

# Dynamic Modeling Process of Neuro Fuzzy System to Control the Inverted Pendulum System

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## Abstract

The analysis and control of complex plants often requires the principles of qualitative process models since quantitative, namely analytical process models are not available. Qualitative modeling is one promising approach to the solution of difficult tasks automation if qualitative process models are not available. This contribution presents a new concept of qualitative dynamic process modeling using so called Dynamic Adaptive Neuro fuzzy Systems. This yields the framework of a new systems theory the essentials of which are given in further section of the paper. First, an identification method is presented, using a combination of linguistic knowledge. Next, a stability definition for dynamic neuro fuzzy systems as well as methods for stability analysis is given. Finally, a neuro fuzzy model-based neuro fuzzy controller design method is developed. The identification of real problems and neuro fuzzy controller design for inverted pendulum system demonstrate the significance of the new systems theory.

## Keywords:

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## 1. Introduction

In contrast to common approaches of Adaptive Neuro Fuzzy modeling [1], the dynamic system is completely described in the neuro fuzzy domain: the neuro fuzzy information about the previous state is directly applied to compute the system's current state, i.e. the delayed neuro fuzzy output is feedback to the input without defuzzification. Knowledge processing in such dynamic neuro fuzzy systems requires a new inference method, the inference with interpolating rules.

The analysis and control of complex plants often requires the introduction of qualitative process models since quantitative, namely analytical process models are not available. An examination of the quantitative and qualitative paradigms will help to identify their strengths and weaknesses and how their divergent approaches can complement each other. However, human experts as operators usually are capable of accomplishing control tasks, taking into consideration only imprecise knowledge about the process which may describe by a set of rules like **IF** valve is "open wide" **THEN** liquid level is "rising fast".

Thus, the behavior of an operator analyzing or controlling a process stimulates the new approach of neuro fuzzy modeling, systems analysis, and controller design pursued in this contribution. The new concept allows integrating qualitative process knowledge into models of these processes like they are found e.g. in process or manufacturing industries as well as in automotive systems [2].

Modeling is achieved using a particular class of dynamic neuro fuzzy systems where the nonlinear static characteristics of the process and-in contrast to common approaches [1]-as well its dynamics are represented in the neuro fuzzy domain. To be more specific, Figure 1 shows an autonomous first order dynamic neuro fuzzy system. The rule base may consist of rules like **IF**  $y_k-1$  is "small" **Then**  $y_k$  is "big".

Linguistic terms like "small" are modeled by neuro fuzzy sets. The knowledge propagation is carried out by a neuro fuzzy inference method. Since the neuro fuzzy output is feed back without a prior defuzzification, the linguistic information about the system is completely modeled in the neuro fuzzy domain. As a consequence, a new inference scheme

has to be derived for the following reasons: An inference method is expected to evaluate a set of neuro fuzzy rules corresponding to the human way of approximate reasoning. Human beings are able to process only such neuro fuzzy sets that might be properly adjoined to linguistic values. Therefore, only these kinds of interpretable neuro fuzzy sets are appropriate inputs of neuro fuzzy systems. Since the neuro fuzzy output of a dynamic neuro fuzzy system has to be processed by the inference in subsequent steps, it has to be guaranteed that the inference maps interpretable neuro fuzzy inputs onto an interpretable neuro fuzzy output.

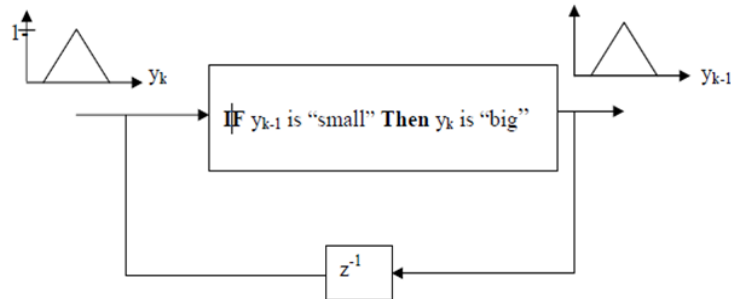


Figure 1. Autonomous First Order Dynamic Neuro Fuzzy System

In the sequel, neuro fuzzy numbers with triangular shaped membership functions, which are often used to characterize linguistic values like “small” or “big”, will be used as interpretable neuro fuzzy sets.

