mode is different from the traditional methods [12].

According to (1) and (2), we have:

$$\left. \begin{aligned} \frac{dx_1(t)}{dt} &= -(\lambda_2 + v_1 + \rho_1\mu_1)x_1(t) + (\lambda_1 - \rho_1\mu_1)x_2(t) + (\rho_2\mu_2 - \rho_1\mu_1)x_4(t) - u_1(t)x_1(t) + \rho_1\mu_1 \\ \frac{dx_2(t)}{dt} &= (\lambda_2 - \rho_1\mu_1)x_1(t) - (\lambda_1 + v_2 + \rho_1\mu_1)x_2(t) + (\rho_2\mu_2 - \rho_1\mu_1)x_4(t) - u_2(t)x_2(t) + \rho_1\mu_1 \\ \frac{dx_4(t)}{dt} &= v_1x_1(t) + v_2x_2(t) - \mu_2x_4(t) \end{aligned} \right\} \quad (17)$$

Writing (17) as the Matrix form, then:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_1\mathbf{x}(t) - \mathbf{U}_0(t)\mathbf{x}(t) + \mathbf{b} = \mathbf{A}_1\mathbf{x}(t) - \mathbf{X}(t)\mathbf{u}_0(t) + \mathbf{b} \quad (18)$$

Where,

$$\mathbf{A}_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} -(\lambda_2 + v_1 + \rho_1\mu_1) & \lambda_1 - \rho_1\mu_1 & \rho_2\mu_2 - \rho_1\mu_1 \\ \lambda_2 - \rho_1\mu_1 & -(\lambda_1 + v_2 + \rho_1\mu_1) & \rho_2\mu_2 - \rho_1\mu_1 \\ v_1 & v_2 & -\mu_2 \end{bmatrix}, \quad \mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_4(t) \end{bmatrix},$$

$$\mathbf{X}(t) = \begin{bmatrix} x_1(t) & 0 & 0 \\ 0 & x_2(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{u}_0(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ 0 \end{bmatrix}, \quad \mathbf{U}_0(t) = \begin{bmatrix} u_1(t) & 0 & 0 \\ 0 & u_2(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \rho_1\mu_1 \\ \rho_1\mu_1 \\ 0 \end{bmatrix}.$$

Projecting the system (1) onto a plane of x_1 and x_2 , we then have:

$$x_1(t) + x_2(t) \leq 1; \quad t \geq 0 \quad (19)$$

As x_1 or x_2 tends to zero, to ensure the boundness of $\mathbf{u}(t)$, where $\mathbf{u}(t) = [u_1(t) \quad u_2(t)]^T$,

we divide the area of x_1 and x_2 into four sets which are denoted with ① ~ ④ as shown in Figure 2, where $0 < \rho < 1$.

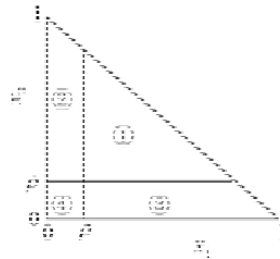


Figure 2. Valid Areas of Control Variable

Theorem 4: In the area ①, $\rho \leq x_1 \leq 1 - \rho$, $\rho \leq x_2 \leq 1 - \rho$, and $x_1 + x_2 \leq 1$. We do the rule control for $u(t)$ below.

$$u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{x_1(t)} ((a_{11} + k_{11})x_1(t) + (a_{12} + k_{12})x_2(t) + (a_{13} + k_{13})x_4(t) + b_1 - k_{11}\bar{x}_1 - k_{12}\bar{x}_2 - k_{13}\bar{x}_4) \\ \frac{1}{x_2(t)} ((a_{21} + k_{21})x_1(t) + (a_{22} + k_{22})x_2(t) + (a_{23} + k_{23})x_4(t) + b_2 - k_{21}\bar{x}_1 - k_{22}\bar{x}_2 - k_{23}\bar{x}_4) \end{bmatrix} \quad (20)$$

where $\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_4 \end{bmatrix} = \bar{x}$, $\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = K$, \bar{x} is the expected steady-state probability, K matches

