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A New Particle Filter Algorithm with Correlative Noises

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Abstract

The standard particle filter (SPF) requirements system noise and measurement noise must be independent. In order to overcome this limit, a new kind of correlative noise particle filter (CN-PF) algorithm is proposed. In this new algorithm, system state model with correlative noise is established, and the noise related proposal distribution function characteristics were analyzed in detail. At last, the concrete form of the best proposal distribution function is derived based on the condition of the minimum variance of importance weight with the assumption of gaussian noise. Theoretical analysis and experimental results show the effectiveness of the proposed new algorithm.

Keywords: *nonlinear system, correlative noises, particle filter, proposal distribution function*

1. Introduction

Particle filter (PF) is a new kind of nonlinear filtering method. The core idea of it is to use the weight which given a series of corresponding information of random sampling particles to approximate the system state of a posteriori probability density function with weighted sum method [1-2]. The system state estimation is realized with the minimum mean square error criterion implementation. PF method no need the assumption that the characteristics of the system are linear and gauss distribution compare with the current widely used Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) which use linear approximation of nonlinear filtering methods. So, the PF can adapt to any non-linear non-gauss systems in theory. In recent years, with increasing ability of the computer processing, the PF algorithm has been widely used in the field of target tracking [3-8].

Traditional PF algorithm usually chooses one step system state transition probability as the proposal distribution function to sample. The main goal of the method is for convenience of sampling and calculating. Although this method is easy to implement; but its filtering precision is heavily dependent on the system model. Especially when the model error is large, due to the lack of the latest observation information correction proposal distribution function, it is easy to cause the system model mismatch error increases after many iterations. Eventually produce a so-called "particle weight degradation problems" and the filter estimation precision is greatly reduced, or even divergence. Therefore, how to select good proposal distribution function is a core content in the research of the algorithm.

In recent years, literatures [2-4] present a series of improved algorithms in order to solve the problem of the proposal distribution function selection. Although these improved algorithm weak the degradation problems of sampling to a certain extent, and increase the overall accuracy of the algorithm in the concrete application; but these studies only are useful under the assumption of gauss white noise with the system noise and measurement noise is independent of each other. In a real environment, the independent of system and measurement noise for each other is very difficult to satisfy the conditions because of the discrezation processing of measuring information [9]. Therefore, it is meaningful in theoretical and practical in practice to develop the noise related cases PF algorithm.

Based on this, this paper proposed a new kind of Correlative Noises Particle Filter (CNPF). The remainder of this paper is organized as follows. Section 2 gives the background of

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the problem. The proposed new algorithm with correlative noises is derived in detail in section 3. Experimental results and analysis are reported in section 4. We conclude this paper in section 5.

2. Background of the Problem

2.1. The System Model

For filtering problem of nonlinear systems, usually adopt nonlinear discrete systems as shown in the following [10]:

$$\begin{cases} x_{k+1} = f_k(x_k, u_k) + \Gamma_k w_k \\ y_k = h_k(x_k) + v_k \end{cases} \tag{1}$$

Where, x_k and y_k denote the system state and measure value at k. Dynamic function $f(\cdot)$ and $h(\cdot)$ determine the overall dynamic model of the system with the initial state x_0 of the system. u_k is the control input vector of the system. Where w_k and v_k denote the system process and measurement noise respectively. Γ_k denotes the input matrix of process noise. This article mainly aimed at the research of correlative noise filtering method, so here first give the following two hypotheses.

Hypothesis 1: Noise satisfy the following features:

$$\begin{cases} E(w_k) = q_k, Cov(w_k, w_j^T) = Q_k \delta_{kj} \\ E(v_k) = r_k, Cov(v_k, v_j^T) = R_k \delta_{kj} \\ Cov(w_k, v_j^T) = S_k \delta_{kj} \end{cases} \tag{2}$$

Where Q_k and R_k represent the system process and measurement noise covariance respectively, and here δ_{kj} meets the following value:

$$\delta_{ij} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases} \tag{3}$$

Hypothesis 2: The system initial state x_0 is unrelated with w_k and v_k, and meets the following features.

$$\begin{cases} \hat{x}_0 = E(x_0) \\ P_0 = Cov(x_0) = | E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] | \end{cases} \tag{4}$$

This paper describe the nonlinear model of noise filtering problem of related cases in the premise of the above assumptions of Equation (1).