Conventional reasoning methods like “max-min- inference” [3] do not generate an interpretable neuro fuzzy output. Therefore, a new neuro fuzzy inference method, the “inference with interpolating rules” was developed which is outlined in the second section. This method is the central element of a new system theory covering processes represented by a set of neuro fuzzy rules. Within the scope of this systems theory an identification procedure is developed in the second section. Measurements as well as heuristic knowledge are used to determine a linguistic representation of the process dynamics. After that, the stability definition for dynamic Neuro fuzzy Systems is given and approach for stability analysis is briefly outlined.

The third section focuses on a new design strategy for neuro fuzzy controllers. This new approach enables the synthesis of neuro fuzzy controllers exclusively based on qualitative process knowledge. Finally, in the fourth section the main characteristics of the new Systems theory are demonstrated for an inverted pendulum system. First, the process is identified. Then, the resulting neuro fuzzy system model is applied to neuro fuzzy controller.

## 2. Identification of Dynamic Neuro fuzzy Systems

Identification of dynamic neuro fuzzy systems requires the transfer of crisp process measurements into the domain of neuro fuzzy modeling. An identification procedure for dynamic neuro fuzzy systems can be developed based on the inference with interpolating rules Figure 1 illustrates the identification concept. The delayed inputs and outputs of the process are used as inputs of the neuro fuzzy inference (serial-parallel structure). The neuro fuzzy error is calculated following Zadeh’s extension principle as the difference between the crisp process output and the neuro fuzzy model output:

$$ER = y_k - \hat{Y}_k$$

Minimizing both, the mean squared center of the error.

$$J_1 = \sum_i (c(ER_i))^2 \rightarrow \min \quad (1)$$

And the fuzziness of the error.

$$J_1 = \sum_i \int \mu_{ER_i}(er) der \rightarrow \min \tag{2}$$

Yields the process model. In Equation (2), the integral over the error membership function defines a measure of its fuzziness [11]. The identification is carried out in two steps. First, the significant delays of the input and the output of the process are determined to fix the structure of the neuro fuzzy model. Second, the rule base of the neuro fuzzy process model is identified minimizing Equation (1) and Equation (2).

**2.1. Determining the Structure of Dynamic Neuro fuzzy Systems**

The significant delays of the neuro fuzzy model can be determined applying a procedure similar to nonlinear system identification algorithms represented by neural nets [5; 7]:

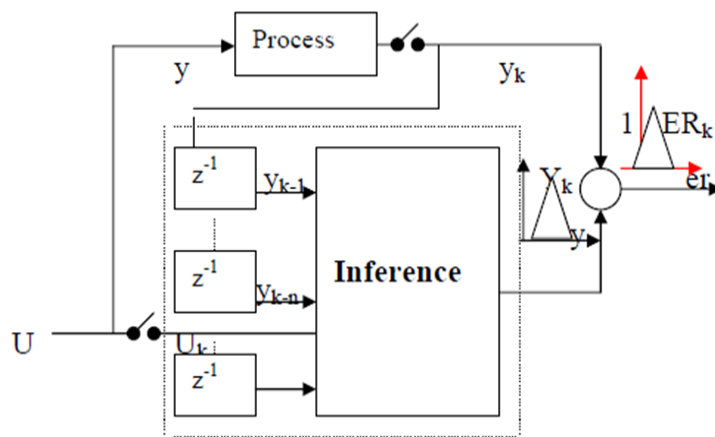


Figure 2. Neuro Fuzzy Model in Parallel-serial Structure

Tangent planes of the systems nonlinearity are estimated on different points of the operating domain. To calculate the tangent planes, matrices are built up from measurements. Using data of the output with a delay exceeding the process' order results in rank deficient matrices. Hence, the maximum required delay of the outputs of lower than maximum order can be determined from the tangent planes. These are parallel to axis spanned by insignificant delayed outputs [7].

**2.2. Identifying the Rule base**

To illustrate the basic ideas of the identification procedure of the rule base, it sufficient to consider the static Neuro fuzzy System depicted in Figure 3. It can be shown that the center of the neuro fuzzy output of the inference only depends on the centers of the neuro fuzzy inputs

[5;5]. This relationship is expressed by the center equation  $\hat{c}(Y) = f(c(E))$ .

