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Compressed Sensing High-accuracy Detection for Electric Power Interharmonics

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Abstract

Interharmonics frequencies are not integer multiple of the fundamental frequency, and interharmonics amplitudes are far less than fundamental amplitude and harmonics amplitudes, which mean high sensitivity to desynchronization problems, so it's dificult to estimate interharmonics. In this paper, a new method based on random sparse sampling and compressed sensing (CS) Bregman technique was proposed to estimate the interharmonics. The random sam pling has following advantages; alias-free, sampling frequency need not obey the Nyquist limit, and higher frequency resolution. So the random sampling can measure the signals which their frequencies component are close, and can implement the higher frequencies measurement with lower sampling frequency. However, the random sampling exists the noise in spectrum analysis, so it's difficult to estimate the low amplitude signals. Compressed sensing can work out this problem by designing observation matrix and with the sparsity reconstruction of the signal in the Fourier domain; in addition, the application of CS can estimate the amplitudes and phases of the signals exactly. The results ofexperiments show that the proposed method can estimate the interharmonics exactly,even if the interharmonics frequencies are close the fundamental frequency and interharmonics amplitudes are far less than fundamental amplitude and can measure high order interharmonics with lower sampling frequency.

Keywords*: compressed sensing, interharmonics, random sampling, sparse sampling, spectrum analysis*

1. Introduction

With the widespread use of nonlinear loads, power system, a large number of frequency of the fundamental frequency of aneuploidy between the harmonics [1]. Pollution caused by the presence of harmonics and interharmonics on the power system environment must be right to effective governance, and accurate detection of harmonics is that the premise of governance. However, inter-harmonic characteristics determine its detection is difficult for harmonic detection. First, the inter-harmonic frequency of the fundamental frequency of aneuploidy is often difficult to determine the cycle of the waveform contains interharmonics. Harmonic and the fundamental inter-harmonic frequency domain is less than one working frequency, which means that the higher harmonics in the detection of inter-frequency resolution. Between harmonic amplitude is often far less than the amplitude of the fundamental and harmonic components, which means that the harmonic component of spectral leakage with high sensitivity, inter-harmonic and fundamental and harmonic components the frequency is close to, this effect is more pronounced. Therefore, the inter-harmonic analysis method should have the following characteristics: by non-synchronous sampling is small; sampling time should not be too long, so as to avoid before and after the collection of data from the same signal; with high frequency resolution.

Accuracy due to non-synchronous sampling and data truncated, using the Fast Fourier Transform (FFT) algorithm for harmonic analysis to produce spectral leakage and fence effect, the impact of harmonic analysis [2-3]. To reduce such errors, the scholars based on the rectangular window [4], Hanning window [5], the Hamming window [6] Blackman, windows [7], Blackman-Harris window [8], the Kaiser window [9] such as signal windows and interpolated FFT algorithm can reduce encountered alone FFT spectrum leakage and fence effect, improve the detection accuracy of the harmonic parameters, but can not be detected near the integer harmonics asked harmonic; adopt fundamental and harmonic parameter estimation based on

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higher order cosine combination window of the spectrum [5, 7, 10] or more lines [11-12] interpolation FFT algorithm, the solution of higher equations [13-15], computing complex; continuous wavelet transform [16-17] to achieve the detection of inter / harmonic, but there is mutual interference of different scales of the wavelet function in the frequency domain, when the signal to be detected with similar frequency harmonic to detect method will fail; on Prony method [18-19] is harmonic, between harmonic analysis and modeling effective way to accurately estimate the sinusoidal component of frequency, amplitude and phase angle, but it needs to solve two odd equation and polynomial time, the high computational complexity and noise-sensitive; there are other methods [20-22], or limited frequency resolution, or large calculation, there are limitations in the specific application.

Designed a compressed between the perception of harmonic detection methods, time domain is lower than the Nyquist theory of random sampling, Bregman, frequency-domain reconstruction with high accuracy detection signal all the harmonics and interharmonics frequency, amplitude and phase. In this paper, a theoretical analysis and calculation of derivation, random sampling could be circumvented by Fourier domain spectral leakage, picket fence effect, as well as non-integer times a wave phenomenon. The simulation results show that: the proposed algorithm can effectively eliminate all the harmonics interfere with each other to improve the accuracy of signal analysis, harmonic analysis [23-29].

2. Random Sampling and Analysis

2.1. The Drawbacks of Uniform Sampling

Uniform sampling of a function of time is a linear function of the standard, such as sampling time interval distribution. Define the sampled signal $x(t)$, the sampling interval Δt, the sampling time point $t_n = n\Delta t$, the sampling frequency $f_s = \frac{1}{\Delta t}$, and to meet the sampling theorem, is greater than 2 times the highest frequency of the value signal. For a limited length of the sampled signal discretization, ie $x[n] = x([1:N]\Delta t)$, N is sampling points, sampling duration $T = N\Delta t$.

