

# Observer-based State Feedback H-infinity Control for Networked Control Systems

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## Abstract

The observer-based  $H_\infty$  controller design problem is considered for the networked control systems with uncertain network-induced delays and packet dropouts. By taking consideration of the delays and packet dropouts existing in the sensor-to-controller and controller-to-actuator simultaneously and introducing an observer with time-varying delays, a new error augmented model is established, which can reflect the delays and packet dropouts characteristics of the actual physical systems appropriately. Based on the delay-dependent Lyapunov stability theory, a sufficient condition is proposed to guarantee that the closed-loop system is asymptotically stable and has  $H_\infty$  performance level  $\gamma$ . Since the obtained condition is nonlinear, the singular value decomposition method is applied to convert the nonlinear inequalities into LMIs. The delay-dependent approach shows a less conservative result than the delay-independent approach. A numerical example is given to demonstrate the high validity and merit of the proposed approach.

**Keywords:** networked control systems, H-infinity control, linear matrix inequality, singular value decomposition, delay-dependent

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## 1. Introduction

The control systems in which the control loops are closed over real-time network are called the networked control systems (NCSs). Recently, NCSs have been widely applied in automotive, aerospace, mobile sensor networks and industrial manufacturing fields [1-4]. However, the limitations of network bandwidth and the data collision and retransmission in the process of sending information cause the existence of information transmission delays and packet dropouts. The network time-delays and data packet dropouts are commonly existed and unavoidable in real-time NCSs, which will decrease the performance and even make systems unstable. Hence, the control systems with time-varying delays and packet dropouts are closer to the actual systems and the research on this kind of systems has a strong practical background.

Network-based control theories and control methods are springing up in recent years, such as system modeling, stability analysis [5] and robust H-infinity control [6-8]. In fact, time delays are often encountered in practical systems, which may induce instability. Literature [9] proposed a new delayed feedback control design method for uncertain systems with time-varying input delay by introducing some relaxation matrices and turning parameters. Data packet dropouts also occur due to node failures or network congestion. Literature [10] concerned with the stability and controller design of NCSs with packet dropouts. However, the problem of stability or control law design for NCSs has not been fully investigated so far. It is worth noting that not all of the state variables can be measured for practical engineering systems and estimation is needed. The introduction of observer can avoid the serious noise disturbance in the real-time measurement results, but it also makes the system analysis more complex and even leads to some nonlinear problems.

Many attentions have been paid to the research on the stability analysis and observer-based controller design of NCSs with delays and dropouts [11-14]. The work [12] discussed the observer-based H-infinity controller design problem for NCSs with packet dropouts in the

multiple channels case, and [13] concerned with the continuous-time networked control system with random sensor-delay. Among these works, it should be mentioned that the Lyapunov functional adopted in [12] only has partial information of the closed-loop states. This can simplify the analysis and synthesis but also make the result conservative for the absence of delay information in the other side. Nowadays, more and more efforts are directed towards reducing the conservatism, which motivate this present paper.

Most of these previous works assumed the delays to be constant and the obtained results were delay-independent. In this present paper, our purpose is to design an observer-based controller for a continuous-time NCSs with consideration of the network-induced delays and packet dropouts. It is shown that a new augmented system model is constructed by introducing an observer and we apply an appropriate delay-dependent Lyapunov functional for the stability analysis. The important point is that the sensor-to-controller and controller-to-actuator network-induced delays and packet dropouts are considered and treated to be equivalent to time-varying delays in this paper. Moreover, the singular value decomposition method is adopted for the obtained nonlinear condition. This will bring a less conservative result.

The notation used in this paper is standard.  $\mathbb{R}^n$  denotes the  $n$ -dimensional real Euclidean space,  $\mathbb{R}^{n \times m}$  is the set of  $n \times m$  real matrices.  $\mathbb{N}$  denotes the natural numbers set. The notation  $X^T$  and  $X^{-1}$  denote its transpose and inverse when it exists, respectively. Given a symmetric matrix  $X = X^T$ , the notation  $X > 0$  ( $X \geq 0$ ) means that the matrix  $X$  is real positive definiteness (semi-definiteness). By *diag* we denote a block diagonal matrix with its input arguments on the diagonal.  $I$  denotes the identity matrix. The symbol  $*$  within a matrix represents the symmetric entries.  $\|\cdot\|$  stands for either the Euclidean vector norm or its induced matrix 2-norm.

