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# Improve fractal interpolation function with Sierspinski triangle

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**ABSTRACT**

Interpolation techniques can be used to determine the approximate value of a parameter if it is known that two values are bound to a certain interval. Interpolation can be done numerically or fractal. The fractal interpolation value is influenced by the vertical scale factor and the fractal interpolation function (FIF). This research introduces fractal interpolation technique with FIF which is constructed from Sierspinski triangles. As an example of application, the interpolation technique is applied to determine the approximate value of the rice demand parameter in the inventory model. The accuracy of the interpolation results is determined using the mean absolute percentage error (MAPE). The number of triangles obtained and the interpolation values for each successive iteration are $3^n$ and $3^{n+1}$. MAPE values from 6 to 9 iteration were 24.603%, 24.603%, 23.858%, 23.772% respectively. There is a decrease in the value of MAPE, this indicates an increase in the value of the accuracy of the interpolation results. It can be concluded that the MAPE value is also influenced by the number of iterations of the interpolation technique.

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## 1. INTRODUCTION

Interpolation is one of the methods for determining approximate values with known two interval boundary values. There are two interpolation techniques, namely numerical interpolation and fractals interpolation. The application of numerical interpolation techniques has been widely carried out in various fields, including the financial sector [1]. Estimate and predict the gas content of hydrogen and iodine in the formation of iodic acid reactions using newton and cubic splines interpolation method [2]. Kriging interpolation with nonlinear functions is employed for predicting rock joint shear strength [3], while the co-kriging interpolation method is utilized for multi-fidelity analysis and uncertainty quantification of beam vibration [4]. Low-cost dive altitude sensor in remotely operated underwater vehicle (ROV) using the Newton's polynomials interpolation based-error correction method [5]. Risk evaluation of investment with cubic spline interpolation [6]. A lagrange-quadratic spline optimal collocation method for the time tempered fractional diffusion equation [7]. Lagrange interpolation to adaptive differential evolution [8]. Analysis of adaptive decision based inverse distance weighted interpolation (DBIDWI) algorithm for salt and pepper noise [9]. Deterministic interpolation methods to climate change detection in Penang Island [10]. The second interpolation technique is fractal interpolation. The application of fractal interpolation has also been widely carried out in various fields. Analysis of coronavirus disease 2019 (COVID-19) spread using fractal interpolation [11]. Analysis random data set using fractal interpolation function (FIF) [12]–[14]. Seismic traces using fractal interpolation with vertical scale factor and residual behavior [15]. In this study, fractal interpolation was applied to determine the value of the demand parameter approach in the inventory model.

The fractal interpolation value is influenced by the vertical scale factor and FIF. Many studies on fractal interpolation have been carried out. Vertical scaling factor improvement in fractal interpolation to predict in navigation system [16]. Improve FIF using box-counting dimension with function contractivity factors [17], [18]. Construct FIF with iterated function system (IFS) [19]. Rational cubic trigonometric FIF that are the generalized fractal version of the classical rational cubic trigonometric polynomial spline [20]. Generalized trigonometric function as FIF [21]. Ri [22], [23] introduce a new idea to construct the nonlinear FIF. Construct FIF with multivariate affine function [24]. FIF with non-affine function [25]. Construct FIF with gasket Sierspinski [26], [27] introduce new surface FIF. FIF construction is a very important part in fractal interpolation. Previous studies have shown that several functions can be used as FIF, including nonlinear and trigonometric functions. This research introduces fractal interpolation techniques with the development of FIF constructed from the Sierpinski triangle. The Sierpinski triangle is a fractal shape constructed from affine functions. The FIF of the Sierpinski triangle consists of three affine functions that map the initial triangle into a new triangle with half the size of the previous triangle. In fractal interpolation, a guarantee of the existence of the attractor is required. Ri [26], [27] has guaranteed the existence of an attractor from gasket Sierpinski. The Sierpinski triangle is part of the Sierpinski gasket, thus ensuring that the FIF built from the Sierpinski triangle can interpolate the data. Interpolation techniques can be used to determine the approximate value of parameters that are not known with certainty, such as the demand parameter in the inventory model. Generally, the value of the demand parameter in the inventory model is determined by forecasting techniques. In this study, the uncertainty approach to the demand parameter is determined using fractal interpolation. For further information, an interpolation calculation using rice supply data is provided.

## 2. METHOD

In this research, fractal interpolation method is introduced with FIF which is constructed from Sierspinski triangle. The following is the FIF formulation of the Sierspinski triangle:

$$F_i: \mathbb{R}^2 \to \mathbb{R}^2, F = \{F_1, F_2, F_3\}$$

$$F_1\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{1}$$

$$F_2\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} \tag{2}$$

$$F_3\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} \tag{3}$$

$F_1$ maps the initial Sierspinski triangle into congruent triangles with a size 0.5 smaller than the initial shape. The $F_2$ function maps the initial shape with a size 0.5 smaller than the initial shape and then shifts it up parallel to the vertical axis by 0.5 units. The $F_3$ function maps the initial shape into a new triangle with a size 0.5 smaller than the initial triangle and shifted to the right parallel to the horizontal axis by 0.5 units.