$(I + K)$ is nonsingular and $(I - K)(I + K)^{-1}$ is convergent, I is unit matrix. Under the action of the control rule (4.4), the steady-state availability of the system is:

$$A_0 = x_1(\infty) + x_2(\infty) = \bar{x}_1 + \bar{x}_2 \quad (21)$$

Proof: Known from the Theorem 1, the desired state equation should possess the following form.

$$\frac{dx(t)}{dt} = A_{m1}x(t) + b_{m1} \quad (22)$$

And so, the steady-state value of the system can be written as:

$$x(\infty) = \bar{x} = (\bar{x}_1 \quad \bar{x}_2 \quad \bar{x}_4)^T = \bar{x}_{m1}$$

Where \bar{x}_{m1} is the expected steady-state probability, $\bar{x}_4 = \frac{1}{\mu_2} (v_1\bar{x}_1 + v_2\bar{x}_2)$.

Let $A_{m1} = -K$, $b_{m1} = K\bar{x}$ and Combining (4.2) and (4.6), we can get (4.4), and (4.5) by combining (4.2), (4.4) and (4.6). To ensure that the system is asymptotically steady, K must match that $(I + K)$ is nonsingular and $(I - K)(I + K)^{-1}$ is convergent [13-14], and k_{31} , k_{32} and k_{33} match the following condition.

$$(a_{31} + k_{31})x_1(t) + (a_{32} + k_{32})x_2(t) + (a_{33} + k_{33})x_4(t) + b_2 - k_{31}\bar{x}_1 - k_{32}\bar{x}_2 - k_{33}\bar{x}_4 = 0 \quad (23)$$

End.

In the area ②, in order to ensure that $u_1(t)$ is bounded, we let $u_1(t)$ be constant, but $u_2(t)$ remain unchanged. So, the control rule for $\mathbf{u}(t)$ is:

$$\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} a_{11} + \frac{1}{\bar{x}_1}(a_{12}\bar{x}_2 + a_{13}\bar{x}_4 + b_1) \\ \frac{1}{x_2(t)}((a_{21} + k_{21})x_1(t) + (a_{22} + k_{22})x_2(t) + (a_{23} + k_{23})x_4(t) + b_2 - k_{21}\bar{x}_1 - k_{22}\bar{x}_2 - k_{23}\bar{x}_4) \end{bmatrix} \quad (24)$$

So, the dynamic behavior of the system becomes

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_{m2}\mathbf{x}(t) + \mathbf{b}_{m2} \quad (25)$$

Where,

$$\mathbf{A}_{m2} = - \begin{bmatrix} \frac{1}{\bar{x}_1}(a_{12}\bar{x}_2 + a_{13}\bar{x}_4 + b_1) & -a_{12} & -a_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}, \quad \mathbf{b}_{m2} = \begin{bmatrix} \rho_v \mu_1 \\ k_{21}\bar{x}_1 + k_{22}\bar{x}_2 + k_{23}\bar{x}_4 \\ k_{31}\bar{x}_1 + k_{32}\bar{x}_2 + k_{33}\bar{x}_4 \end{bmatrix}$$

k_{31} , k_{32} and k_{33} match (23). In order to ensure that the system is asymptotically steady, \mathbf{A}_{m2} matches that $(\mathbf{I} - \mathbf{A}_{m2})$ is nonsingular and $(\mathbf{I} + \mathbf{A}_{m2})(\mathbf{I} - \mathbf{A}_{m2})^{-1}$ is convergent.

Thus, the steady-state value of the system is:

$$\mathbf{x}(\infty) = \bar{\mathbf{x}} = (\bar{x}_1 \quad \bar{x}_2 \quad \bar{x}_4)^T = \bar{\mathbf{x}}_{m2}$$

In the same way, In the area ③, we set $u_2(t)$ as constant, but $u_1(t)$ remain unchanged. So, we have:

$$\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{x_1(t)}((a_{11} + k_{11})x_1(t) + (a_{12} + k_{12})x_2(t) + (a_{13} + k_{13})x_4(t) + b_1 - k_{11}\bar{x}_1 - k_{12}\bar{x}_2 - k_{13}\bar{x}_4) \\ a_{22} + \frac{1}{\bar{x}_2}(a_{21}\bar{x}_1 + a_{23}\bar{x}_4 + b_2) \end{bmatrix} \quad (26)$$

So, the dynamic behavior of the system becomes:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_{m3}\mathbf{x}(t) + \mathbf{b}_{m3} \quad (27)$$