In the first step, the center equation is:

$i=1, \dots, m:$

$$J_1 = \sum_{i=1}^m (c(ER_i))^2 \sum_{i=1}^m (y_i - \hat{c}(Y_i))^2$$

$$J_2 = \sum_{i=1}^m (y_i - f(c(E)))^2 = \sum (y_i - f(e_i))^2$$

For Neuro fuzzy Systems with multiple inputs the center equation is the piecewise multilinear interpolation function spanned by the centers of the neuro fuzzy premises and the neuro fuzzy conclusion [5]. Thus, in case of a single input system the center equation is a piecewise linear interpolation function. Figure 3 shows an optimization result. Obviously, four rules had to be:

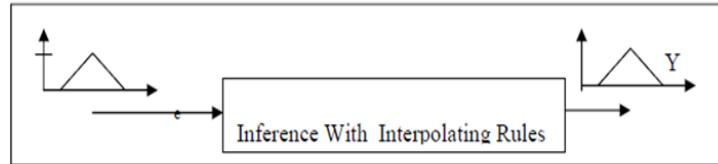


Figure 3. SISO Static Neuro Fuzzy System

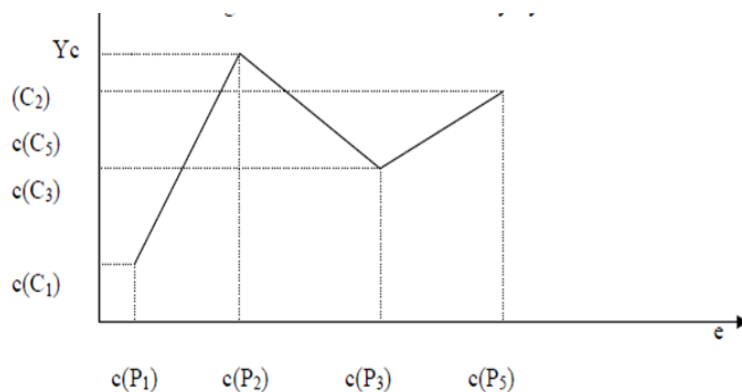


Figure 4. Optimized Center Equation

Identified. Therefore, the centers of four premises  $c(P1), \dots, c(P5)$  and four conclusion  $c(C1), \dots, c(C5)$  were found. The center equation is the linear interpolation function  $f(c(E))$  spanned by  $c(P1), \dots, c(P5)$  and  $c(C1), \dots, c(C5)$ . Because of the unsteadiness of the gradient of  $J1$  gradient-based search strategies may not applicable. For systems of higher order, evolutionary algorithms have been successfully applied [5].

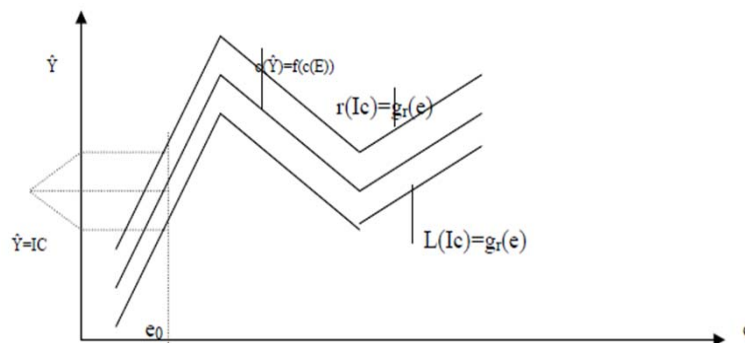


Figure 5. Determining the Neuro Fuzzy Output for a Crisp Input

Having determined the centers of the premises and conclusions, the shapes of the membership functions are determined in the next step with a methodology developed in [5]. This approach guarantees a minimum fuzziness of the error by minimizing Equation (2) under consideration of the straggling of the measurements. Figure 5 shows in extension of Figure 4

the neuro fuzzy output  $\hat{Y}$  computation for a given crisp input  $e_0$  using the identified neuro fuzzy model. Due to the crisp input, the neuro fuzzy output of the model is equivalent to the interpolating conclusion. The left and right foot of the interpolating conclusion are calculated using  $L(IC)=gl(e_0)$  and  $r(IC)=gr(e_0)$  respectively.  $gl(e)$  and  $gr(e)$  are piecewise multilinear interpolation functions spanned by the left and right feet of the conclusion membership functions,  $L(c_1), \dots, L(c_5)$  and  $r(c_1), \dots, r(c_5)$ , respectively, and the centers of the premise membership functions  $c(p_1), \dots, c(p_5)$ . IF the crisp input  $e_0$  belongs to the measurements, i.e.  $e_0 = e_i$ , the membership values of the corresponding measured crisp output  $y_i$  is always greater than zero:

$$e_0 = e_i \rightarrow \mu_{\hat{Y}}(y_i) > 0.$$

Thus, the neuro fuzzy model output might be interpreted as a possibility distribution [8].

