

## Single-winding Regulation Mode of Controllable Reactor of Transformer Type

Yibin Liu, Mingxing Tian, Jianning Yin

School of Automation and Electrical Engineering, Lanzhou Jiaotong University  
Lanzhou, 730070, Gansu province, China, 13659465840  
Corresponding author, e-mail: yanerwuming@126.com

### Abstract

*It is an important part to select the regulation mode in the design and manufacture of a controllable reactor of transformer type (CRT). Based on the circuit equations expressed by the self and mutual inductance, the piecewise expressions of the instantaneous current in work winding in the single-winding regulation mode are obtained, and then the formula for calculating the RMS of each harmonic current in work winding is derived by Fourier series decomposition. At last, the control characteristics of CRT and the curves of the RMS of harmonic currents with reference to the output power in three typical single-winding regulation modes are presented and their advantages and disadvantages are compared in the sample, which provides a reference for the design of CRT.*

**Keywords:** controllable reactor of transformer type, single-winding regulation mode, harmonic current

**Copyright © 2014 Institute of Advanced Engineering and Science. All rights reserved.**

### 1. Introduction

Reactive balance is very important for the secure and economical operation of the power systems[1]. By now, due to the long-term efforts of many scientific workers at home and abroad, there are various kinds of reactive-load compensation equipments have come into use [2-4]. In 1995, G.N.Aleksandrov, a Russian expert, came up with the basic circuit diagram of CRT[5], after that many scholars do further research on it [6-9], and point out that CRT has the advantages of fast response and controllable harmonic contents, which is a reactive-load compensation equipment that can apply to EHV transmission lines.

The key reason the harmonic contents of CRT can be controlled is that there is a restriction among regulation mode, control steps, capacity of each step and harmonic contents. Reference [7] presented 3 regulation modes: step-single-branch mode, fixed-single-branch mode, transfer-single-branch mode, since all of them allow only one control winding to be regulated, they can be called as single-winding regulation mode. Reference [7] did research on the 3 modes in case of neglecting the coupling of the control windings. However, the non-ignorable inductive coupling always exists among the control windings, and each of the control winding current will seriously affect the others, this leads to a great difficulty for the selection of the rated capacity of each control winding.

In this paper, taking into account of the inductive coupling among the control windings, the formulas for calculating the RMS of each harmonic current and the current harmonic coefficients are given, and then the variation trends of the output power and the curves of the RMS of harmonic currents with reference to the output power in the three typical single-winding regulation modes are presented, this will provide a reference for the design of a CRT.

### 2. Instantaneous Current of Working Winding

The basic circuit diagram of a CRT is illustrated in Figure 1, where,  $W_1$  is the high-voltage work winding, and  $W_2, W_3, \dots, W_n$  are control windings. In addition, each control winding is equipped with a current-limiting reactor ( $X_2, X_3, \dots, X_n$ ) and a thyristor switch ( $T_2, T_3, \dots, T_n$ ) which consists of two thyristors in parallel but in opposite directions. Assume that the voltage of

the grid is shown as  $u_1 = \sqrt{2}U_1 \cos(\omega t)$ , and the starting point during each period is the time when the voltage of the grid reached maximum.

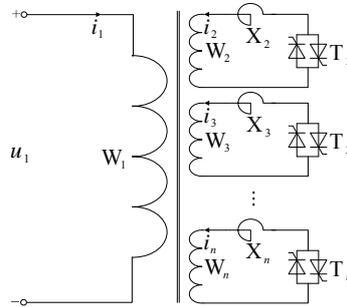


Figure 1. Basic Circuit Diagram of CRT

Including  $W_1$ , There are  $n$  windings in the CRT illustrated in Figure 1. Assume that all the parameters of all windings are referred to  $W_1$ . Neglecting the iron saturation and all resistance, the instantaneous circuit equations for all of the windings while all the  $n-1$  control windings are short-circuited are given by (1), where,  $L_k$  is the self-inductance of  $W_k$  ( $1 \leq k \leq n$ ),  $M_{kq}$  is the mutual-inductance between  $W_k$  and  $W_q$ ,  $L_{Xk}$  is the inductance of  $X_k$ , while  $X_k$  is the current-limiting reactor connected with  $W_k$  as mentioned above.