## 2. Problem Statement

The network-induced delays and packet dropouts are the main features of NCS. Firstly, we will do some explanations for these characteristics of the system under consideration in this paper. The sensor is time-driven and its signal is sampled periodically (sampling period  $h$ ;  $h > 0$ ) at sampling instants. The controller and actuator are event-driven. That means the sensor data arrives at the controller, the control signal is calculated. And the output of the controller arrives at the actuator, the plant inputs are changed immediately.

In view of the network-induced delays and packet dropouts, we use the term  $\tau_k$  referring to the delays between the sensor and the controller ( $\tau_k^{sc}$ ) and delays between the controller and the actuator ( $\tau_k^{ca}$ ), namely,  $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ ,  $0 \leq \tau_k \leq \bar{\tau}$ . Similarly, we use  $d$  referring to the consecutive packet dropouts between the sensor and the controller ( $d^{sc}$ ) and dropouts between the controller and the actuator ( $d^{ca}$ ) namely,  $d = d^{sc} + d^{ca}$ ,  $0 \leq d \leq \bar{d}$ .

Consider a system described as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_1\omega(t) \\ y(t) &= C_1x(t) \\ z(t) &= C_2x(t) + D\omega(t) \end{aligned} \quad (1)$$

Where  $x(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}^m$  is the input,  $y(t) \in \mathbb{R}^r$  is the measured output of the plant,  $\omega(t)$  and  $z(t)$  are the disturbance input and controlled output, respectively. And  $\omega(t) \in L_2[0, \infty)$ . The system matrices  $A$ ,  $B$ ,  $B_1$ ,  $C_1$ ,  $C_2$ , and  $D$  are constant matrices with appropriate dimensions.

This paper views the packet dropouts as a delay which grow beyond the defined bounds. Then the total system delays denoted by  $\eta(t)$  satisfy  $\eta(t) \in \left[ \min_k \tau_k, (\bar{d} + 1)h + \max_k \tau_{k+\bar{d}+1} \right] = [0, \bar{\eta}]$ , for any  $k \in \mathbb{N}$ , where  $\bar{\eta} = (\bar{d} + 1)h + \bar{\tau}$ . Then the original system with delays and packet dropouts is equivalent to a system with time-varying delays.

**Remark 1.** By the definition,  $\eta(t)$  is a piecewise continuous function, which changes whenever the sensor signal reaches the controller. The derivative of  $\eta(t)$  is always equal to one, except at the transition point.

Due to that not all states can be measured, the dynamic observer and controller are constructed as follows:

$$\begin{aligned}\dot{x}_o(t) &= Ax_o(t) + Bu(t) + L(y(t) - C_1x_o(t)) \\ u(t) &= Kx_o(t - \eta(t))\end{aligned}\quad (2)$$

Where  $x_o(t) \in \mathbb{R}^n$  is the state of the observer,  $K$  and  $L$  are the gains of the controller and observer, respectively.

By introducing the estimation error  $e(t) = x(t) - x_o(t)$ , we get the following augmented system.

$$\begin{aligned}\dot{\xi}(t) &= \bar{A}\xi(t) + \bar{B}\xi(t - \eta(t)) + \bar{B}_1\omega(t) \\ z(t) &= \bar{C}\xi(t) + \bar{D}\omega(t)\end{aligned}\quad (3)$$

Where,

$$\begin{aligned}\xi^T(t) &= [x^T(t) \quad e^T(t)] \\ \bar{A} &= \begin{bmatrix} A & 0 \\ 0 & A - LC_1 \end{bmatrix}, \bar{B} = \begin{bmatrix} BK & -BK \\ 0 & 0 \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} B_1 \\ B_1 \end{bmatrix}, \bar{C} = [C_2 \quad 0], \bar{D} = D\end{aligned}\quad (4)$$

This paper aims to design an observer-based  $H_\infty$  control law such that the closed-loop system (3) satisfies the following properties simultaneously:

- The close-loop system (3) is asymptotically stable;
- Subject to the zero initial condition and all nonzero  $\omega(t)$ , the controlled output  $z(t)$  satisfies  $\|z(t)\|_2^2 \leq \gamma^2 \|\omega(t)\|_2^2$ .