By Fourier transform analysis of sampled signals $x(t) = \sin(2\pi ft), (f = 185Hz, N = 256, f_s = 256Hz)$. Signal spectrum analysis results shown in Figure 1.

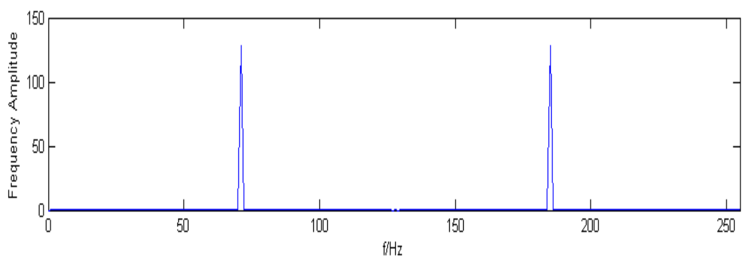

Figure 1. Signal Spectrum Analysis of Uniformly Sampling (fs=256Hz)

It can be seen from Figure 1, the sampling frequency is less than 2 times of the signal real frequency value, frequency value $f = 185Hz$ of the aliasing signal 71Hz. The real signal spectrum is not distinguished because the aliasing signal spectrum is equal to the true signal. Also noted that the cases in the frequency resolution of 1Hz, the signal frequency is an integer multiple of the frequency resolution, so the ability to accurately measure the frequency value.

In the previous cases, other parameters constant, the changing sampling frequency is $f_s = 512Hz$, to meet the limitations of the sampling theorem. Signal spectrum analysis results shown in Figure 2.

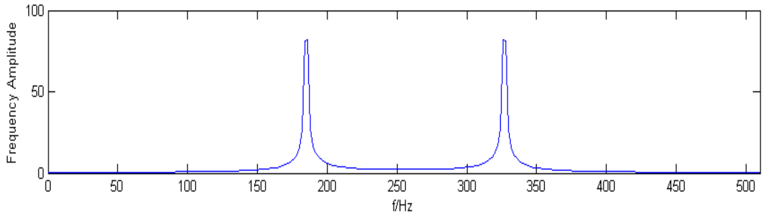

Figure 2. Signal Spectrum Analysis of Uniformly Sampling (fs=512Hz)

Figure 2 shows the aliasing signal (0, $f_s/2$) band, but due to changes in sampling frequency, the frequency resolution becomes 2Hz, signal the true frequency of families 185Hz is not an integer multiple of the frequency resolution, thus leading to spectral leakage and fence phenomenon, so that the measured frequency value is $f = 188Hz$ which is deviated from the correct value.

As can be seen from the above analysis, uniform sampling is limited by the sampling frequency limit; aliasing frequency; frequency resolution is not high, there is the problem of spectral leakage and fence phenomena.

2.2. Random Sampling and its Fourier Transform

Random sampling, sometimes called non-uniform sampling, as opposed to uniform sampling of a sampling method. The sampling interval random sampling is random, the time interval is generally set to unequal intervals, not a linear function of the sampling points and sampling time. Random sample from the sampling theorem limit, increasing the frequency detection range can be detected in the short length of the data, low sampling frequency to the higher order frequency, allowing real-time to quickly meet the requirements of a particular occasion. The most important thing is that the random sampling of non-uniform sampling can eliminate signal aliasing problems caused by uniform sampling; also has the advantage of high frequency resolution, reducing the spectrum leak to eliminate the problem of the fence phenomena.

In the example above, the other parameters constant, $t_n = rand(0,1)T = g(n), x(n) = x(t_n)$,the switch to random sampling, where rand (0,1) random number between (0,1), $n = 1,2,\cdots,N$,g (n) is a nonlinear function of n. Fourier transform:

$$X(\omega) = \sum_{n=1}^{N} x(n)\exp(-j\omega t_n) \quad (1)$$

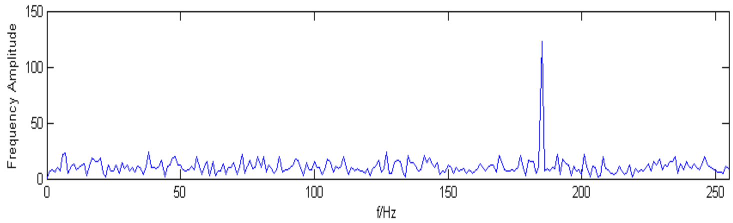

Figure 3. Signal Spectrum Analysis of Random Sampling (Average fs=256Hz)

Random sample (N = 256), Fourier transform spectrum analysis results shown in Figure 3. The use of random sampling time sampling interval increases, the frequency resolution to eliminate the phenomenon of the fence. As a result of random sampling, the aliasing signal will no longer concentrated on some special points and the sampling frequency, but evenly distributed to all of the signal frequency band. In addition, spectral leakage will cause the spectrum noise. The spectrum noise can be reduced with the increase of sampling points.

