

QR-based Channel Estimation for Orthogonal Frequency Division Multiplexing Systems

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Abstract

Channel estimation is the key technology for Orthogonal Frequency Division Multiplexing (OFDM), which has direct impact on the performance of OFDM. In this paper, we present a novel QR-based algorithm to update the channel impulse response (CIR) for DFT-based channel estimation. The discrete Fourier transform (DFT) estimation reduces the noise power that exists outside of the CIR part, because the estimated CIR from LS has most of its power concentrated on the first L samples. To reduce the noise power that exists inside of the first L samples, the CIR is further processed by QR decomposition in proposed algorithm. The simulation results show that the bit-error-rate (BER) of our estimator has reduced significantly compared with the conventional DFT-based channel estimator and LS-linear estimation.

Keywords: OFDM, channel estimation, QR

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1. Introduction

OFDM is an efficient technique for high data rate wireless communication system due to its strong anti-interference capability, high spectrum efficiency and robustness to frequency selective fading channel [1-2]. In the single carrier transmission system, the adaptive equalization is applied to eliminate inter-symbol interference (ISI), which increases the difficulty of receiver design for the complexity of the equalizer and the algorithm structure. However, the ISI caused by multipath propagation can be effectively reduced by inserting cyclic prefix (CP) in the OFDM system. Being one of the most suitable transmission technology of broadband communication, OFDM has been adopted as a standard for digital audio broadcasting (DAB), Digital Video Broadcasting-Terrestrial (DVB-T), wireless LAN (802.11), wireless WAN (802.16) [3] and so on since the 1980s. Furthermore, with the development of modern digital signal processing technology, OFDM has become an efficient modulation scheme for the fourth-generation (4G) wireless mobile communications.

Channel estimation is an important issue for OFDM system. Since the communication data distributes in frequency domain and time domain, the channel estimation is carried out in frequency domain and time domain as well. The time-domain-based channel estimation obtains CIR by processing the time-domain data, then transforms it to channel frequency response (CFR) utilizing DFT. Channel estimation in frequency domain for OFDM systems is often carried out by using pilots, which are some known sequences. As shown in Figure 1, there are two classical pilot patterns, which are the block-type pattern by inserting pilot tones into all of the subcarriers of OFDM symbols and the comb-type pattern by inserting pilot tones into each OFDM symbols. The former one is susceptible to fast fading channel, the later one is robust to frequency selective fading environment.

The LS-based algorithm, as the simplest channel estimation method in frequency, is susceptible to Gaussian noise dramatically [4-5]. While the minimum mean-square error (MMSE) estimator shows good performance [6], it requires knowledge of the channel statistics and the signal-to-noise ratio (SNR), which inevitably increases computational complexity. Some studies turned to LMMSE [7] and SVD [8] decomposition to reduce the computational complexity, Zhou W and Lam WH reduce complexity by using the fast Fourier transform (FFT) operation [9], however the computational complexity remains high. As [10] showed, the DFT-based estimator improves CIR available from the LS estimator by limiting the number of channel

taps in time domain. The performance is improved by deciding significant channel taps adaptively without information on the channel statistics in [11]. Nevertheless, all the methods above only deal with noise outside of L taps, noise inside of L taps is ignored.