Where,

$$\mathbf{A}_{m3} = - \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ -a_{21} & \frac{1}{\bar{x}_2}(a_{21}\bar{x}_1 + a_{23}\bar{x}_4 + b_2) & -a_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}, \quad \mathbf{b}_{m3} = \begin{bmatrix} k_{11}\bar{x}_1 + k_{12}\bar{x}_2 + k_{13}\bar{x}_4 \\ \rho'_v \mu_1 \\ k_{31}\bar{x}_1 + k_{32}\bar{x}_2 + k_{33}\bar{x}_4 \end{bmatrix}$$

k_{31} , k_{32} and k_{33} match (23). In order to ensure that the system is asymptotically steady, \mathbf{A}_{m3} matches that $(\mathbf{I} - \mathbf{A}_{m3})$ is nonsingular and $(\mathbf{I} + \mathbf{A}_{m3})(\mathbf{I} - \mathbf{A}_{m3})^{-1}$ is convergent.

Thus, the steady-state value of the system is:

$$\mathbf{x}(\infty) = \bar{\mathbf{x}} = (\bar{x}_1 \quad \bar{x}_2 \quad \bar{x}_4)^T = \bar{\mathbf{x}}_{m3}.$$

In the same way, In the area ④, we set the control rule for $\mathbf{u}(t)$ as:

$$\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} a_{11} + \frac{1}{\bar{x}_1}(a_{12}\bar{x}_2 + a_{13}\bar{x}_4 + b_1) \\ a_{22} + \frac{1}{\bar{x}_2}(a_{21}\bar{x}_1 + a_{23}\bar{x}_4 + b_2) \end{bmatrix} \quad (28)$$

So, the dynamic behavior of the system becomes:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_{m4}\mathbf{x}(t) + \mathbf{b}_{m4} \quad (29)$$

Where,

$$\mathbf{A}_{m4} = - \begin{bmatrix} \frac{1}{\bar{x}_1}(a_{12}\bar{x}_2 + a_{13}\bar{x}_4 + b_1) & -a_{12} & -a_{13} \\ -a_{21} & \frac{1}{\bar{x}_2}(a_{21}\bar{x}_1 + a_{23}\bar{x}_4 + b_2) & -a_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}, \quad \mathbf{b}_{m4} = \begin{bmatrix} \rho_1 \mu_1 \\ \rho'_1 \mu_1 \\ k_{31}\bar{x}_1 + k_{32}\bar{x}_2 + k_{33}\bar{x}_4 \end{bmatrix}$$

k_{31} , k_{32} and k_{33} match (23). In order to ensure that the system is asymptotically steady, \mathbf{A}_{m4} matches that $(\mathbf{I} - \mathbf{A}_{m4})$ is nonsingular and $(\mathbf{I} + \mathbf{A}_{m4})(\mathbf{I} - \mathbf{A}_{m4})^{-1}$ is convergent.

Thus, the steady-state value of the system is:

$$\mathbf{x}(\infty) = \bar{\mathbf{x}} = (\bar{x}_1 \quad \bar{x}_2 \quad \bar{x}_4)^T = \bar{\mathbf{x}}_{m4}$$

Under the action of the control rules (20), (24), (26) and (28), the form of the motion equation of the system is as follows.

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_{mi}\mathbf{x}(t) + \mathbf{b}_{mi} \quad (i = 1, 2, 3, 4)$$

Then, solving the above equation, we get:

$$\mathbf{x}(t) = \exp(\mathbf{A}_{mi}t)\mathbf{x}(0) + \int_0^t \exp(\mathbf{A}_{mi}(t - \tau))d\tau \cdot \mathbf{b}_{mi}$$

According to \mathbf{A}_{mi} , we can know that the system is asymptotically steady. So,

$$\mathbf{x}(t) = \exp(\mathbf{A}_{mi}t) \left(\mathbf{x}(0) + \mathbf{A}_{mi}^{-1}\mathbf{b}_{mi} \right) - \mathbf{A}_{mi}^{-1}\mathbf{b}_{mi}$$

Thus, the steady-state value of the system is:

$$\mathbf{x}(\infty) = -\mathbf{A}_{mi}^{-1}\mathbf{b}_{mi} = \bar{\mathbf{x}}_{mi}$$