$$L_n(p\mathbf{i}_n) = \mathbf{u}_n \tag{1}$$

Where,  $L_n = \begin{bmatrix} L_1 & M_{12} & M_{13} & \dots & M_{1n} \\ M_{21} & L_2 + L_{X2} & M_{23} & \dots & M_{2n} \\ M_{31} & M_{32} & L_3 + L_{X3} & \dots & M_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & M_{n3} & \dots & L_n + L_{Xn} \end{bmatrix}$ ,  $p = d/dt$ ,  $\mathbf{i}_n = [i_1 \ i_2 \ i_3 \ \dots \ i_n]^T$ ,  $\mathbf{u}_n = [u_1 \ 0 \ 0 \ \dots \ 0]^T$ .

Assume that there are  $h-1$  ( $1 \leq h \leq n$ ) control windings are involved in the operation, and only one of them is being regulated, whose triggered angle is equal to  $\alpha$  ( $0 < \alpha < \pi/2$ ), while the other  $h-2$  are short-circuited during the period. Since the symmetry of current waveform in  $W_1$ , we just have to work out the expression for the instantaneous current in  $[0, \pi/2]$ .

The circuit equations for all of the windings in  $(0, \alpha]$  can be rewritten as:

$$L_{h-1}(p\mathbf{i}_{h-1}) = \mathbf{u}_{h-1} \tag{2}$$

Where,  $L_{h-1}$ ,  $\mathbf{i}_{h-1}$ ,  $\mathbf{u}_{h-1}$  are the submatrixes of  $L_n$ ,  $\mathbf{i}_n$ ,  $\mathbf{u}_n$  respectively, which can be obtained by removing the elements of  $L_n$ ,  $\mathbf{i}_n$ ,  $\mathbf{u}_n$  that corresponding to the regulated and open-circuited windings.

From(2), the differential equation for the work winding current is derived as:

$$\frac{di_1}{dt} = \frac{u_1}{L_{1,h-1}} \tag{3}$$

Where,  $1/L_{1,h-1}$  is the first element of the first column of  $L_{h-1}^{-1}$ , and  $L_{h-1}^{-1}$  is the inverse matrix of  $L_{h-1}$ . Since the initial condition in this time segment is  $i_1|_{\omega t=0} = 0$ , from (3),  $i_1$  is figure out as (4).

$$i_1 = \frac{\sqrt{2}U_1}{\omega L_{1,h-1}} \sin(\omega t) \quad (4)$$

In the same way, the circuit equation for work winding current in  $(\alpha, \pi/2]$  can be rewritten as:

$$\frac{di_1}{dt} = \frac{u_1}{L_{1,h}} \quad (5)$$

Where,  $1/L_{1,h}$  is the first element of the first column of  $L_h^{-1}$ , and  $L_h^{-1}$  is the inverse matrix of  $L_h$ , and  $L_h$  is the submatrixes of  $L_n$ , which can be obtained by removing the elements of  $L_n$  that corresponding to the open-circuited control windings. Since the final value of the former time segment is the initial condition of the current time segment, from (5),  $i_1$  can be figure out as (6).

$$i_1 = \frac{\sqrt{2}U_1}{\omega L_{1,h}} \sin(\omega t) - \left( \frac{\sqrt{2}U_1}{\omega L_{1,h}} - \frac{\sqrt{2}U_1}{\omega L_{1,h-1}} \right) \sin \alpha \quad (6)$$

### 3. Formulas for Calculating Harmonics

The waveform of  $i_1$  is symmetric among 4 quarter periods, which contains only fundamental and odd harmonics. Hence,  $i_1$  can be expressed as a Fourier series like this.

$$i_1 = \sum_{k=1}^{\infty} \{ b_{2k-1} \sin[(2k-1)\omega t] \} \quad (7)$$

Where,

$$b_{2k-1} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} i_1(\omega t) \sin[(2k-1)\omega t] d(\omega t) \quad (k=1,2,3,\dots) \quad (8)$$