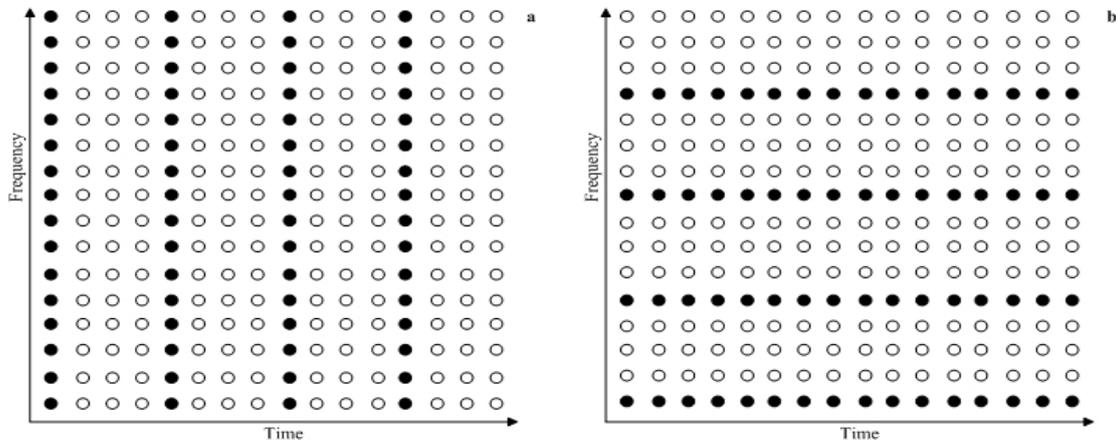


Figure 1. Pilot Sub-carrier Arrangement: Block-type (a) and Comb-type (b)

In this paper, we propose a novel QR-based [12] algorithm to reduce the noise inside of L taps. In order to avoid high computational complexity, we choose the LS estimator, then reduce the noise outside of L taps by DFT estimator, finally update CIR by processing data matrix in time domain by QR decomposition. The results of computer simulations show the proposed algorithm achieves lower BER and better communication performance.

2. System Model

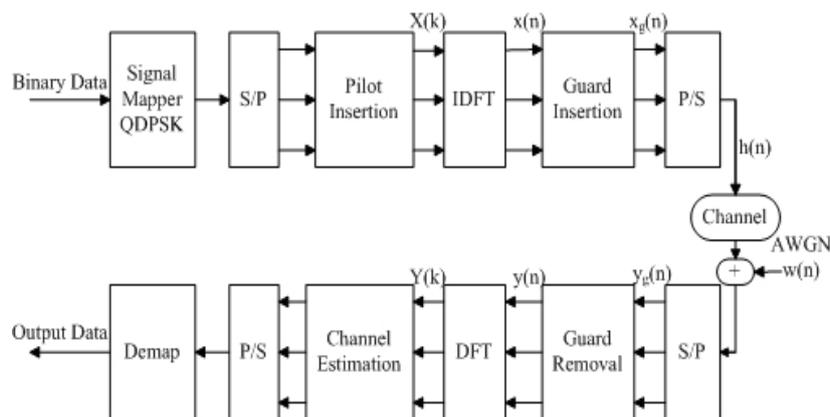


Figure 2. Baseband Model of a Typical OFDM System

Figure 2 shows a typical block diagram of OFDM system with pilot signal assisted. The binary information data are grouped and mapped into multi-amplitude-multi-phase signals. After comb-type pilot insertion, the modulated data $X(k)$ are sent to an IDFT block and are multiplexed into $x(n)$.

$$x(n) = IDFT\{X(k)\} = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \exp(j2\pi \frac{nk}{N}), 0 \leq n \leq N-1, \quad (1)$$

Assuming the number of subcarriers is N . In order to avoid ISI and consequent inter-carrier interference (ICI), a CP is appended to the OFDM symbol. Assuming the length of CP N_g is greater than maximum time delay, then the ISI is removed.

$$x_g(n) = \begin{cases} x(N_g + n), n = -N_g, N_g + 1, \dots, -1 \\ x(n), n = 0, 1, \dots, N - 1 \end{cases} \quad (2)$$

After passing through a multipath channel, one received discrete-time domain OFDM signal $y_g(n)$ is represented by:

$$y_g(n) = x_g(n) \otimes h(n) + w(n) \quad (3)$$

Where \otimes denotes cyclic convolution operation, $w(n)$ is Additive White Gaussian Noise (AWGN) and $h(n)$ is CIR. It can be represented by:

$$h(n) = \sum_{i=0}^{\tau-1} h_i \exp(j2\pi f_{D_i} T \frac{n}{N}) \cdot \delta(\lambda - r_i), 0 \leq n \leq N - 1 \quad (4)$$