The following is a fractal interpolation algorithm with FIF constructed from Sierspinski triangles.

a) Given the initial data $\{(x_i, y_i)^T \in \mathbb{R}^2 : i = 1,2,3\}, x_1 < x_2 < x_3$ and the number of iterations desired. The initial conditions $(x_i, y_i)^T$ are taken based on the data. The selection of initial conditions is based on the consideration that the triangular region described from the three initial conditions will include all values in the data. This is related to the calculation of the error value.

b) Determine distance $x_1, x_2, x_3$ and distance $y_1, y_2, y_3$:

$$d_1 = |x_1 - x_2|, d_2 = |x_1 - x_3|, d_3 = |x_2 - x_3|$$

$$d_4 = |y_1 - y_2|, d_5 = |y_1 - y_3|, d_6 = |y_2 - y_3|$$

Distance calculations are needed to determine the values $\begin{bmatrix} x^* \\ y^* \end{bmatrix}$.

c) Determine $F_1\left(\begin{bmatrix} x^* \\ y^* \end{bmatrix}\right)$ based on (1), $F_2\left(\begin{bmatrix} x^* \\ y^* \end{bmatrix}\right)$ based on (2), and $F_3\left(\begin{bmatrix} x^* \\ y^* \end{bmatrix}\right)$ based on (3).

d) We get a new triangle $S^* = \bigcup_{i=1}^{3} F_i(S^*) = F_1(S^*) \cup F_2(S^*) \cup F_3(S^*)$
e) The iteration is continued and returns to Step 2.
f) $S^*$ is a new triangle with the interpolated points. A new triangle is obtained three triangle that is half the size of the previous triangle. Iterations can be continued until the expected error value is obtained.

## 3. RESULTS AND DISCUSSION

The application of fractal interpolation technique aims to determine the approximate value of the demand parameter in the inventory model. In this study, fractal interpolation techniques were applied to determine the value of rice demand parameters in a company.

Based on the data in Table 1 taken (10000, 3138679), (10800, 1177733), (12000, 362400) as the initial condition to perform fractal interpolation with FIF built from Sierspinski triangles. This value is taken from the lowest and highest prices and one more point is taken by considering the initial triangle formed which can cover the data area. The initial condition selection is not single, three other pairs of points in the data can also be selected, such as (10000, 3138679), (10150, 1512299), (12000, 362400) as the initial condition. However, after carrying out the calculations, it can be seen from the number of iterations that selection (10000, 3138679), (10800, 1177733), (12000, 362400) as the starting point is more optimal. The following is the calculation stage for iteration 1.

a) Let $x_1 = 10000$; $y_1 = 3138679$; $x_2 = 10800$; $y_2 = 1177733$; $x_3 = 12000$; $y_3 = 362400$. A sketch of the Sierpinski triangle image based on the initial conditions given in Figure 1.

Table 1. Data on prices and demand

| Period (month) | Price (IDR) | Demand (kilogram) |
|---|---|---|
| 1 | 10500 | 1877819 |
| 2 | 10150 | 1512299 |
| 3 | 10800 | 1177733 |
| 4 | 11550 | 1405435 |
| 5 | 11400 | 2419069 |
| 6 | 12000 | 362400 |
| 7 | 10250 | 1966169 |
| 8 | 10500 | 1848084 |
| 9 | 10400 | 1449570 |
| 10 | 11800 | 923049 |
| 11 | 10350 | 2119598 |
| 12 | 10000 | 3138679 |

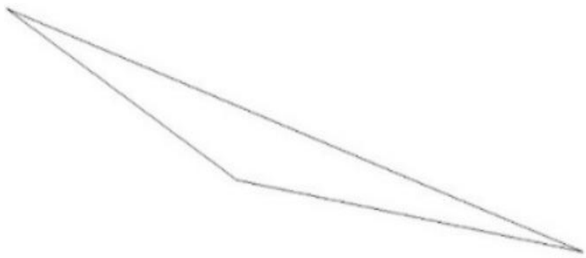

Figure 1. Sierpinski triangle iteration 0

Figure 1 is an initial triangle formed based on pairs of points as initial conditions. Next, using (1)-(3) the Sierspinski triangle will be formed.

b) Determine the distance to the $x$ and $y$ axes

$$d_1 = |x_1 - x_2| = |10000 - 10800| = 800$$
$$d_2 = |x_1 - x_3| = |10000 - 12000| = 2000$$
$$d_3 = |x_2 - x_3| = |10800 - 12000| = 1200$$
$$d_4 = |y_1 - y_2| = |3138679 - 1177733| = 1960946$$
$$d_5 = |y_1 - y_3| = |3138679 - 362400| = 2776279$$
$$d_6 = |y_2 - y_3| = |1177733 - 362400| = 815333$$

The distances $d_1$, $d_2$ and $d_3$ are used to determine the translation about the horizontal axis. While the distances $d_4$, $d_5$, and $d_6$ are used to determine the translation about the vertical axis. Using FIF (1)-(3) we will get $\begin{bmatrix} x^* \\ y^* \end{bmatrix}$ as the new initial condition.

c) Determine interpolation value based on FIF (1), (2), and (3).