Then, we have:

$$\mathbf{x}(t) = \exp(\mathbf{A}_{mi}t) \left(\mathbf{x}(0) - \bar{\mathbf{x}}_{mi} \right) + \bar{\mathbf{x}}_{mi}$$

Under the condition that the initial value $x(0)$ and the expected steady-state value \bar{x}_{mi} are given out, it is quite obvious that the motion equation of the system is only related to $\exp(\mathbf{A}_{mit})$. So, we use the method of resolvent matrix to solve $\exp(\mathbf{A}_{mit})$ [13]. Thus, we have:

$$\exp(\mathbf{A}_{mit}) = L^{-1} \left((s\mathbf{I} - \mathbf{A}_{mi})^{-1} \right)$$

Firstly, we conduct the following definitions.

$$\begin{aligned} w_1 &= \frac{1}{\bar{x}_1} (a_{12}\bar{x}_2 + a_{13}\bar{x}_4 + b_1), \quad w_2 = \frac{1}{\bar{x}_2} (a_{21}\bar{x}_1 + a_{23}\bar{x}_4 + b_2) \\ p_1(s) &= s^3 + s^2(k_{11} + k_{22} + k_{33}) + s(k_{11}k_{33} + k_{11}k_{22} + k_{22}k_{33} - k_{12}k_{21} - k_{23}k_{32} - k_{31}k_{13}) + \\ &\quad k_{11}k_{22}k_{33} + k_{12}k_{23}k_{31} + k_{13}k_{32}k_{21} - k_{13}k_{31}k_{22} - k_{12}k_{21}k_{33} - k_{32}k_{23}k_{11} \\ p_2(s) &= s^3 + s^2(w_1 + k_{22} + k_{33}) + s(w_1(k_{22} + k_{33}) + k_{22}k_{33} + a_{12}k_{21} - k_{23}k_{32} + a_{13}k_{31}) + \\ &\quad w_1(k_{22}k_{33} - k_{32}k_{23}) - a_{12}(k_{23}k_{31} - k_{21}k_{33}) - a_{13}(k_{32}k_{21} - k_{31}k_{22}) \\ p_3(s) &= s^3 + s^2(k_{11} + k_{33} + w_2) + s(w_2(k_{11} + k_{33}) + k_{11}k_{33} + k_{12}a_{21} + a_{23}k_{32} - k_{31}k_{13}) + \\ &\quad w_2(k_{11}k_{33} - k_{13}k_{31}) - a_{23}(k_{12}k_{31} - k_{32}k_{11}) - a_{21}(k_{13}k_{32} - k_{12}k_{33}) \\ p_4(s) &= s^3 + s^2(w_1 + w_2 + k_{33}) + s\{w_1(k_{33} + w_2) + w_2k_{33} - a_{12}a_{21} + k_{32}a_{23} + a_{13}k_{31}\} + \\ &\quad w_2(a_{13}k_{31} + w_1k_{33}) + k_{32}a_{23}w_1 - a_{12}a_{21}k_{33} + a_{12}a_{23}k_{31} + a_{13}k_{32}a_{21} \\ q_1(s) &= s^2 + s(k_{22} + k_{33}) + k_{22}k_{33} - k_{23}k_{32}, \quad q_2(s) = s^2 + s(k_{11} + k_{33}) + k_{11}k_{33} - k_{13}k_{31} \\ q_3(s) &= s^2 + s(k_{11} + k_{22}) + k_{11}k_{22} - k_{12}k_{21}, \quad q_4(s) = -k_{21}(s + k_{33}) + k_{23}k_{31} \\ q_5(s) &= -k_{31}(s + k_{22}) + k_{21}k_{32}, \quad q_6(s) = -k_{12}(s + k_{33}) + k_{13}k_{32} \\ q_7(s) &= -k_{32}(s + k_{11}) + k_{12}k_{31}, \quad q_8(s) = -k_{13}(s + k_{22}) + k_{12}k_{23} \\ q_9(s) &= -k_{23}(s + k_{11}) + k_{13}k_{21}, \quad f_1(s) = a_{12}(s + k_{33}) - a_{13}k_{32} \\ f_2(s) &= s^2 + s(w_1 + k_{33}) + w_1k_{33} + a_{13}k_{31}, \quad f_3(s) = -k_{32}(s + w_1) - a_{12}k_{31} \\ f_4(s) &= a_{13}(s + k_{22}) - a_{12}k_{23}, \quad f_5(s) = -k_{23}(s + w_1) - a_{13}k_{21} \\ f_6(s) &= s^2 + s(w_1 + k_{22}) + w_1k_{22} + a_{12}k_{21}, \quad g_1(s) = s^2 + s(w_2 + k_{33}) + w_2k_{33} + a_{23}k_{32} \\ g_2(s) &= a_{21}(s + k_{33}) - a_{23}k_{31}, \quad g_3(s) = -k_{31}(s + w_2) - a_{21}k_{32} \\ g_4(s) &= -k_{13}(s + w_2) - a_{23}k_{12}, \quad g_5(s) = a_{23}(s + k_{11}) - a_{21}k_{13} \\ g_6(s) &= s^2 + s(w_2 + k_{11}) + w_2k_{11} + a_{21}k_{12}, \quad h_1(s) = a_{13}(s + w_2) + a_{12}a_{23} \\ h_2(s) &= a_{23}(s + w_1) + a_{13}a_{21}, \quad h_3(s) = s^2 + s(w_1 + w_2) + w_1w_2 - a_{12}a_{21}. \end{aligned}$$