From (4), (6), (8), the RMS of the  $2k-1$  ( $k=1,2,3,\dots$ )th harmonic current in the work winding can be derived as (9):

$$I_{2k-1} = \frac{U_1}{\pi \omega} \left| \frac{1}{L_{1,h}} f_k(\pi/2) + \left( \frac{1}{L_{1,h-1}} - \frac{1}{L_{1,h}} \right) f_k(\alpha) \right| \quad (9)$$

Where,

$$f_k(\alpha) = \frac{1}{2k-1} \left[ \frac{\sin 2(k-1)\alpha}{k-1} + \frac{\sin 2k\alpha}{k} \right] \quad (k=2,3,4,\dots) \quad (10)$$

The formula(9) clarifies the relationship not only between the RMS of each current harmonic and triggered angle, but also between the harmonic components and the parameters of self and mutual inductance, which is very important for the design of CRT.

The  $2k-1$  th current harmonic coefficient for the work winding can be derived

$$k_{H,2k-1} = \frac{I_{2k-1}}{I_1} = \left| \frac{f_k(\alpha_h)}{\pi L_{1,h-1} / (L_{1,h} - L_{1,h-1}) + f_1(\alpha_h)} \right| \quad (11)$$

**4. Analysis of Examples**

Taking CRT described in [7] as an illustration, the 3 single-winding regulation modes are analysed in the case of taking account of the coupling of the control windings in this paper. The CRT described in [7] has 6 windings(including  $W_1$ ), the rated voltage is  $U_N=500/\sqrt{3}kV$ . Based on the self and mutual impedance which can be converted into the self and mutual inductance and the inductance of current-limiting reactors which can be calculated by the recursion algorithm,  $L_n$  can be expediently obtained, in this case,  $n=6$ , Hence,  $L_6$  is (unit:H)

$$L_6 = \begin{bmatrix} 452.83 & 451.40 & 451.36 & 451.27 & 451.17 & 450.98 \\ 451.40 & 538.36 & 452.67 & 452.57 & 452.41 & 452.16 \\ 451.36 & 452.67 & 524.34 & 452.64 & 452.48 & 452.22 \\ 451.27 & 452.57 & 452.64 & 482.61 & 452.57 & 452.32 \\ 451.17 & 452.41 & 452.48 & 452.57 & 462.89 & 452.48 \\ 450.98 & 452.16 & 452.22 & 452.32 & 452.48 & 453.99 \end{bmatrix}$$

Referring to [7], a 5-digit binary code is used to represent the state of each thyristor switch, there is a one-to-one correspondence between per bit of this code in sequence from high to low and each control winding of CRT from  $W_6$  to  $W_2$ , where, "0" means the corresponding control winding is open-circuited, and the triggered angle of its thyristor is equal to  $90^\circ$ ; "1" means short-circuited, and the triggered angle of the corresponding thyristor is equal to  $0^\circ$ ; "1" means regulation, the corresponding triggered angle falls somewhere in between those two.

During the process that the output power of CRT changes from no-load to the rated, the variation range of output power can be divided into 5 steps in step-single-branch mode, 16 steps in fixed-single-branch mode, and 15 steps in transfer-single-branch mode in this case.

According to the above description, the regulating processes of the 3 operation modes are as follows. The double arrow in the processes indicates that the process is reversible, where, " $\leftrightarrow$ " presents a instantaneously shifted process of winding currents, while " $\rightleftharpoons$ " means a smooth power regulation due to the change of triggered angle of the thyristor switch.

(1) Step-single-branch mode

$$00000 \rightleftharpoons 00001 \rightleftharpoons 00011 \rightleftharpoons 00111 \rightleftharpoons 01111 \rightleftharpoons 11111$$