3. Compressed Sensing Principle

3.1. Compressed Sensing Representation

Compressed Sensing (CS) theory main idea is: Suppose a length Nof the signal x on an orthogonal basis or tight frame coefficients Ψ is sparse (ie only a few non-zero coefficient), the coefficients of projection to another $\Phi : M \times N$ ($M << N$)not related to a transform-based observations Ψ, the collection of the observations $y{:}M \times 1$. Signal x is accurately recovered by solving an optimization problem in virtue of these observations.

First, if the signal $x \in R^N$ on an orthogonal basis or tight frame Ψ is compressible, the obtained transform coefficients $\alpha = \Psi^T x$, α is the equivalent or sparse approximation of x; the second step, to design a stable, not related to the transform-based Ψ, $M \times N$ dimension observation matrix Φ to observe x the upcoming projected onto the M-dimensional space, observing a collection $y = \Phi x$ of the process for the compression of the sampling process, namely the taking of samples [26-29]. Finally, the use of optimization problem solving the x's exact or approximate approximation $\hat{x}$.

When the noise z observations,

$$y = \Phi x + z \tag{2}$$

It can be transformed for the sake,

$$\min_x \| \Psi^T x \|_1 \qquad s.t. \| y - \Phi x \|_2 < \varepsilon \tag{3}$$

Or,

$$\hat{x} = \arg\min_x \frac{1}{2} \| y - \Phi x \|_2 + \lambda \| \Psi^T x \|_1 \tag{4}$$

3.2. Separable Bregman Iterative Algorithm to Restore the Signal

Problem (4) to solve the first converted to the sparse vector (5) to solve, $A = \Phi\Psi$, then:

$$\hat{\alpha} = \arg\min_\alpha \frac{\lambda}{2} \| y - A\alpha \|_2^2 + \| \alpha \|_1 \tag{5}$$

Bregman algorithm [17-20], specific steps are as follows:

1) Calculate: $B = (\lambda A^T A + I_N)^{-1}$, I_N is N-dimensional unit matrix, $F = \lambda A^T y$; b_0, d_0 are for the N-dimensional zero vector.
2) Given $\lambda(=10)$, iteration termination conditions $\delta(=0.001)$, the number of iterations $n = 1$.

3) Calculate: $\alpha_n = B(F + d_{n-1} - b_{n-1})$, $d_n = sign(\alpha_n + b_{n-1})\max(|\alpha_n + b_{n-1}| - 1, 0)$,
4) $b_n = b_{n-1} + \alpha_n - d_n$.
5) If $\| \alpha_n - \alpha_{n-1} \| \geq \delta$, $n = n + 1$, Go to Step 3); Otherwise, stop the iteration, $\hat{\alpha} = \alpha_n$

3.3. Signal Low-speed Sampling Design with the Observation Matrix

The design of the observer is to design efficient observation matrix can capture the design of a sparse signal useful information efficiency of the observations (ie, sampling) protocol, which the sparse signal is compressed into a small amount of data. These agreements is non-adaptive, only need a small amount of the fixed waveform and the original signal linking these fixed waveform and signal to provide a compact representation of the base. In addition, the observation process is independent of the signal itself. Using an optimized reconstructed signal can gather a small number of observations.

Sampling interval [0, T] in this interval were collected randomly M points, $t_i = rand(0,1)T, i = 1,2,\cdots,M$, $rand(0,1)$ are random points between (0,1) ; $x = [x(t_1), x(t_2), \cdots, x(t_M)]^T$, $y = x \in R^M$; Interval reconstruction of the complex frequency domain N-dimensional $\alpha \in C^N, M << N$. Compressed sensing harmonic detection is to find a mapping: $F: R^M \rightarrow C^N$.

Design a random observation matrix Φ :

$$\Phi(m,n) = \frac{1}{N} \sum_{l=-N/2+1}^{N/2} \exp(2\pi i \frac{l}{N}(\frac{t_m}{T_s} - n)), m = 1,2,\cdots,M; n = 1,2,\cdots,N \quad (6)$$

T_s is uniform time-domain reconstruction of the equivalent of N-point sampling interval, ψ is Fourier-based, $\psi_j(t) = N^{-1/2} e^{i2\pi jt/N}, j = 1,2,\cdots,N$. This design to meet irrelevant Φ and ψ limitations of isometric resistance. $A = \Phi\Psi$, This random sample of observations random characteristics. Observation matrix of random unrelated characteristic is a sufficient condition for the right to restore the signal, the height of the observation matrix and signal irrelevant to ensure the effective restoration of the signal.