Where τ is the total number of transmit paths, h_i is the complex impulse response of the i th path, f_{D_i} is the i th path Doppler frequency shift, λ is the delay spread index, and r_i is the i th path delay time normalized by sampling time. Removing the CP in the receiving side:

$$y(n) = y_g(n + N_g), 0 \leq n \leq N - 1 \quad (5)$$

Then $y(n)$ is sent to DFT block for the following operation:

$$Y(k) = DFT\{y(n)\} = \frac{1}{N} \sum_{k=0}^{N-1} y(n) \exp(-j2\pi \frac{nk}{N}), 0 \leq n \leq N - 1 \quad (6)$$

Assuming there is no ISI, I is the ICI caused by Doppler frequency shift, the system frequency-domain model is represented by:

$$Y(k) = X(k)H(k) + I(k) + W(k), 0 \leq k \leq N - 1 \quad (7)$$

Where:

$$H(k) = \sum_{i=0}^{\tau-1} h_i \exp(j2\pi f_{D_i} T \frac{\sin(\pi f_{D_i} T)}{\pi f_{D_i} T}) \cdot \exp(-j2\pi \frac{r_i}{N} k) \quad (8)$$

$$I(k) = \frac{1}{N} \sum_{i=0}^{\tau-1} \sum_{\substack{K=0, \\ K \neq k}}^{N-1} h_i X(K) \cdot S \cdot \exp(\frac{j2\pi r_i}{N} K)$$

$$S = \frac{1 - \exp[j2\pi (f_{D_i} T - k + K)]}{1 - \exp[j \frac{2\pi}{N} (f_{D_i} T - k + K)]} \quad (9)$$

The pilot signal $Y(p)$ is extracted from receiving signals, and the CFR $\tilde{H}(p)$ for pilot sub-channels by LS estimator is:

$$\tilde{H}(p) = \frac{Y(p)}{X(p)} \quad (10)$$

Where p is the index of pilot sub-channels. The CFR for data sub-channels is obtained by interpolation.

3. QR-based Channel Estimation

Figure 3 gives a block diagram of DFT-based estimator. Since the estimated CIR from LS has most of its power concentrated on the first L samples, the DFT-based estimation reduces the noise power that exists outside of the CIR part.

The CFR $\tilde{H}_{LS}(k)$ obtained from LS-based estimator is sent to IDFT:

$$\tilde{h}_{LS}(n) = IDFT\{\tilde{H}_{LS}(k) = h(n) + \tilde{w}(n)\} = [h_0, h_1, \dots, h_{N-1}]^T \tag{11}$$

Assuming L is less than Ng , the CIR in traditional approach is:

$$h(n) = IDFT\{\tilde{H}_{LS}(k)\}, 0 \leq n \leq L \tag{12}$$

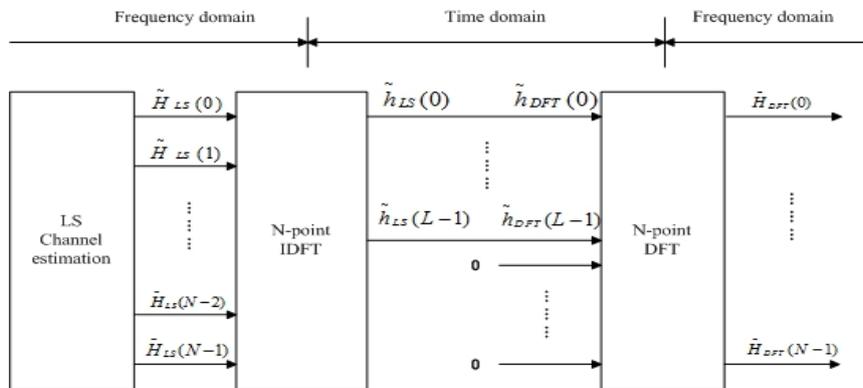


Figure 3. Block Diagram of the DFT-based Channel Estimation

By using Equation (11) and Equation (12), CIR can be divided into two parts:

$$\tilde{h}_{LS}(n) = \begin{cases} h(n) + \tilde{w}(n), & 0 \leq n \leq L \\ \tilde{w}(n), & otherwise \end{cases} \tag{13}$$