2.2. The Standard Particle Filter Algorithm

In view of the system state equation described by type (1), we can summarize the SPF as "forecast" and "update" two steps [2]. $\left\{x_{0:k}^i, \omega_k^i\right\}_{i=1}^N$ denote the sampling particles collection of the system posterior probability density $p(x_{0:k} | Y_k)$, where, $\left\{x_{0:k}^i\right\}_{i=1}^N$ denote the sampling particles collection which given to the corresponding weight information, and the weights meet

$\sum_{i=1}^{N}\omega_k^i = 1$, $x_{0:k} = \{x_0, x_1, ..., x_k\}$ just denote the collection of the system state at k. Based on the idea of SPF, when we get measurement information $Y_k = \{y_1, y_2, ..., y_k\}$, we have:

$$p(x_{0:k} \mid Y_k) \approx \sum_{i=1}^{N} \omega_k^i \delta(x_{0:k} - x_{0:k}^i) \tag{5}$$

Where:

$$\omega_k^i \propto \omega_{k-1}^i \frac{p(y_k \mid x_k^i)\, p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid x_{k-1}^i, z_k)} \tag{6}$$

$q(x_k \mid \cdot)$ denotes the proposal distribution function, and usually, Equation (7) is take as the prior distribution of the proposal distribution function,

$$q(x_k^i \mid x_{k-1}^i, z_k) = p(x_k^i \mid x_{k-1}^i) \tag{7}$$

Bring (7) into (6), we have:

$$\omega_k^i \propto \omega_{k-1}^i p(y_k \mid x_k^i) \tag{8}$$

3. Particle Filter Algorithm with Correlative Noises

3.1. Situation Analysis of Correlative Noises

In the research of algorithm, we usually considered the observation noise as additive noise. For the sake of simplicity, here we rewrite the dynamic model of (1) as:

$$\begin{cases} x_{k+1} = f(x_k, v_k) \\ y_l = h(x_l) + e_l \end{cases} \tag{9}$$

The observed quantity and process condition between the joint posterior probability density $p(X_k \mid Y_k)$ can be represented as:

$$p(X_k \mid Y_k) \propto p(y_k \mid X_k, Y_{k-1}) \times p(x_k \mid X_{k-1}, Y_{k-1})\, p(X_{k-1} \mid Y_{k-1}) \tag{10}$$

For the standard of Markov models, in a separate process noise and measurement noise we have:

$$p(v_i, e_j) = p(v_i)\, p(e_j) \tag{11}$$

$$p(x_k \mid X_{k-1}, Y_{k-1}) = p(x_k \mid x_{k-1}) \tag{12}$$

$$p(y_k \mid X_k, Y_{k-1}) = p(y_k \mid x_k) \tag{13}$$

Where, the dynamic system of (9) can be shown in Figure 1 with the evolution of the graphics.

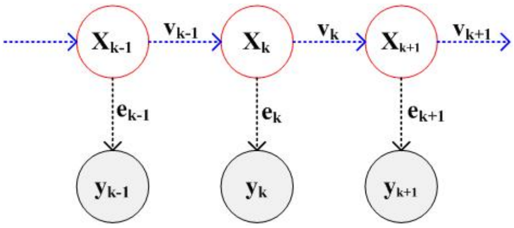

Figure 1. State Space Model

From Figure 1, we can get the relationship between the process noise and measurement noise which can be denoted by Figure 2 [11]. As we can see from Figure 2, the correlation main performance on the time association. The main purpose of considering the correlation of v_{k-1} and e_{k-1} is to find the noise appropriate decomposition form of joint probability density function $p(v_i, e_j)$.

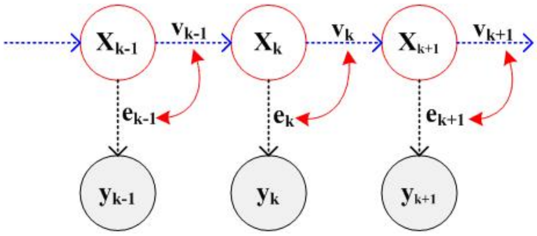

Figure 2. Process and Measurement Noise Correlation Diagram

Assuming that the noise vector sequence $(v_{k-1}, e_{k-1})^T$ is independent, according to the relationship of the Figure 2 shows, there are:

$$p(x_k \mid X_{k-1}, Y_{k-1}) = p(x_k \mid x_{k-1}, y_{k-1}) \tag{14}$$

$$p(y_k \mid X_k, Y_{k-1}) = p(y_k \mid x_k) \tag{15}$$

Then the process and measurement noise joint probability density function can be decomposed as follows:

$$p(v_{k-1}, e_{k-1}) = p(v_{k-1} \mid e_{k-1})\, p(e_{k-1}) \tag{16}$$

3.2. Derive of Optimal Proposal Distribution Function when Noise Related

For the convenience of formula derivation, here described the state space model of (9) as (17).