(2) Fixed-single-branch mode

$$\begin{aligned} & \overbrace{00000 \rightleftharpoons 00001}^{(1)} \rightleftharpoons \overbrace{00010 \rightleftharpoons 00011}^{(2)} \rightleftharpoons \overbrace{00100 \rightleftharpoons 00101}^{(3)} \rightleftharpoons \overbrace{00110 \rightleftharpoons 00111}^{(4)} \rightleftharpoons \overbrace{01000 \rightleftharpoons 01001}^{(5)} \rightleftharpoons \overbrace{01010 \rightleftharpoons 01011}^{(6)} \\ & \rightleftharpoons \overbrace{01100 \rightleftharpoons 01101}^{(7)} \rightleftharpoons \overbrace{01110 \rightleftharpoons 01111}^{(8)} \rightleftharpoons \overbrace{10000 \rightleftharpoons 10001}^{(9)} \rightleftharpoons \overbrace{10010 \rightleftharpoons 10011}^{(10)} \rightleftharpoons \overbrace{10100 \rightleftharpoons 10101}^{(11)} \rightleftharpoons \overbrace{10110 \rightleftharpoons 10111}^{(12)} \\ & \rightleftharpoons \overbrace{11000 \rightleftharpoons 11001}^{(13)} \rightleftharpoons \overbrace{11010 \rightleftharpoons 11011}^{(14)} \rightleftharpoons \overbrace{11100 \rightleftharpoons 11101}^{(15)} \rightleftharpoons \overbrace{11110 \rightleftharpoons 11111}^{(16)} \end{aligned}$$

(3) Transfer-single-branch mode

$$\begin{aligned} & \overbrace{00000 \rightleftharpoons 00001}^{(1)} \rightleftharpoons \overbrace{00010 \rightleftharpoons 00011}^{(2)} \rightleftharpoons \overbrace{00100 \rightleftharpoons 00101}^{(3)} \rightleftharpoons \overbrace{00110 \rightleftharpoons 00111}^{(4)} \rightleftharpoons \overbrace{01000 \rightleftharpoons 01001}^{(5)} \rightleftharpoons \overbrace{01010 \rightleftharpoons 01011}^{(6)} \\ & \rightleftharpoons \overbrace{10010 \rightleftharpoons 10011}^{(7)} \rightleftharpoons \overbrace{10100 \rightleftharpoons 10101}^{(8)} \rightleftharpoons \overbrace{10110 \rightleftharpoons 10111}^{(9)} \rightleftharpoons \overbrace{11000 \rightleftharpoons 11001}^{(10)} \rightleftharpoons \overbrace{11010 \rightleftharpoons 11011}^{(11)} \rightleftharpoons \overbrace{11100 \rightleftharpoons 11101}^{(12)} \\ & \rightleftharpoons \overbrace{11110 \rightleftharpoons 11111}^{(14)} \rightleftharpoons \overbrace{11110 \rightleftharpoons 11111}^{(15)} \end{aligned}$$

Figure 3 shows the piecewise curves of the RMS of fundamental current in work winding with reference to the triggered angle during each step, namely, the control characteristic. The number in each parenthesis in Figure 3 represents the step number, which is the same with what in the regulating processes mentioned above. The corresponding triggered angle during each step reduces from  $90^\circ$  to  $0^\circ$  in both Figure 3(a) and Figure 3(b), which is different from Figure 3(c). In Figure 3(c), the triggered angle of the  $i$ th step reduces from a specific angle  $\theta_i$ , called as starting regulating angle that varies among different steps. In this case, the calculated values of  $\theta_i$  are given in Table 1.

Table 1. Starting Regulating Angle of each Step in Transfer-single-branch Mode

$i$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\theta_i(^{\circ})$	90.0	7.19	27.1	29.1	31.8	90.0	3.90	23.1	25.0	90	3.40	22.4	90.0	3.20	90.0