Finally, it has to be emphasized the in general linguistic knowledge is applied in combination with the measurements. On the one hand, linguistic knowledge may be used for situations where no measurements are available. On the other hand, rules given by human experts can be taken as starting conditions for the optimization procedure. For example, the starting conditions for the optimization whose results are illustrated in Figure 4 are the centers of the four premises and conclusions of the respective rules.

### 3. Stability Analysis of Dynamic Neuro Fuzzy Systems

To show the typical behavior of Dynamic Neuro fuzzy Systems and to obtain an appropriate stability definition, it is sufficient to consider two a simple autonomous Neuro fuzzy System represented by the following two rules:

**IF**  $y_{k-1}$  is "negative" **Then**  $y_k$  is "positive"  
**IF**  $y_{k-1}$  is "positive" **Then**  $y_k$  is "negative"

The membership functions defined on the input domain are shown in Figure 6. Depending on the output membership functions, the system exhibits different dynamic behavior. Given the output membership functions of Figure 7, we obtain system 1 which is stable since the output converges to the neuro fuzzy number with the center 0, the left foot  $-2$  and the right foot  $+2$ . Figure 8 depicts the neuro fuzzy output resulting from a crisp initial state  $y_0=2$ . The output membership functions of system 2 shown in Figure 9 cause an unstable system behavior. Although the center of the output converges to 0 for any initial state, its left and right foot moves to infinity (Figure 10). Since the output becomes fuzzier with every step, the specificity of the output vanishes for  $k \rightarrow \infty$ .

These simple examples suggest the following stability definition for Dynamic Neuro fuzzy Systems: An equilibrium point of a Dynamic Neuro Fuzzy System marked by a crisp value  $R_0$  is stable if:

- $R_0$  is an asymptotically stable equilibrium point for the center of the output  $c(Y_k)$
- The feet of the neuro fuzzy output stay in a bounded environment of  $R_0$ .

In the examples above  $R_0=0$  marks the equilibrium point.

System 1 has a stable equilibrium point, whereas the equilibrium point of system 2 is unstable. Since it is sufficient to examine the mapping of the crisp parameters of the neuro fuzzy input onto the crisp parameters of the neuro fuzzy output, conventional methods for the stability analysis of nonlinear systems can be applied. If all interpolating premises defined on  $y_{k-1}, \dots, y_{k-n}$  are fuzzier than the interpolating conclusion with the same center defined on  $y_k$ , it is only necessary to analyze the mapping of the centers of the neuro fuzzy input onto the neuro fuzzy output [5].

With a constant neuro fuzzy  $U_k$  results a discrete nonlinear system described by the center equation  $c(Y_k)=f(c(Y_{k-1}), \dots, c(Y_{k-n}))$ . With the centers  $c(Y_k), c(Y_{k-1}), \dots, c(Y_{k-n})$  of the neuro fuzzy output  $Y_k$  and its delays  $Y_{k-1}, \dots, Y_{k-n}$ . To analyze such a system, methods based on common stability analysis approaches may be used. The "Convex Decomposition" [9, 10] as an efficient numerical stability analysis method and an approach based "integral Ljapunov Function" [11] have been successfully applied t Dynamic Neuro fuzzy Systems.

Considering first order Dynamic Neuro fuzzy Systems, the region of attraction of an equilibrium point can even be analytically determined [5, 12].