$$\begin{cases} x_{k+1} = f_k(x_k) + G_k v_k \\ y_k = h_k(x_k) + e_k \end{cases} \tag{17}$$

Where, v_k and e_k is correlated. According to Figure 2, we can represent the dependence relation of noises as $p(y_k, x_{k+1} \mid x_k)$ further, then:

$$p(y_k, x_{k+1} \mid x_k) \neq p(x_{k+1} \mid x_k)\, p(y_k \mid x_k) \tag{18}$$

In the situation of giving x_k, y_k and x_{k+1} are independent. In the SPF, the form of a proposal distribution function is $q(x_k \mid X_{k-1}, Y_k)$, and we can get (19) according to the dependent variable Y_{k-1} and x_k [12].

$$q(x_k \mid X_{k-1}, Y_k) = q(x_k \mid x_{k-1}, y_k) \tag{19}$$

According the interdependence of y_{k-1} and x_k, we can get (20) as follow:

$$q(x_k \mid X_{k-1}, Y_k) = q(x_k \mid x_{k-1}, y_k, y_{k-1}) \tag{20}$$

There are：

***Theorem 1*：** When noise related PF the optimal proposal distribution function is：

$$q(x_k \mid x_{k-1}, y_k, y_{k-1}) = \frac{p(y_k \mid x_k)\, p(x_k \mid x_{k-1}, y_{k-1})}{p(y_k \mid y_{k-1}, x_{k-1})} \tag{21}$$

Proof: According to the rule of bayesian inference, the advice of the posterior distribution can be distribution function can be expressed as:

$$\begin{aligned} q(x_k \mid x_{k-1}, y_k, y_{k-1}) &= p(x_k \mid x_{k-1}, y_k, y_{k-1}) \\ &= \frac{p(x_k, y_k \mid y_{k-1}, x_{k-1})}{p(y_k \mid y_{k-1}, x_{k-1})} \\ &= \frac{p(y_k \mid x_k)\, p(x_k \mid x_{k-1}, y_{k-1})}{p(y_k \mid y_{k-1}, x_{k-1})} \end{aligned} \tag{22}$$

Theorem 1 is proven.

3.3. Optimal Proposal Distribution Function of Gaussian Noise

Usually, the system and measurement noise meet the assumption of gaussian, and the noise adapted by SPF meets the follow needs just as (23).

$$x_0 \sim N(\hat{x}_{1|0}, P_{1|0}) \tag{23}$$

$$\begin{pmatrix} v_k \\ e_k \end{pmatrix} \in N\left(0, \begin{bmatrix} Q_k & S_k = 0 \\ S_k^T & R_k \end{bmatrix}\right) \tag{24}$$

And can also be expressed as：

$$p\left(\begin{pmatrix} x_{k+1} \\ y_k \end{pmatrix} \mid x_k\right) = N\left(\begin{pmatrix} f(x_k) \\ h(x_k) \end{pmatrix}, \begin{bmatrix} G_k Q_k G_k^T & G_k S_k \\ S_k^T G_k^T & R_k \end{bmatrix}\right) \tag{25}$$

According to (25), we can give the rule of judging the noise correlation as:

$$\begin{cases} S_k = 0, no \\ S_k \neq 0, yes \end{cases} \tag{26}$$

***Theorem 2*:** The gaussian model optimal proposal distribution function of type (25) can express as follows:

$$q(x_k \mid x_{k-1}, y_k, y_{k-1}) \propto N(f(x_{k-1}) + G_{k-1}S_{k-1}R_{k-1}^{-1}(y_{k-1} - h(x_{k-1})), G_{k-1}(Q_{k-1} - S_{k-1}R_{k-1}^{-1}S_{k-1}^T)G_{k-1}^T)N(h(x_k), R_k) \quad (27)$$

Proof: According to the rule of (21), we can decomposed the optimal proposal distribution function into two factors as (27), and $p(y_k \mid x_k)$ can be got by the measurement model of (9), and $p(x_k \mid x_{k-1}, y_{k-1})$ can be got by the following lemma 1.