**Lemma 1 [15].** For any vectors  $a, b$  and matrices  $N, X, Y, Z$  with appropriate dimensions,

where  $X$  and  $Z$  are symmetric. If  $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0$  then:

$$-2a^T N b \leq \inf_{X, Y, Z} \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

### 3. Result and Analysis

Firstly, we have the  $H_\infty$  performance analysis, a sufficient condition of the asymptotically stability with  $H_\infty$  performance level for the system in (3) is given as follows:

**Theorem 1.** Consider the closed-loop system (3) and for the given positive constants  $\gamma$  and  $\bar{\eta}$ ,  $K$  and  $L$  are the gains of the controller and observer, respectively. if there exist matrices with appropriate dimensions  $P > 0, Q > 0, R > 0$  and  $X_1 > 0, X_2, X_3 > 0, Y_1, Y_2$  satisfying:

$$\Omega = \begin{bmatrix} \Pi_{11} & \Pi_{12} & P\bar{B}_1 + \bar{\eta}\bar{A}^T R \bar{B}_1 & \bar{C}^T \\ * & \Pi_{22} & \bar{\eta}\bar{B}^T R \bar{B}_1 & 0 \\ * & * & \bar{\eta}\bar{B}_1^T R \bar{B}_1 - \gamma^2 I & \bar{D}^T \\ * & * & * & -I \end{bmatrix} < 0\quad (5)$$

$$\Theta = \begin{bmatrix} X & Y \\ * & R \end{bmatrix} \geq 0\quad (6)$$

Where,

$$\begin{aligned}\Pi_{11} &= P\bar{A} + \bar{A}^T P + Q + \bar{\eta}X_1 + Y_1 + Y_1^T + \bar{\eta}\bar{A}^T R\bar{A} \\ \Pi_{12} &= P\bar{B} + \bar{\eta}X_2 - Y_1 + Y_2^T + \bar{\eta}\bar{A}^T R\bar{B} \\ \Pi_{22} &= -Q + \bar{\eta}X_3 - Y_2 - Y_2^T + \bar{\eta}\bar{B}^T R\bar{B} \\ X &= \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix}, Y = \begin{bmatrix} Y_1^T & Y_2^T \end{bmatrix}^T\end{aligned}$$

Then the system (3) is asymptotically stable with an  $H_\infty$  performance level  $\gamma$ .

**Proof:** Construct a Lyapunov-Krasovskii functional as:

$$V = \xi^T(t)P\xi(t) + \int_{t-\eta(t)}^t \xi^T(t)Q\xi(t)dt + \int_{-\eta}^0 \int_{t+\theta}^t \dot{\xi}^T(s)R\dot{\xi}(s)dsd\theta$$

Where  $P > 0$ ,  $Q > 0$ ,  $R > 0$

Taking the time derivative of  $V$  to obtain:

$$\begin{aligned}\dot{V} &= \xi^T(t) [P\bar{A} + \bar{A}^T P + Q + \bar{\eta}\bar{A}^T R\bar{A}] \xi(t) + \xi^T(t) [2P\bar{B} + 2\bar{\eta}\bar{A}^T R\bar{B}] \xi(t - \eta(t)) \\ &\quad + \xi^T(t) [2P\bar{B}_1 + 2\bar{\eta}\bar{A}^T R\bar{B}_1] \omega(t) + \xi^T(t - \eta(t)) [-Q + \bar{\eta}\bar{B}^T R\bar{B}] \xi(t - \eta(t)) \\ &\quad + \xi^T(t - \eta(t)) [2\bar{\eta}\bar{B}^T R\bar{B}_1] \omega(t) + \omega^T(t) [\bar{\eta}\bar{B}_1^T R\bar{B}_1] \omega(t) - \int_{t-\eta}^t \dot{\xi}^T(v)R\dot{\xi}(v)dv\end{aligned}$$