3.4. Implementation Steps of the Harmonic Detection in Compressed Sensing

1) In the time domain given interval were collected randomly M points, the point sequence for the observation vector;
2) Reconstruct the N points in this interval T_s, which have frequency-domain resolution

$$f = \frac{1}{NT_s} Hz$$

3) By (6), the design of the $M \times N$ observation matrix Φ, the $N \times N$ order inverse Fourier transform matrix Ψ;
4) Bregman algorithm for reconstruction of complex N-point frequency domain;
5) Given the magnitude of the threshold, when the reconstruction of the frequency domain is larger than the threshold, find the appropriate frequency, amplitude and phase.

4. Experimental Evaluation

Signal contains the fundamental, harmonics and harmonics, and their parameters in Table 1, the expression:

Table 1. Truth Component of the Signal & their Testing Result

	Actual value			Detection value		
Waveform	Freq./Hz	Margin/V	Phase/℃	Freq./Hz	Margin/V	Phase/℃
Fundamental	50.00	35.0000	0.0000	50.00	34.9816	0.0056
Interharmonic	75.00	5.0000	155.0000	75.00	4.9715	154.9602
Harmonic	150.00	7.0000	35.0000	150.00	6.9858	34.9871
Interharmonic	175.00	3.0000	50.0000	175.00	2.9770	49.9232
Harmonic	250.00	4.0000	70.0000	250.00	3.9800	69.9572
Harmonic	350.00	1.1250	115.0000	350.00	0.9840	115.0625

$$x(t) = \sum_{i=1}^{6} A_i \cos(2\pi f_i t + \phi_i) \quad (7)$$

The highest signal frequency: $f_{\max} = 350Hz$, Were collected randomly in one second of time M = 256 points, its random equivalent sampling frequency $f_s = 256Hz < f_{\max}$, Random equivalent sampling frequency is much less than 2 times the highest signal frequency, does not meet the Nyquist sampling theorem; If the frequency of such uniform sampling, the Fourier transform of the existence of a certain spectrum aliasing and leakage, is not possible to detect tothe signal harmonics. Mining M= 256 using this method reconstruct the frequency domain N = $3 \times 256 = 768$ point, resolution 1Hz, test results are in Table 1 the right side of Figure 4. Harmonic frequency, amplitude, initial phase of the true values and measured values are plotted on the same plot, the results are very accurate.

Figure 4 shows the original signal, the sampling points and the reconstruction of time-domain signal, the picture shows the original signal and reconstructed signal amplitude-frequency diagram, the lower part of the original signal and reconstructed signal frequency diagram; time domain reconstruction of the signal relative error Relativeerror = 0.0014.

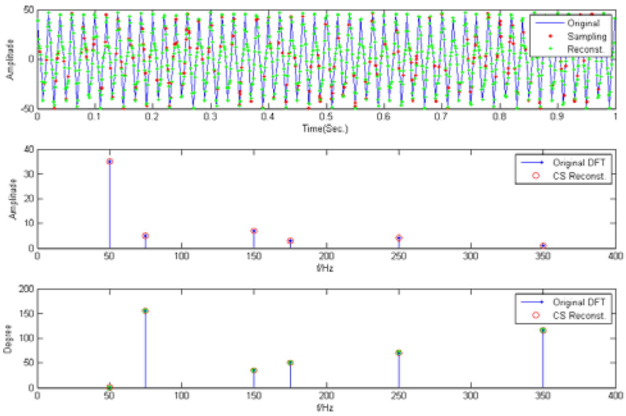

Figure 4. The Inter-harmonic Compressed Sensing Detection

5. Conclusion

Random sampling technique as a non-uniform sampling method can effectively improve the sampling rate of the sampling system. In random sampling, the sampling time interval of non-uniform distribution can not be collected enough sample values for signal reconstruction. The high precision of the power harmonic analysis for electric metering, harmonic power flow calculation, equipment, network testing, power system harmonic compensation, and inhibition of great significance. In this paper, the signal is sparse in the Fourier transform, to design a random observation matrix, then sparse sampling; the use of Bregman iterative algorithm successfully restored the signal. This method without adding any hardware costs on the basis of the limited random sampling value reconstruction frequency domain signal. The experiments showed that frequency-domain sparse signal well below the sampling rate of the signal Nyquist frequency sampling, compressed sensing signal reconstruction algorithm can accurately reconstruct the frequency-domain signal through the method of this paper, high-precision detection of signal of each harmonic and inter-harmonic frequency, amplitude and phase. A theoretical analysis and calculation of derivation of this method to circumvent the Fourier domain spectrum leakage, picket fence effect, and non-integer times a wave phenomenon. The proposed algorithm can effectively eliminate all the harmonics interfere with each other to improve the accuracy of signal analysis, suitable for high accuracy harmonic analysis.

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