And then, we have:

$$\begin{aligned} \exp(\mathbf{A}_{m1t}) &= L^{-1} \begin{bmatrix} \frac{q_1(s)}{p_1(s)} & \frac{q_4(s)}{p_1(s)} & \frac{q_7(s)}{p_1(s)} \\ \frac{q_2(s)}{p_1(s)} & \frac{q_5(s)}{p_1(s)} & \frac{q_8(s)}{p_1(s)} \\ \frac{q_3(s)}{p_1(s)} & \frac{q_6(s)}{p_1(s)} & \frac{q_9(s)}{p_1(s)} \end{bmatrix}, \quad \exp(\mathbf{A}_{m2t}) = L^{-1} \begin{bmatrix} \frac{q_1(s)}{p_2(s)} & \frac{f_1(s)}{p_2(s)} & \frac{f_4(s)}{p_2(s)} \\ \frac{q_2(s)}{p_2(s)} & \frac{f_2(s)}{p_2(s)} & \frac{f_5(s)}{p_2(s)} \\ \frac{q_3(s)}{p_2(s)} & \frac{f_3(s)}{p_2(s)} & \frac{f_6(s)}{p_2(s)} \end{bmatrix}, \\ \exp(\mathbf{A}_{m3t}) &= L^{-1} \begin{bmatrix} \frac{g_1(s)}{p_3(s)} & \frac{q_4(s)}{p_3(s)} & \frac{g_4(s)}{p_3(s)} \\ \frac{g_2(s)}{p_3(s)} & \frac{q_5(s)}{p_3(s)} & \frac{g_5(s)}{p_3(s)} \\ \frac{g_3(s)}{p_3(s)} & \frac{q_6(s)}{p_3(s)} & \frac{g_6(s)}{p_3(s)} \end{bmatrix}, \quad \exp(\mathbf{A}_{m4t}) = L^{-1} \begin{bmatrix} \frac{g_1(s)}{p_4(s)} & \frac{f_1(s)}{p_4(s)} & \frac{h_1(s)}{p_4(s)} \\ \frac{g_2(s)}{p_4(s)} & \frac{f_2(s)}{p_4(s)} & \frac{h_2(s)}{p_4(s)} \\ \frac{g_3(s)}{p_4(s)} & \frac{f_3(s)}{p_4(s)} & \frac{h_3(s)}{p_4(s)} \end{bmatrix}. \end{aligned}$$

Based on the above discussions, under the condition that the initial value $x(0)$ and the expected steady-state value \bar{x}_{mi} are given out, we can get the motion equation $x(t)$ of the system by combining $x(\infty) = -A_{mi}^{-1}b_{mi} = \bar{x}_{mi}$ and $\exp(A_{mi}t)$.