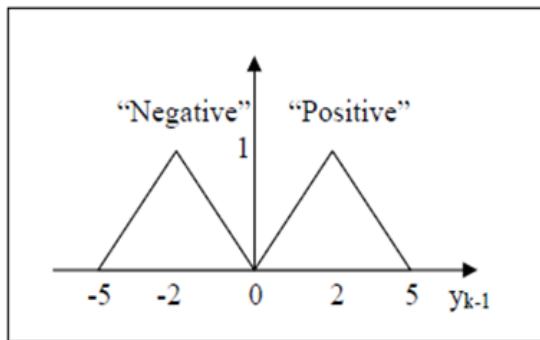


Figure 6. Membership Functions Defined for  $y_{k-1}$

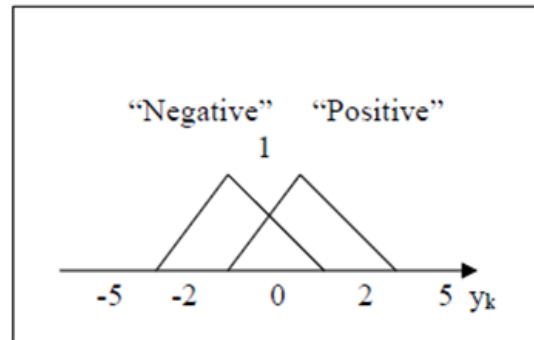


Figure 7. Output Membership Function of System 1

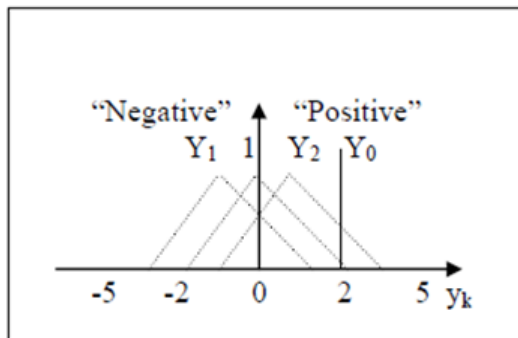


Figure 8. Dynamic Behavior of System 1

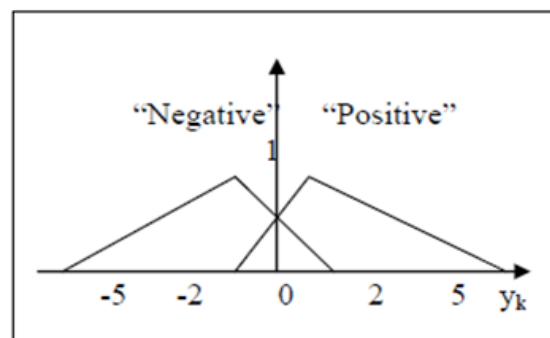


Figure 9. Output Membership Functions of System 2

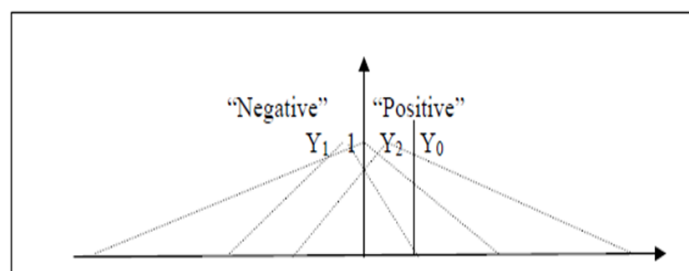


Figure 10. Dynamic Behavior of System 2

#### 4. Neuro Fuzzy Model Based Neuro Fuzzy Controller Design

This section outlines a new neuro fuzzy controller synthesis approach using a qualitative (neuro fuzzy) process model. In Figure 11, the structure of the controlled neuro fuzzy system is depicted. The plant is modeled by the second order Dynamic Neuro fuzzy System. The controller determines the control signal  $U_k$  from the neuro fuzzy model output  $Y_k$  and the command variables  $W_k$ . As mentioned above, the center of the neuro fuzzy model output exclusively depends on the centers of the inference inputs. Therefore, the center of the output can only be manipulated by the center of control signals. Thus, given the center of the neuro fuzzy

model output and the center of the command an appropriate crisp control signal can be determined. Consequently, the center equation of the neuro fuzzy controller is determined from the center equation of the neuro fuzzy process model. For a dynamic neuro fuzzy system of order  $n$ , the center equation is given by  $c(Y_k) = f(c(Y_{k-1}), \dots, c(Y_{k-n}), c(U_{k-\delta}), \dots, c(U_{k-m}))$ .  $c(Y_k), c(Y_{k-n}), c(U_{k-\delta}), \dots, c(U_{k-m})$  represent the centers of the neuro fuzzy inputs and outputs and their delay.  $\delta$  is the difference order of the center equation. To deduce the center equation of the neuro fuzzy controller, approaches for controller synthesis of time-discrete nonlinear systems can be applied. In [5] the center equation is determined by input/output linearization. The problem of handling a zero dynamics which may occur when using this methods is discussed [5, 13].