Define  $\varepsilon^T(t) = [\xi^T(t) \quad \xi^T(t - \eta(t))]$  combining Leibniz-Newton formula and the Lemma 1, we have:

$$\begin{aligned}- \int_{t-\eta}^t \dot{\xi}^T(v)R\dot{\xi}(v)dv &\leq \bar{\eta} \begin{bmatrix} \xi(t) \\ \xi(t - \eta(t)) \end{bmatrix}^T \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t - \eta(t)) \end{bmatrix} \\ &\quad + 2 \begin{bmatrix} \xi(t) \\ \xi(t - \eta(t)) \end{bmatrix}^T \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} [\xi(t) - \xi(t - \eta(t))]\end{aligned}$$

Combining all of the above, we obtain:

$$\dot{V} \leq \begin{bmatrix} \xi(t) \\ \xi(t - \eta(t)) \end{bmatrix}^T \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t - \eta(t)) \end{bmatrix}$$

Using the Schur complement [16], the inequality (5) implies  $\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0$ . Thus, the closed loop system (3) is asymptotically stable. And then we will discuss the  $H_\infty$  performance of the system.

Letting:

$$J_{z\omega} = \int_0^\infty [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)] dt \quad (7)$$

Under zero initial condition,  $V(0) = 0$  and  $V(\infty) \geq 0$ . Thus:

$$J_{z\omega} \leq \int_0^\infty [z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t)] dt$$

For any nonzero  $\omega(t) \in L_2[0, \infty)$ , we have:

$$J_{z\omega} \leq \int_0^{\infty} \zeta^T(t) \Xi \zeta(t) dt$$

Where,

$$\zeta^T(t) = [ \xi^T(t) \quad \xi^T(t - \eta(t)) \quad \omega^T(t) ]$$

$$\Xi = \begin{bmatrix} \Pi_{11} + \bar{C}^T \bar{C} & \Pi_{12} & P\bar{B}_1 + \bar{\eta} \bar{A}^T R \bar{B}_1 + \bar{C}^T \bar{D} \\ * & \Pi_{22} & \bar{\eta} \bar{B}^T R \bar{B}_1 \\ * & * & \bar{\eta} \bar{B}_1^T R \bar{B}_1 - \gamma^2 I + \bar{D}^T \bar{D} \end{bmatrix}$$

Applying the Schur complement to (5) yields  $\Xi < 0$ . Using the zero initial condition.

$$\int_0^{\infty} z^T(t) z(t) dt < \int_0^{\infty} \gamma^2 \omega^T(t) \omega(t) dt$$

We have  $\|z(t)\|_2^2 \leq \gamma^2 \|\omega(t)\|_2^2$  namely, the system has a prescribed  $H_{\infty}$  performance level  $\gamma$ . The proof is completed.

**Remark 2.** The gains  $K$  and  $L$  are given in the stability analysis, condition (5) is LMI which can be solved easily. However, when considering the controller design, the parameters  $K$  and  $L$  become unknown variables, namely, the matrix inequality in (5) is nonlinear. Therefore, we cannot solve it directly. This paper extends the singular value decomposition method proposed in [17] to deal with this problem.

**Theorem 2.** For the given positive constants  $\gamma$  and  $\bar{\eta}$ ,  $K$  and  $L$  are the gains of the controller and observer, respectively. The closed-loop system (3) is asymptotically stable with an  $H_{\infty}$  performance level  $\gamma$ , if there exist matrices with appropriate dimensions  $W > 0$ ,  $W_{21} > 0$ ,  $W_{22} > 0$ ,  $Q_i > 0$ ,  $S_i > 0$ ,  $R_i > 0$  and  $X_{1i}$ ,  $X_{2i}$ ,  $X_{3i}$ ,  $Y_{1i}$ ,  $Y_{2i}$ ,  $M$ ,  $N$  ( $i = 1, 2, 3$ ) satisfying the matrix inequalities (8), (9) and the matrix Equation (10), (11).