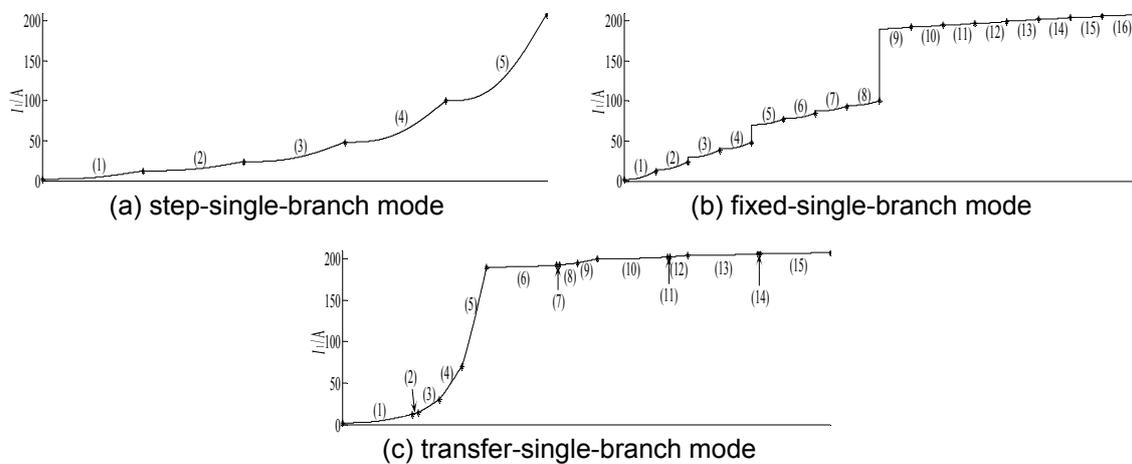


Figure 3. RMS of Fundamental Current vs Step Numbers

Figure 3(b) shows that the output power is intermittent when the CRT operating in fixed-single-branch mode, which is mainly due to that the capacity assignment of each control winding can't satisfy the design requirement for continuation, in fact, the capacity of each control winding mainly depends on the value of its current-limiting reactor. Hence, it is of great importance to select an appropriate value for each current-limiting reactor in order to satisfy the continuation of output power. Figure 3(a) and Figure 3(c) shows that the output power is continuous when CRT operating in step-single-branch mode and transfer-single-branch mode, but the regulation of output power mainly depends on the fourth and fifth step in this case, and the capacity of the regulating control windings in these two steps are larger than others, this is unreasonable in the practical application.

Figure 4 shows respectively the variation tendency of the RMS of the 5th, 7th, 11th, 13th harmonic current and the total harmonic current components (denoted as  $I_H$ ) that poured into power system with reference to the RMS of the fundamental current in work winding.

The curves of the RMS of each harmonic current with reference to the output power have a trend of increase in both step-single-branch mode and transfer-single-branch mode, while they are steady in fixed-single-branch mode. Furthermore, the waveform distortion in transfer-single-branch mode is the most serious among 3 modes, and the waveform distortion in fixed-single-branch mode is much less than the other two. Analysing the trends of the above curves combining with each step, we can find that it will lead to larger harmonic current in work winding if a control winding of larger capacity is regulated.