By ignoring the noise outside of L taps, the DFT-based estimator diminish the effect of noise:

$$\tilde{h}_{DFT}(n) = \begin{cases} h(n) + \tilde{w}(n), & 0 \leq n \leq L \\ 0, & otherwise \end{cases} \tag{14}$$

Then $\tilde{h}_{DFT}(n)$ is transformed to frequency domain:

$$\tilde{H}_{DFT}(k) = DFT\{\tilde{h}_{DFT}(n)\} \tag{15}$$

As shown in Equation (12), noise inside of L taps still exists in DFT-based method. We propose a novel time-domain algorithm based on QR decomposition to further reduce noise. The estimated transmitter data \tilde{X}_e can be recovered by simply dividing the received signal by the channel response:

$$\tilde{X}_e = \frac{Y(k)}{\tilde{H}_{DFT}(k)}, 0 \leq k \leq N - 1 \tag{16}$$

From Equation (3), the time-domain receiving signals $y(n)$ without CP is denoted as:

$$y = A \cdot h + w \tag{17}$$

\tilde{x} transformed from \tilde{X}_e by DFT is sent back channel estimation block to form A:

$$A = [x]_{N \times L} = \begin{bmatrix} x_0 & x_{N-1} & \cdots & x_{N-L} \\ x_1 & x_0 & \cdots & x_{N-L+1} \\ \vdots & \vdots & \cdots & \vdots \\ x_{N-1} & x_{N-2} & \cdots & x_{N-L-1} \end{bmatrix} \tag{18}$$

The solution of Equation (17) is:

$$\tilde{h} = \arg \min \|A \cdot h - y\|_2^2 \tag{19}$$

It is assumed that $A=Q \begin{bmatrix} R \\ O \end{bmatrix}$, where Q is a orthogonal matrix, R is an upper triangular matrix.

$$Q^T A = \begin{bmatrix} R \\ O \end{bmatrix} \tag{20}$$

$y(n)$ is processed by Q^T :

$$Q^T y = \begin{bmatrix} \bar{y}_n \\ O \end{bmatrix} \tag{21}$$

Considering Equation (18), (19), (20), (21):

$$R \cdot \tilde{h}_{new} = \bar{y}_n \tag{22}$$

Now, we get new CIR \tilde{h}_{new} , then sent it back to Equation (12)-Equation (15) to diminish the noise which exits inside of L taps again. We give the block diagram of the QR-based channel estimation in Figure 4.

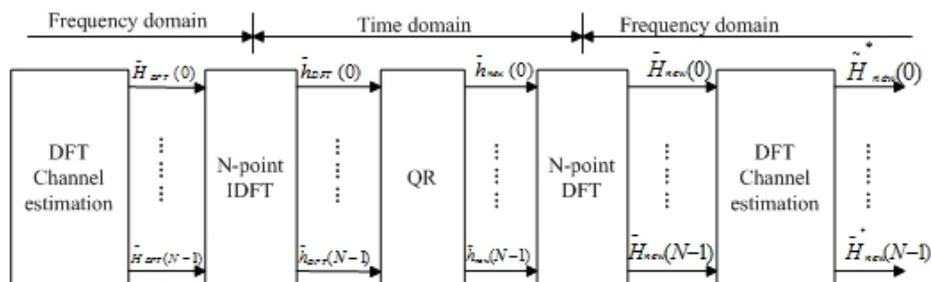


Figure 4. Block Diagram of the QR-based Channel Estimation

4. Simulation Results

In this section, we investigate the BER performance of the proposed channel estimation algorithm on multipath Rayleigh fading and Doppler spread channel. The parameters of channel is shown in Table 1.