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & B_1 & W_1 C_2^T & W_1 A^T & 0 \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} & B_1 & W_2^T C_2^T & W_2^T A^T & W_3 A^T - C_1^T N_3^T \\ * & * & \Phi_{33} & \Phi_{34} & 0 & 0 & \Phi_{37} & 0 \\ * & * & * & \Phi_{44} & 0 & 0 & \Phi_{47} & 0 \\ * & * & * & * & -\gamma^2 I & D^T & B_1^T & B_1^T \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\bar{\eta}^{-1} S_1 & -\bar{\eta}^{-1} S_2 \\ * & * & * & * & * & * & * & -\bar{\eta}^{-1} S_3 \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} X_{1i} & X_{2i} & Y_{1i} \\ * & X_{3i} & Y_{2i} \\ * & * & R_i \end{bmatrix} \geq 0 \quad (9)$$

$$W_2 C_1^T = C_1^T \tilde{W}_2 \quad (10)$$

$$W_3 C_1^T = C_1^T \tilde{W}_3 \quad (11)$$

Where,

$$\begin{aligned}
\Phi_{11} &= AW_1 + W_1A^T + Q_1 + \bar{\eta}X_{11} + Y_{11} + Y_{11}^T \\
\Phi_{12} &= AW_2 + W_2A^T - C_1^T N_2^T + Q_2 + \bar{\eta}X_{12} + Y_{12} + Y_{12}^T \\
\Phi_{13} &= BM_1 - BM_4 + \bar{\eta}X_{21} - Y_{11} + Y_{21}^T \\
\Phi_{14} &= BM_2 - BM_3 + \bar{\eta}X_{22} - Y_{12} + Y_{22}^T \\
\Phi_{22} &= AW_3 + W_3A^T - N_3C_1 - C_1^T N_3^T + Q_3 + \bar{\eta}X_{13} + Y_{13} + Y_{13}^T \\
\Phi_{23} &= \bar{\eta}X_{22} - Y_{12}^T + Y_{22}, \Phi_{24} = \bar{\eta}X_{23} - Y_{13} + Y_{23}^T \\
\Phi_{33} &= -Q_1 + \bar{\eta}X_{31} - Y_{21} - Y_{21}^T \\
\Phi_{34} &= -Q_2 + \bar{\eta}X_{32} - Y_{22} - Y_{22}^T \\
\Phi_{37} &= M_1^T B^T - M_4^T B^T, \Phi_{47} = M_2^T B^T - M_3^T B^T \\
\Phi_{44} &= -Q_3 + \bar{\eta}X_{33} - Y_{23} - Y_{23}^T \\
W &= \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}, X_{ki} = WX_{ki}W, Y_{ji} = WY_{ji}W, i, k = 1, 2, 3, j = 1, 2
\end{aligned}$$

Furthermore, the controller parameters are given as  $K = M_1W_1^{-1}$ ,  $L = N_2\tilde{W}_2^{-1}$ .

**Proof:** By using the Schur complement, (5) can be rewritten as follows:

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & P\bar{B}_1^T & \bar{C}^T & \bar{A}^T \\ * & \Psi_{22} & 0 & 0 & \bar{B}^T \\ * & * & -\gamma^2 I & \bar{D}^T & \bar{B}_1^T \\ * & * & * & -I & 0 \\ * & * & * & * & -\bar{\eta}^{-1}R^{-1} \end{bmatrix} < 0 \quad (12)$$

Where,

$$\begin{aligned}
\Psi_{11} &= P\bar{A} + \bar{A}^T P + Q + \bar{\eta}X_1 + Y_1 + Y_1^T \\
\Psi_{12} &= P\bar{B} + \bar{\eta}X_2 - Y_1 + Y_2^T \\
\Psi_{22} &= -Q + \bar{\eta}X_3 - Y_2 - Y_2^T
\end{aligned}$$

Defining  $W = P^{-1} = \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}$ ,  $\Delta_1 = \text{diag}\{W, W, I, I, I\}$ ,  $\Delta_2 = \text{diag}\{W, W, W\}$ .

Performing congruence transformations to (12) by  $\Delta_1$  and substituting (4) into (12), since the matrix  $C_1^T$  is of full column rank, we denote:

$$\begin{aligned}
M_i &= KW_i, W_j C_1^T = C_1^T \tilde{W}_j, N_j^T = \tilde{W}_j L^T \\
i &= 1, 2, 3, 4, j = 2, 3
\end{aligned}$$

Then we can obtain (8).