Lemma 1: Assume that the vector X and Y is joint gaussian distribution, and there are (28) as follows:

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left(\begin{pmatrix} u_x \\ u_y \end{pmatrix}, \begin{bmatrix} P_{xx} & P_{xy} \\ P_{xy}^T & P_{yy} \end{bmatrix}\right) = N\left(\begin{pmatrix} u_x \\ u_y \end{pmatrix}, P\right) \quad (28)$$

When get the measurement $Y = y$ value, then the conditional distribution can be express as the following forms of gaussian distribution.

$$(X \mid Y = y) \sim N(u_x + P_{xy}P_{yy}^{-1}(y - u_y), P_{xx} - P_{xy}P_{yy}^{-1}P_{yx}) \quad (29)$$

Let $X = x_k \mid x_{k-1}$, $Y = y_{k-1} \mid x_{k-1}$, and combine the joint distribution of X and Y as (28), there are:

$$\begin{aligned} & p(x_k \mid x_{k-1}, y_{k-1}) \\ & = N(f(x_{k-1}) + G_{k-1}S_{k-1}R_{k-1}^{-1}(y_{k-1} - h(x_{k-1})), G_{k-1}(Q_{k-1} - S_{k-1}R_{k-1}^{-1}S_{k-1}^T)G_{k-1}^T) \end{aligned} \quad (30)$$

Then:

$$\begin{aligned} & q(x_k \mid x_{k-1}, y_{k-1}) \\ & = N(f(x_{k-1}) + G_{k-1}S_{k-1}R_{k-1}^{-1}(y_{k-1} - h(x_{k-1})), G_{k-1}(Q_{k-1} - S_{k-1}R_{k-1}^{-1}S_{k-1}^T)G_{k-1}^T) \end{aligned} \quad (31)$$

4. Simulation Analysis

In this paper, we adapt the following model just as literature [10] to simulate the performance of the new method. The model just as follows:

$$\begin{cases} \mathbf{x}_{k+1} = \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{bmatrix} = \begin{bmatrix} 3\sin(x_{2,k}) \\ x_{1,k} + x_{3,k} \\ 0.2E^{-0.5x_{1,k}}(x_{2,k} + x_{3,k}) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \omega_k \\ z_k = x_{1,k} + E^{-3x_{2,k}x_{3,k}} + v_k \end{cases} \quad (32)$$

We can see (32) has high nonlinear. Here ω_k and v_k are all white gaussian noise, and the statistical characteristics of them meets the following:

$$q_k = 0.2, Q_k = 0.04, r_k = 0.3, R_k = 0.09 \quad (33)$$

The initial parameter value of state is set to:

$$x_0 = [-0.7, 1, 1]^T \quad (34)$$

$$\hat{x}_0 = [-0.7,1,1]^T, P_0 = I \tag{35}$$

4.1. Analysis of Independent Noise

We can know that the uncorrelated of ω_k and v_k can denotes as $S_k = 0$. In order to compare the performance of this paper new method, traditional PF and the new method of CN-PF was used to estimate the state of x_1 respectively in experiments. The estimate results just as Figure 3, Figure 4 and Figure 5.

The experiment results denote that the traditional PF and CN - PF can track the system state effectively when the noise is independently. Both methods to keep the good tracking precision and less error. The filtering precision of the two methods are almost the same in the case of noise are independent of each other, which can be seen in figure 5. Can be understood as in the case of noise independent, this method is approximate to the traditional PF algorithm.