Table 1. Parameters of Rayleigh Channel

Rayleigh Channel		
Path Number	Average Power(dB)	Delay(μ s)
1	0	0
2	-1.0857	1
3	-2.1715	2
4	-3.2572	3
5	-4.3429	4

The modulation & demodulation method of pilot symbols and data symbols is QDPSK. The basic system parameters for the simulations are summarized in Table 2.

Table 2: Simulation Environment

Parameter	Value
Carrier frequency	2(GHz)
FFT size	1024
Length of CP	8
Pilot subcarrier spacing	18
Modulation order	QDPSK

Here we assume that the guard intervals are longer than the maximum delay spread of the channel. The proposed algorithm, LS-based method and DFT-based algorithm are simulated and compared in Figure 5.

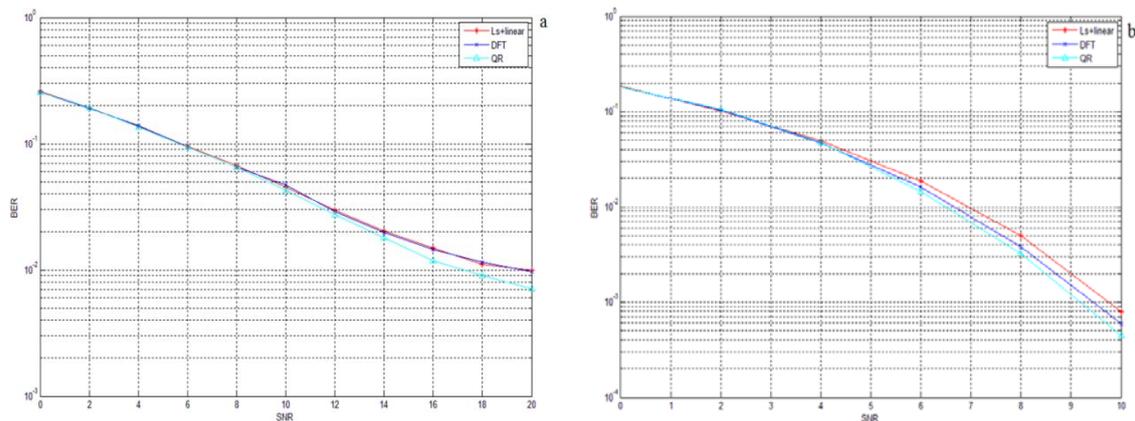


Figure 5. BER Performance in Different Channel

The different Doppler Frequency Shift are used in the simulation. Figure 5(a) shows the performance when the maximum Doppler frequency is 132Hz, the DFT-based estimator is 0.9dB better than LS-linear estimator and about 2dB from the proposed method. Fig.5(b) depicts the SER simulations for the maximum Doppler frequency is 264Hz. In the Figure, the

performance of proposed approach is 2.5dB better than DFT-based estimator which is similar to the LS-linear estimator. A comprehensive analysis of Figure 5 shows that while not in Figure 5(a), the BER still exists in Figure 5(b) when $\text{SNR} > 10\text{dB}$, that means a bigger Doppler Frequency Shift led to poorer performance. Above all, the QR-based algorithm renovating CIR from DFT estimator obtains lower BER and improves OFDM communication system performance.

5. Conclusion

To overcome the shortcoming that DFT estimator ignores noise outside of L taps, we present and analyze a novel QR-based algorithm to update CIR from DFT estimator and reduce noise inside of L taps. The simulation shows that the BER performance is 2-2.5dB better than conventional DFT-based method and LS-linear algorithm, hence, the OFDM communication system is improved.

Acknowledgements

This work was supported by Natural Science Foundation of Hunan project Ring-resonator-spectroscopic-based detection mechanism and methods of gas pollution, project number 14JJ2013 and Natural Science Foundation of Xinjiang project Detection theory and methods of gas pollution, project number 2013211A035.

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