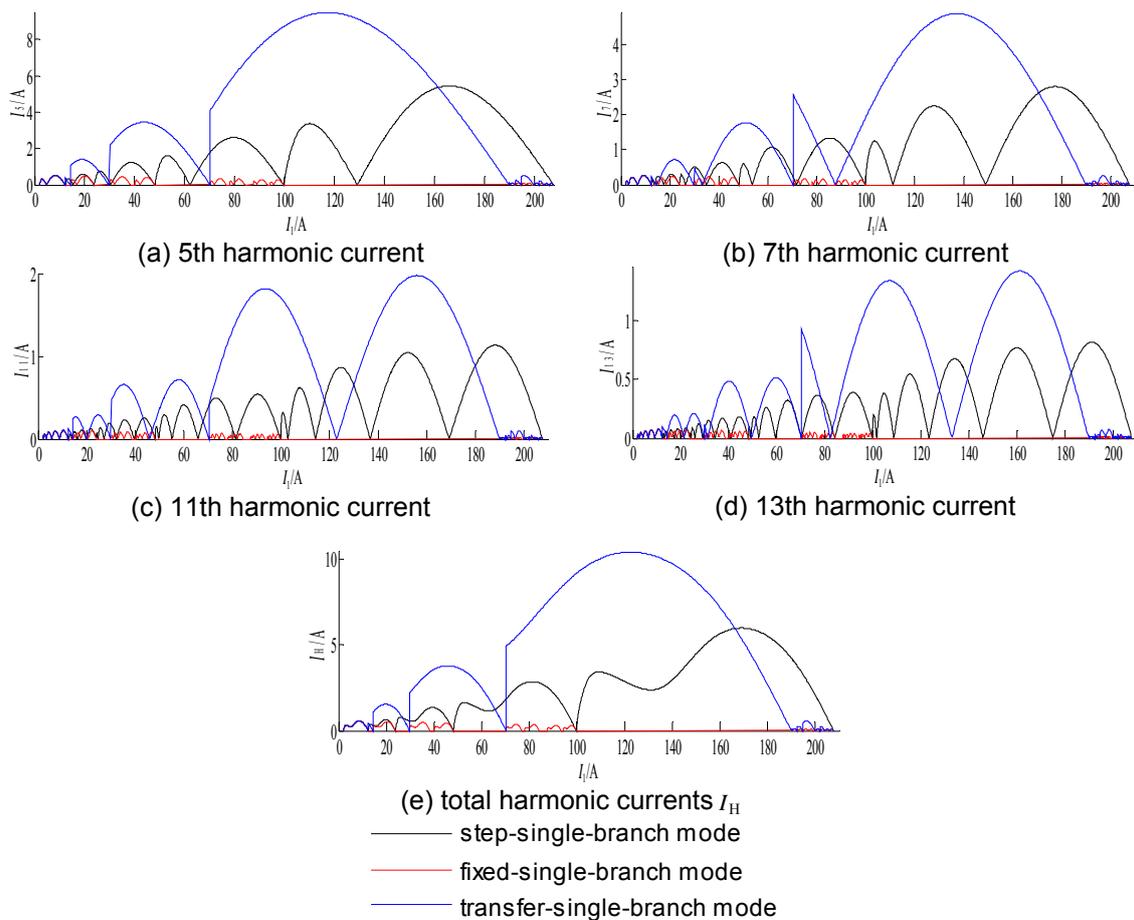


Figure 4. RMS of every Harmonic Currents and  $I_H$  vs output Power

#### 4. Conclusion

(1) The RMS of harmonic currents in work winding of all kinds of single-winding regulation mode in the case of considering the coupling among control windings can be calculated by an unified formula.

(2) Operating in the single-winding regulation mode, the control winding of small capacity should be regulated to reduce the harmonic contents in the work winding on the premise that the continuation of the output power have been ensured.

#### Acknowledgements

This study is Supported by National Natural Science Foundation of China (51167009), National Natural Science Foundation of China (51367010) and Science and Technology Program of Gansu Province (1304WCGA181).

#### References

- [1] Zhaoan Wang, Jun Yang, Jinjun Liu. Harmonic Suppression and Var Compensation. Second Edition. Beijing: Machine Press. 2005: 6-11 [in Chinese].
- [2] Nie Hongzhan, Wang Zhenhao. *Development of Multi-function Var Compensation Controller for Low-voltage Distribution Transformer*. Electric Power Automation Equipment. 2004; 24(8): 50-52 [in Chinese].
- [3] Yan Guoqi, Hang Naishan, Li Ruqi, et al. *A New Type Reactive Power Compensation Equipment*. Electric Power Automation Equipment. 2004; 24(6): 68-71 [in Chinese].

- 
- [4] Liu Shuming, Li Qionglin, Du Xizhou, et al. *Parameter Matching of Series Reactor in Reactive Compensation Capacitor Bank*. *Electric Power Automation Equipment*. 2012; 32(4): 145-150 [in Chinese].
  - [5] G.N.Aleksandrov, BI Al'bertinskij, IA Shkuropat. Operational Principles of a Controlled Shunting Reactor of the Transformer Type. *Russian Electrical Engineering*, 1995; 66(11): 42-47.
  - [6] Tian Mingxing, Li Qingfu, Li Qunfeng. A Controllable Reactor of Transformer Type. *IEEE Transactions on Power Delivery*. 2004; 19(4): 1718-1726.
  - [7] Tian Mingxing. Basic Theoretical Research on Controllable Reactors of Transformer Type. PhD Thesis. Xi'an: Xi'an Jiaotong University. 2005 [in Chinese].
  - [8] Zhou Lawu. The Theory and It's Application on New Type Ultra-high Voltage Controlled Reactor. PhD Thesis. Changsha: Hunan University. 2008 [in Chinese].
  - [9] Zhang Yu. Research on a Novel Transformer-type Controllable Reactor. PhD Thesis. Wuhan: Huazhong University of Science & Technology. 2009 [in Chinese].