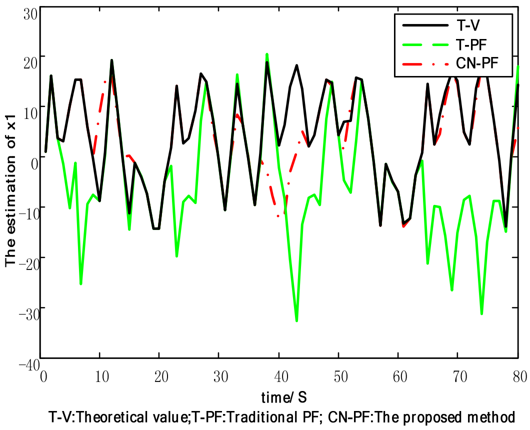

Figure 3. State Estimation Curve of x_1

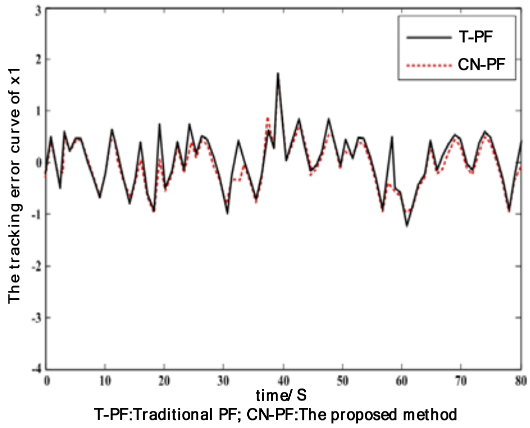

Figure 4. The Tracking Error Curve of x_1

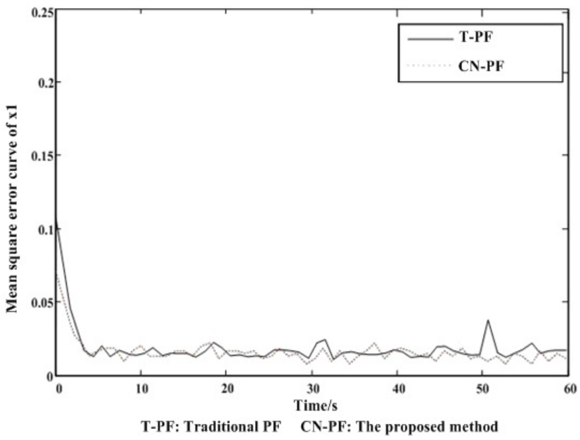

Figure 5. Mean Square Error Curve of x_1

4.2. Noise Related Situation Simulation Analysis

We can know that the correlated of ω_k and v_k can denotes as $S_k \neq 0$. In order to compare the performance of this paper new method, the value is $S_k = 0.1$ The estimate results just as Figure 6, Figure 7 and Figure 8. We can see that the system model error increases gradually when the noise correlation which can be get from Figure 6 and Figure 7. The tracking errors of state x_3 increases gradually, but this paper method can keep a good tracking performance.

The mean square error of the traditional PF algorithm with a cumulative as the increase of time, but this paper has maintained a good track effect and the root mean square error curve gradually converge to zero, which can be seen in the Figure 8. The results show that the proposed method named CN - PF have fast convergence rate, and high precision, strong stability, when the noise is related, and fully proves the feasibility and effectiveness of the new method.

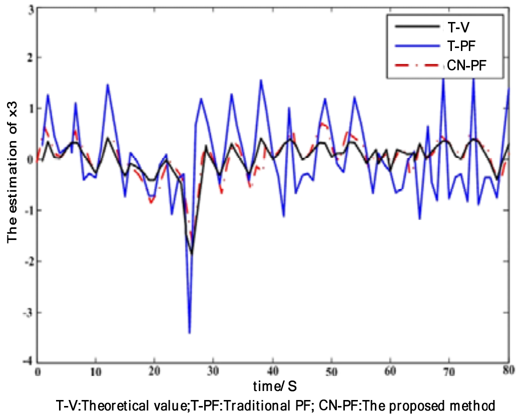

Figure 6. State Estimation Curve of x_3

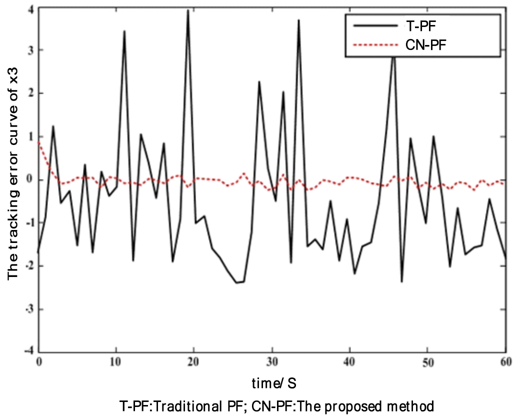

Figure 7. The Tracking Error Curve of x_3

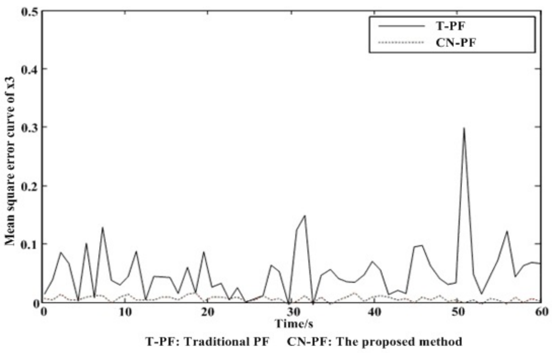

Figure 8. Mean Square Error Curve of x_3

5. Conclusion

Aimed at the limitation of the traditional PF algorithm under the condition of noise related, this paper proposed a noise related particle filter algorithm, mainly do the following several aspects: 1) The system state model of noise related situation is established, and the nature of the proposed distribution function when the noise correlation is deduced in detail; 2) Gives the decomposition expression of joint probability density in the related noise case; 3) The optimal proposal distribution function in the noise related cases was deduced under the conditions of importance weight minimum variance significance based on the gaussian noise background, and the effectiveness of the new algorithm is verified by computer simulation. Because of the proposed method is a expansion of the scope of the traditional PF algorithm, therefore, it is easy to combine the optimization of the current existing application accuracy of the algorithm in different fields. In the next step of study, the optimal filtering problem in the case of unknown system noise statistical characteristics will be further studied.

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