The example depicted in Figure (11) demonstrates the basic ideas of the neuro fuzzy model based controller synthesis. The underlying set of rules is:

**IF**  $Y_{k-1} = A$  and  $U_{k-1} = X$  **Then**  $Y_k = AX$   
**IF**  $Y_{k-1} = B$  and  $U_{k-1} = X$  **Then**  $Y_k = BX$

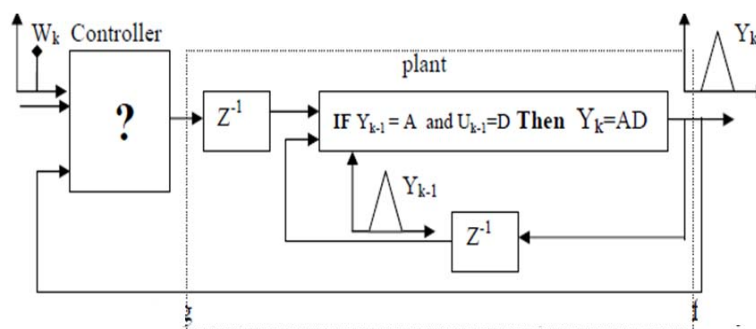


Figure 11. Structure of a Controlled Dynamic Neuro Fuzzy System

**IF**  $Y_{k-1} = C$  and  $U_{k-1} = X$  **Then**  $Y_k = CX$   
**IF**  $Y_{k-1} = A$  and  $U_{k-1} = Z$  **Then**  $Y_k = AZ$   
**IF**  $Y_{k-1} = B$  and  $U_{k-1} = Z$  **Then**  $Y_k = BZ$   
**IF**  $Y_{k-1} = C$  and  $U_{k-1} = Z$  **Then**  $Y_k = CZ$

The premises A, B and C are defined for the delayed output  $y_{k-1}$  and the premises X and Z are defined for  $U_{k-1}$ . The conclusions AX, ..., CZ defined on  $y_k$  are assumed not to be fuzzier than one of the premises A, B or C. Therefore, it is sufficient to consider the mapping of the centers  $c(Y_{k-1})$  and  $c(U_{k-1})$  onto the center  $c(Y_k)$ . Using the inference with interpolating rules to evaluate the neuro fuzzy rule set, it can be shown [5, 12] that  $c(Y_k) = r(c(Y_{k-1})) + h(c(Y_{k-1}), c(U_{k-1}))$ .

Holds. Assuming  $h(c(Y_{k-1}), c(U_{k-1})) \neq 0$ , the control law:

$$c(U_{k-1}) = \frac{y_R - r(c(Y_{k-1}))}{h(c(Y_{k-1}))}$$

Ensures that the center of the output  $c(Y_k)$  reaches a desired equilibrium point  $y_R$  within a single step. Without a bounded control signal, the region of attraction equals the domain of definition. Due to the maximum difference order ( $\delta = n = 1$ ) a zero dynamics does not occur [5, 13]. However, in practical applications a bounded control signal must be considered. Now, a region of attraction of the equilibrium point might be determined using a Lyapunov function, e.g.

$$V(c(Y_k)) = (c(Y_k) - y_R)^2$$

Thus, the first step of the controller design is to formulate the control law Equation (3). Next, the region of the attraction of the desired equilibrium point is determined considering the bounds of the control signal. From Equation (3), an adequate manipulation variable might be calculated for each center  $c(Y_k)$  of the previous determined region of attraction.