5. Stability Analysis

As the checking rate is constant, we have the linear system as follows.

$$\frac{dx(t)}{dt} = Ax(t) \quad (30)$$

Where,

$$A = \begin{bmatrix} -(\lambda_2 + v_1 + \rho_1\mu_1 + u_1) & \lambda_1 - \rho_1\mu_1 & \rho_2\mu_2 - \rho_1\mu_1 \\ \lambda_2 - \rho'_1\mu_1 & -(\lambda_1 + v_2 + \rho'_1\mu_1 + u_2) & \rho'_2\mu_2 - \rho'_1\mu_1 \\ v_1 & v_2 & -\mu_2 \end{bmatrix}$$

According to the linear control theory [13] and [14], if all eigenvalues of the matrix A possess the negative real parts, and then system (5.1) is asymptotically steady. In other words, the matrix A must match the following conditions.

- (1) $(I - A)$ is nonsingular.
- (2) $(I + A)(I - A)^{-1}$ is convergent.

As the checking rate is not constant, we have the linear time-varying system as follows.

$$\frac{dx(t)}{dt} = A(t)x(t) \quad (31)$$

Where,

$$A(t) = \begin{bmatrix} -(\lambda_2 + v_1 + \rho_1\mu_1 + u_1(t)) & \lambda_1 - \rho_1\mu_1 & \rho_2\mu_2 - \rho_1\mu_1 \\ \lambda_2 - \rho'_1\mu_1 & -(\lambda_1 + v_2 + \rho'_1\mu_1 + u_2(t)) & \rho'_2\mu_2 - \rho'_1\mu_1 \\ v_1 & v_2 & -\mu_2 \end{bmatrix}$$

$u_1(t)$ and $u_2(t)$ are bounded for $t \in [0, \infty)$, the real numbers δ_1 and δ_2 are existent and satisfy:

$$0 \leq u_1(t) \leq \delta_1 < \infty; \quad 0 \leq u_2(t) \leq \delta_2 < \infty.$$

In order to quote lemmas, we conduct definitions as follows.

$\dot{A}(t)$: It denotes the matrix that is made up of the elements that are the derivative of everyone element on t in the matrix $A(t)$.

$\|\square\|$: It denotes the norm of vectors or matrices.

A^* : It denotes the gone transposition matrix of matrix A .

$Re(\square)$: It denotes the real part.

According to [15], [16] and [17], we have the following lemmas.

Lemma 1: If the eigenvalue of the n order square matrix $A(t)$ that are $\lambda_1(t), \lambda_2(t), \dots, \lambda_n(t)$ match:

$$\left| Re(\lambda_i(t) + \lambda_j(t)) \right| \geq 2\delta > 0 \quad (i, j = 1, 2, \dots, n)$$

And $\|A(t)\| \leq k$, then, for anyone n order nonsingular Hamilton symmetric square matrix C , having Hamilton symmetric square matrix $G(t)$ makes:

$$A^*(t)G(t) + G(t)A(t) = C \quad (32)$$

If $C = -I$, then $\|G(t)\| \leq k_0$; if $\|C\| \leq S$, then $\|G(t)\| \leq k_0 S$, where:

$$k_0 = \frac{2^{n-1}}{2\delta} \left(1 + \frac{k}{2\delta} \right)^{\frac{(n+2)(n-1)}{2}} \quad (33)$$

Lemma 2: If the eigenvalue $\lambda_i(t)$ of the n order square matrix $A(t)$ not only match the conditions in lemma 1 but also match:

$$\operatorname{Re}(\lambda_i(t)) \leq -\sigma < 0 \quad (i, j = 1, 2, \dots, n)$$

then $G(t)$ as showed in lemma 1 is positive definite.

Lemma 3: If $A(t)$ matches the conditions in lemma 1 and $\|\dot{A}(t)\| \leq k_1$, then having Hamilton symmetric square matrix $G(t)$ makes:

- (1) $A^*(t)G(t) + G(t)A(t) = -I$.
- (2) $\|\dot{G}(t)\| \leq 2k_1 k_0^2$, where k_0 as defined in Lemma 1.