Similarly, (9) is given through performing congruence transformations to (6) by  $\Delta_2$ . The proof is completed.

**Remark 3.** For solving the equation constraint (10), literature [17] proposed a singular value de- composition method (SVD). For the matrix  $C_1^T$ , there always have:

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} C_1^T V = \begin{bmatrix} \Sigma \\ 0 \end{bmatrix}$$

Where  $U = [U_1 \ U_2]^T$  and  $V$  are two orthogonal matrices and  $\Sigma$  is a diagonal matrix with positive diagonal elements.

If matrix  $W$  satisfying,

$$W_2 = U_1^T W_{21} U_1 + U_2^T W_{22} U_2$$

Where  $W_{21} > 0$ ,  $W_{22} > 0$  and  $U_1, U_2$  are defined above, then there exists  $\tilde{W}_2$  satisfying  $W_2 C_1^T = C_1^T \tilde{W}_2$ . It is equivalent to:

$$W_2 U^T \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V^T = U^T \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V^T \tilde{W}_2$$

Thus,

$$\tilde{W}_2^{-1} = V \Sigma^{-1} W_{21}^{-1} \Sigma V^T$$

We have the gains of the controller and observer.

$$K = M_1 W_1^{-1}, L = N V \Sigma^{-1} W_{21}^{-1} \Sigma V^T$$

**Remark 4.** By using the singular value decomposition method, we have a good solution to deal with the nonlinear terms BKW and LC<sub>1</sub>W. This implies that the feasible values of K and L can be obtained easily and the observer-based H<sub>∞</sub> controller is designed.

#### 4. A Numerical Example

In this part, we will use a numerical example to demonstrate the validity of the proposed approach. Consider system (1) with:

$$\begin{aligned} A &= \begin{bmatrix} -0.8 & 0 \\ 1 & 0.1 \end{bmatrix}, B = \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix} \\ C_1 &= [1 \ 0], C_2 = [1 \ 0], D = 0.01 \end{aligned}$$

According to Remark 3, we use the matrix singular value decomposition and obtain:

$$U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, V = 1, \Sigma = 1$$

Suppose that the sampling interval is  $h = 0.1$  and  $\gamma = 0.625$ . When the time delay variable initial condition are given with  $\bar{\eta} = 0.9$  and  $x_0 = [0.5 \ 1]^T$ , the control gain  $K$  and observer gain  $L$  can be obtained by applying Theorem 2 as:

$$K = [ -31.6777 \quad -24.6816 ], L = \begin{bmatrix} -18.4192 \\ -1.9466 \end{bmatrix} \quad (13)$$

The disturbance input presented in this example is  $\omega(t) = e^{-t}$ . Figure 1 shows the maximum singular value plot of the closed-loop system by the obtained controller and observer (13) and Figure 2 shows the state response of the closed-loop system. It is clear that the states become convergent to zero and the system can work well with the proposed method in this paper.