We start with the neuro fuzzy controller rule set:

**IF  $Y_k = A$  Then  $U_k = u_A$**   
**IF  $Y_k = B$  Then  $U_k = u_B$**   
**IF  $Y_k = C$  Then  $U_k = u_C$**

The premises A, B and C are known from the rule set representing the process behavior, whereas the crisp conclusions  $u_A$ ,  $u_B$  and  $u_C$  are calculated using Equation (3). Thus, the conclusion of the first rule is given by:

$$u_A = \frac{y_R - r(c(A))}{h(c(A))}$$

Due to the singletons used as conclusions, crisp controller inputs lead to a crisp control output. Only for the crisp inputs  $c(A)$ ,  $c(B)$ , and  $c(C)$  the evaluation of the controller rule set using the inference with interpolating rules yields the same output as the crisp control law Equation (3). If  $c(Y_k)$  is somewhere between these particular values, the controller output is determined by interpolation. It might be necessary to add more rules if the characteristic of the neuro fuzzy controller differs too much from the nonlinear characteristics of the neuro fuzzy control law Equation (3). With the inference with interpolating rules, the neuro fuzzy rule set resulting so far may be used to design a neuro fuzzy controller; the neuro fuzzy rule set resulting so far may be used to design a neuro fuzzy controller. Due to the crisp conclusions, a defuzzification is not required for crisp inputs of the controller. Because of the piecewise multilinear center equation, such a controller has characteristics consisting of regions where multilinear functions are defined. However, we obtain a controller with the same characteristics if the neuro fuzzy rule set is evaluated with the conventional sum-prod-inference combined with a center of singletons defuzzification. Only the premise membership functions have to be manipulated in the following way: the centers of all premises are kept but the feet are moved to the centers of the adjacent premises [5]. The result is a neuro fuzzy rule set with triangular membership functions for the premises and singletons for the conclusions. This set of rules is used for a neuro fuzzy controller, which can be evaluated with well-known methods. Thus, the final tuning of the controller in the closed loop with the real process might be accomplished with common software tools.

## 5. Identification and Control of an Inverted Pendulum System

The considered inverted pendulum system is depicted in Figure 12. Input and output of the process are the force and the angle of the pendulum, respectively.

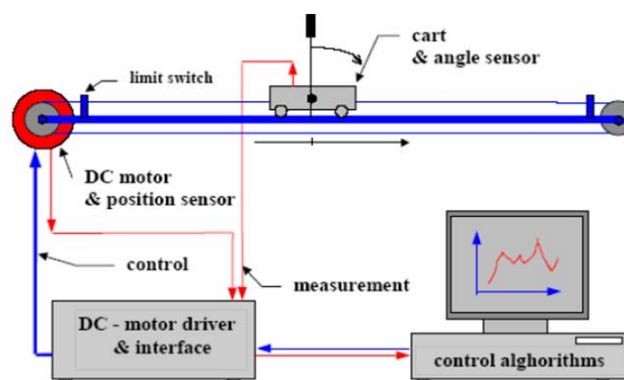


Figure 12. Inverted Pendulum System



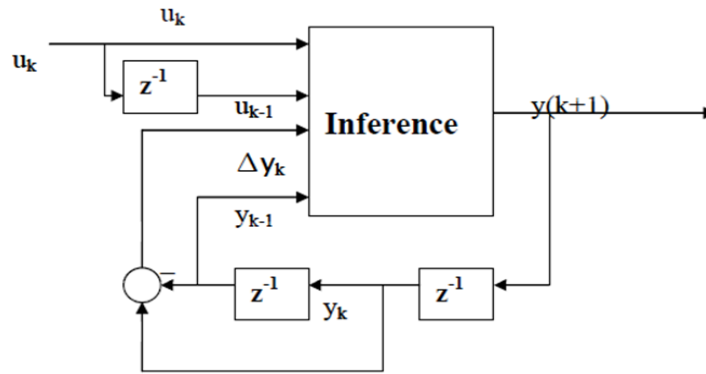


Figure 13. Structure of the Neuro Fuzzy Process Model

In the first step, the structure of the neuro fuzzy process model is identified Figure 12. The identification of the rule base was carried out in the second step and yielded 35 rules. From the dynamic neuro fuzzy model a neuro fuzzy controller with 55 rules was designed following the procedure outlined in section 4. The resulting closed system is given in Figure 14 (a, b, c).