Definition 1: Let the linear time-varying system be:

$$\frac{dx(t)}{dt} = A_{11}(t)x(t) \quad (34)$$

Where,

$$A_{11}(t) = \begin{bmatrix} -(\lambda_2 + v_1 + \rho_1 \mu_1 + u_1(t)) & \lambda_1 - \rho_1 \mu_1 & \rho_2 \mu_2 - \rho_1 \mu_1 \\ \lambda_2 - \rho_1' \mu_1 & -(\lambda_1 + v_2 + \rho_1' \mu_1 + u_2) & \rho_2' \mu_2 - \rho_1' \mu_1 \\ v_1 & v_2 & -\mu_2 \end{bmatrix}$$

$\|A_{11}(t)\| \leq k_{11}$, $\|\dot{A}_{11}(t)\| = |u_i'(t)| \leq \delta_3$ and the eigenvalue $\lambda_i(t)$ of the matrix $A_{11}(t)$ match:

- (1) $|\operatorname{Re}(\lambda_i(t) + \lambda_j(t))| \geq 2\delta_{11} > 0$, $(i, j = 1, 2, 3)$.
- (2) $\operatorname{Re}(\lambda_i(t)) \leq -\sigma_{11} < 0$, $(i = 1, 2, 3)$.

If this system is asymptotically steady, then we call $A_{11}(t)$ a strong stable matrix and call the system (5.5) a strong stable system. Let $g_{01} = \frac{8\delta_3}{\delta_{11}^2} \left(1 + \frac{k_{11}}{2\delta_{11}} \right)^{10}$.

Theorem 5: If $g_{01} < 1$, then system (5.5) is a strong stable system.

Proof: According to Definition 1, we know that matrix $A_{11}(t)$ matches the conditions in Lemma 1, Lemma 2 and Lemma 3. And therefore we establish LyaPunov function as follows.

$$V(x, t) = x^* G x$$

Where $G(t)$ is determined by Lemma 3, and is positive definite by Lemma 2. So, we have:

$$V(x, t) > 0$$

According to Lemma 3, we get:

$$A_{11}^*(t)G(t) + G(t)A_{11}(t) = -I$$

$$\|G(t)\| \leq g_{01}$$

The derivative of $V(x,t)$ on t is:

$$\begin{aligned} \frac{dV(x,t)}{dt} &= x^* G(t)x + x^* \dot{G}(t)x + x^* G(t)\dot{x} = x^* [A_{11}^*(t)G(t) + \dot{G}(t) + G(t)A_{11}(t)]x \\ &= x^* [-I + \dot{G}(t)]x = -\|x\|^2 + x^* \dot{G}(t)x \leq |x^* \dot{G}(t)x| - \|x\|^2 \leq (\|\dot{G}(t)\| - 1)\|x\|^2 \end{aligned}$$

If $g_{01} < 1$, then $\frac{dV(x,t)}{dt} < 0$, we can easily know that system (5.5) is asymptotically steady.

Definition 2: Let the linear time-varying system be:

$$\frac{dx(t)}{dt} = (A_{11}(t) + \varepsilon_1 U_1(t))x(t) \quad (35)$$

Where $U_1(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -u_2(t) + u_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\|U_1(t)\| \leq m_1$, $A_{11}(t)$ is a strong stable matrix, $\varepsilon_1 U_1(t)$ is a

disturbance matrix about $A_{11}(t)$, and ε_1 is disturbance coefficient. This system is called a disturbance system of strong stable system. Let $g_{11} = 2m_1 k_{01}$, where $k_{01} = \frac{2}{\delta_{11}} \left(1 + \frac{k_{11}}{2\delta_{11}}\right)^5$.

Theorem 6: If $|\varepsilon_1| < \frac{1-g_{01}}{g_{11}}$, then system (5.6) is asymptotically steady.

Proof: According to Definition 1, we know that matrix $A_{11}(t)$ matches the conditions in lemma 1, lemma 2 and lemma 3. So, we establish LyaPunov function as follows.

$$V(x,t) = x^* Gx$$

Where $G(t)$ is determined by Lemma 3.