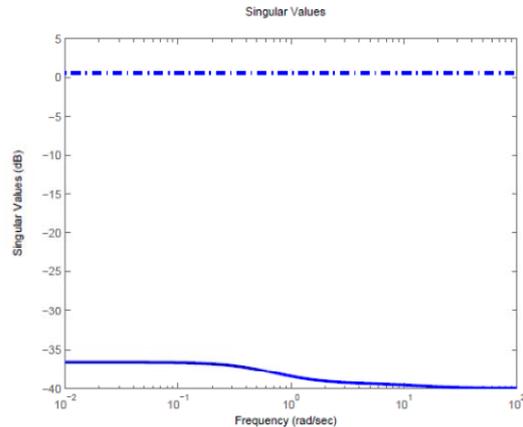


Figure 1. Maximum Singular Value of the System with Observer-based Control

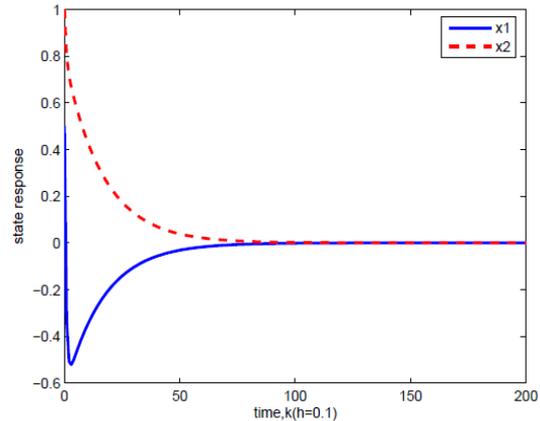


Figure 2. The State Response of the System with Observer-based Control

## 5. Conclusion

The design problem of an observer-based  $H_\infty$  controller for a linear continuous-time NCSs with time delays and packet dropouts has been investigated in this paper. It is shown that a new augmented system model is constructed by introducing an observer. Based on the delay-dependent Lyapunov-Krasovskii functional, a sufficient condition is derived to guarantee the closed-loop system asymptotically stable with  $H_\infty$  performance level  $\gamma$ . The SVD method is used to deal with the nonlinear problem existed in the obtained condition. Compared with the existing results, we employ tighter bounding of the cross terms in deriving stability condition and obtain a delay-dependent result. The method proposed in this paper is less conservative and a numerical example has shown its simplicity and effectiveness.

## References

- [1] MBG Cloosterman, N van de Wouw, WFMH Heemels, H Nijmeijer. Stability of networked control system with uncertain time-varying delays. *IEEE Transactions on Automatic Control*. 2009; 54: 1575-1580.
- [2] Y Tipsuwan, MY Chow. Control methodologies in networked control systems. *Control Engineering practice*. 2003; 11: 1099-1111.
- [3] M Trivellato, N Benvenuto. State control in networked control systems under packet drops and limited transmission. *IEEE Transactions on Communications*. 2010; 58: 611-622.
- [4] TC Yang. Networked control system: A brief survey. *IEE Proceedings of Control Theory Application*. 2006; 153: 403-412.
- [5] HJ Gao, XY Meng, T Chen. Stabilization of networked control systems with a new delay characterization. *IEEE Transactions on Automatic control*. 2008; 53: 2142-2148.
- [6] J Wu, T Chen, L Wang. Delay-dependent robust stability and h-control for jump linear systems with delays. *Systems and Control Letters*. 2006; 51: 511-518.
- [7] W Zhang, MS Branicky, SM Phillips. *On improved robust stabilization for uncertain systems with unknown input delays*. Proceedings of American Control Conference. 2011; 1656-1661.
- [8] D Yue, QL Han, J Lam. Network-based robust h-control of systems with uncertainty. *Automatica*. 2005; 41: 999-1007.
- [9] D Yue, QL Han. Delayed feedback control of uncertain systems with time-varying input delay. *Automatica*. 2005; 41: 233-240.
- [10] WT Chen. Design of networked control systems with packet dropouts. *IEEE Transactions on Automatic control*. 2007; 52: 1314-1319.
- [11] D Yue, QL Han, C Peng. State feedback controller design of Networked control systems. *IEEE Transactions on Circuits Systems: II*. 2004; 51: 640-644.
- [12] WW Che, JL Wang, GH Yang. Observer-based h-infinity control in multiple channel networked control systems with random packet dropouts. *Journal of Control Theory and Applications*. 2010; 8: 359-367.

- 
- [13] C Lin, Z Wang, F Yang. Observer-based networked control for continuous-time systems with random sensor delays. *Automatica*. 2009; 45: 578-584.
- [14] YL Wang, QL Han. *Observer-based continuous-time networked control system design*. Proceedings of American Control Conference. 2012; 5694-5699.
- [15] YS Moon, P Park, WH Kwon, YS Lee. Delay-dependent robust stabilization of uncertain state-delayed systems. *International Journal of Control*. 2001; 74: 1447-1455.
- [16] L Yu, *Robust Control LMI Approach*. Beijing: Tinghua University Press. 2002.
- [17] FW Yang, ZD Wang, YS Hung, M Gani.  $H_\infty$  control for networked systems with random communication delays. *IEEE Transactions on Automatic control*. 2006; 51: 511-518.