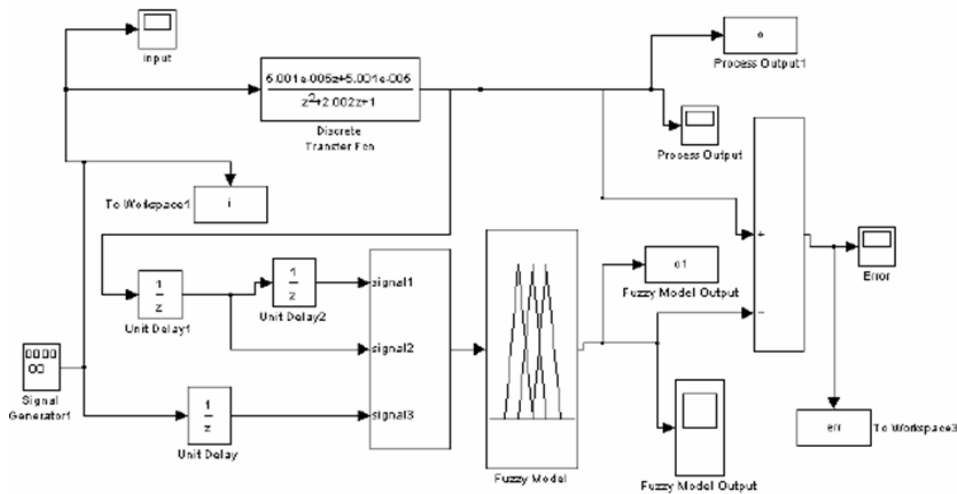


Figure 14(a). Inverted Pendulum System Represented by Fuzzy Model

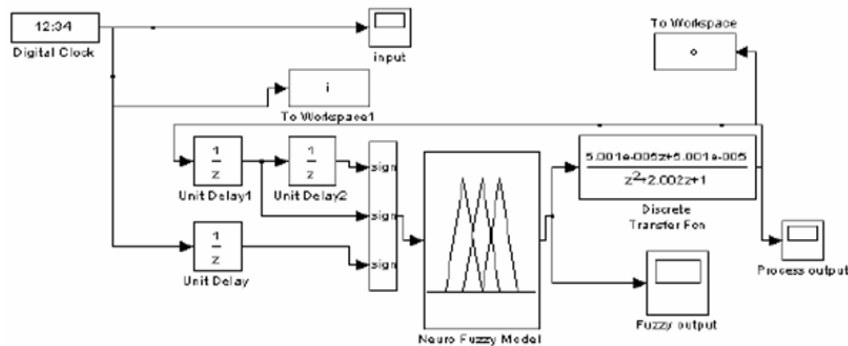


Figure 14(b). Inverted Pendulum System Controlled by a Neuro Fuzzy Model Based Neuro Fuzzy Controller

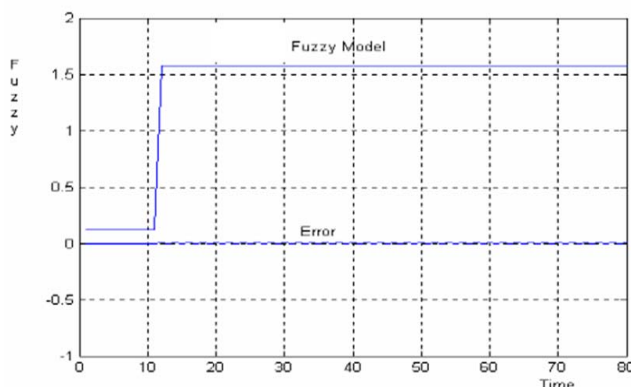


Figure 14(c). System Response of Neuro Fuzzy Model

## 6. Conclusion

This contribution presented the framework of a new qualitative systems theory based on Dynamic Neuro fuzzy Systems where knowledge about the process behavior is described by the a set of rules. The dynamics of the process behavior is modeled by appropriate time delay and feed back of the neuro fuzzy output to the system's input without previous defuzzification. Hence, an important feature of this theory is the particular procedure for rule propagation, which was developed for this class of systems and is called inference with interpolating rules. The essentials of this systems theory were outlined: In addition to rule based modeling by human experts an identification method allows to obtain a Dynamic Neuro fuzzy System from measurements. A new stability definition and different approaches for analytical and numerical stability analysis were briefly described. Moreover, a neuro fuzzy-model based controller synthesis method was given. Finally, as practical demonstration a inverted pendulum system was identified from measurements, a neuro fuzzy controller was designed using the identification neuro fuzzy process model and the closed loop behavior was presented. Concluding, the new systems theory enables qualitative modeling and simulation as well as systems analysis and controller design of complex dynamic processes. Since the qualitative approach is often the only way to obtain an appropriate process representation, the new concept of the qualitative systems theory offers a considerable potential towards the automation of this system class.

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