Known from the proof process in Theorem 5,

$$V(x,t) > 0$$

$$A_{11}^*(t)G(t) + G(t)A_{11}(t) = -I$$

$$\|G(t)\| \leq g_{01}$$

The derivative of $V(x,t)$ on t is:

$$\begin{aligned} \frac{dV(x,t)}{dt} &= x^* G(t)x + x^* \dot{G}(t)x + x^* G(t)\dot{x} = x^* \{A_{11}^*(t)G(t) + \dot{G}(t) + G(t)A_{11}(t) + \varepsilon_1 [U^*(t)G(t) + G(t)U(t)]\}x \\ &= x^* [-I + \dot{G}(t) + \varepsilon_1 [U^*(t)G(t) + G(t)U(t)]]x = x^* \{\dot{G}(t) + \varepsilon_1 [U^*(t)G(t) + G(t)U(t)]\}x - \|x\|^2 \\ &\leq |x^* \{\dot{G}(t) + \varepsilon_1 [U^*(t)G(t) + G(t)U(t)]\}x| - \|x\|^2 \leq (\|\dot{G}(t)\| + 2|\varepsilon_1| \|U(t)\| \|G(t)\| - 1)\|x\|^2 \leq (g_{01} + |\varepsilon_1| g_{11} - 1)\|x\|^2 \end{aligned}$$

If $|\varepsilon_1| < \frac{1-g_{01}}{g_{11}}$, then $\frac{dV(\mathbf{x},t)}{dt} < 0$, it is known that system (5.6) is asymptotically steady.

Definition 3: Let the linear time-varying system be

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}_{12}(t)\mathbf{x}(t) \quad (36)$$

Where,

$$\mathbf{A}_{12}(t) = \begin{bmatrix} -(\lambda_2 + v_1 + \rho_1\mu_1 + u_1) & \lambda_1 - \rho_1\mu_1 & \rho_2\mu_2 - \rho_1\mu_1 \\ \lambda_2 - \rho_1'\mu_1 & -(\lambda_1 + v_2 + \rho_1'\mu_1 + u_2(t)) & \rho_2'\mu_2 - \rho_1'\mu_1 \\ v_1 & v_2 & -\mu_2 \end{bmatrix}$$

$\|\mathbf{A}_{12}(t)\| \leq k_{12}$, and $\|\dot{\mathbf{A}}_{12}(t)\| = |u_2'(t)| \leq \delta_4$. Setting $g_{02} = \frac{8\delta_4}{\delta_{12}^2} \left(1 + \frac{k_{12}}{2\delta_{12}}\right)^{10}$. The eigenvalue $\lambda_i(t)$ of the

matrix $\mathbf{A}_{12}(t)$ match:

$$(1) \quad \left| \operatorname{Re}(\lambda_i(t) + \lambda_j(t)) \right| \geq 2\delta_{12} > 0, \quad (i, j = 1, 2, 3).$$

$$(2) \quad \operatorname{Re}(\lambda_i(t)) \leq -\sigma_{12} < 0, \quad (i = 1, 2, 3).$$

Theorem 7: If $g_{02} < 1$, then system (5.7) is a strong stable system. The proof is similar to Theorem 5, and so the proof process can be ignored.

Definition 4: Let the linear time-varying system be:

$$\frac{d\mathbf{x}(t)}{dt} = (\mathbf{A}_{12}(t) + \varepsilon_2 \mathbf{U}_2(t))\mathbf{x}(t) \quad (37)$$

Where, $\mathbf{U}_2(t) = \begin{bmatrix} -u_1(t) + u_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\|\mathbf{U}_2(t)\| \leq m_2$, $\mathbf{A}_{12}(t)$ is a strong stable matrix, $\varepsilon_2 \mathbf{U}_2(t)$ is a

disturbance matrix about $\mathbf{A}_{12}(t)$, ε_2 is disturbance coefficient. Let $g_{12} = 2m_2k_{02}$, where

$$k_{02} = \frac{2}{\delta_{12}} \left(1 + \frac{k_{12}}{2\delta_{12}}\right)^5.$$

Theorem 8: If $|\varepsilon_2| < \frac{1-g_{02}}{g_{12}}$, then system (5.8) is asymptotically steady.

The proof is similar to Theorem 6, here the proof process is ignored.

6. Conclusion

In this paper, the dynamic behavior of maintenance system based on state transition is analyzed, the differential equations that are used to describe the dynamic behavior of the system is established, and the performance indexes of the system are calculated and given out. We control the dynamic behavior of the system to achieve the expected steady-state performance by selecting checking rate as control variable. And its adaptability and the stability of the corresponding control system are analyzed. The results indicate that the method is more objective and accurate than the traditional method so as to be able to provide the theoretical support for management problems of the large complex equipment by using this method.

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