Interpolation is performed on (10000, 3138679), (10800, 1177733), (12000, 362400) use the $f_1$ function.

$$\begin{aligned} f_1(x_1{}^*, y_1{}^*) &= \left(x_1 + \tfrac{1}{2}|x_2 - x_1|, y_1 - \tfrac{1}{2}|y_2 - y_1|\right) \\ &= \left(10000 + 0{,}5(400), 3138679 - 0{,}5(1960946)\right) \\ &= (10400, 2158206) \end{aligned}$$

Obtained $f_1(x_1{}^*, y_1{}^*) = (10400, 2158206)$

$$\begin{aligned} f_1(x_2{}^*, y_2{}^*) &= (x_2, y_2) = (10800, 1177733) \\ f_1(x_3{}^*, y_3{}^*) &= \left(x_3 - \tfrac{1}{2}|x_3 - x_2|, y_3 + \tfrac{1}{2}|y_3 - y_2|\right) \\ &= \left(12000 - 0{,}5(1200), 362400 + 0{,}5(815333)\right) \\ &= (11400, 770067) \end{aligned}$$

Obtained $f_1(x_3{}^*, y_3{}^*) = (11400, 770067)$
Interpolation is performed on $(10000, 3138679), (10800, 1177733), (12000, 362400)$ use the $f_2$.

$$\begin{aligned} f_2(x_1{}^*, y_1{}^*) &= (x_1, y_1) = (10000, 3138679) \\ f_2(x_2{}^*, y_2{}^*) &= f_1(x_1{}^*, y_1{}^*) = (10400, 2158206) \\ f_2(x_3{}^*, y_3{}^*) &= \left(x_3 - \tfrac{1}{2}|x_1 - x_3|, y_3 + \tfrac{1}{2}|y_1 - y_3|\right) \\ &= \left(12000 - 0{,}5(2000), 362400 + 0{,}5(2776279)\right) \\ &= (11000, 1750539) \end{aligned}$$

Interpolation results are obtained $f_2(x_3{}^*, y_3{}^*) = (11000, 1750539)$.
Interpolation is performed on $(10000{,}3138679), (10800{,}1177733), (12000{,}362400)$ use the $f_3$.

$$\begin{aligned} f_3(x_1{}^*, y_1{}^*) &= f_2(x_2{}^*, y_2{}^*) = (10400, 2158206) \\ f_3(x_2{}^*, y_2{}^*) &= f_1(x_3{}^*, y_3{}^*) = (11400, 770067) \\ f_3(x_3{}^*, y_3{}^*) &= (x_3, y_3) = (12000, 362400) \end{aligned}$$

Using $f_1$ and the initial conditions given, the interpolated values are obtained (10400, 2158206), (10800, 1177733), (11400, 770067). Using $f_2$ we get (10000, 3138679), (10400, 2158206), (11000, 1750539) and with $f_3$ we get (10400, 2158206), (11400, 770067), (12000, 362400). The Sierspinski triangle image resulting from iteration 1 is given in Figure 2.

Figure 1 is a new triangle with half the size of the previous triangle and ignoring the middle part. In iteration 1, each initial condition will be mapped by the functions $f_1$, $f_2$ and $f_3$ so that three new congruent triangles are obtained. Using the same steps in iteration 1, the calculation is continued for iteration 2 and so on. As a visualization for the interpolation stages, the Sierspinski triangle iteration 2 is given.

Figure 3 is the Sierspinski triangle resulting from the interpolation of iteration 2. There are nine new triangles and twenty-seven pairs of points which are the initial conditions for the iteration 2. The complete interpolation results of iteration 1 and iteration 2 are given in Table 2.

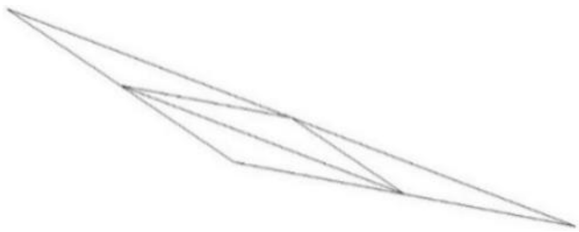

Figure 2. Sierpinski triangle iteration 1

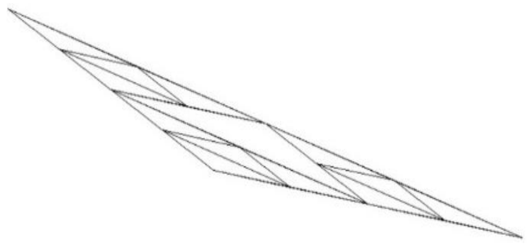

Figure 3. Sierpinski triangle iteration 2

Table 2. Interpolation results of iteration 1 and iteration 2

| Price (IDR) | Demand (kilogram) | Demand interpolation of iteration 1 | Demand interpolation of iteration 1 |
|---|---|---|---|
| 10500 | 1877819 | | |
| 10150 | 1512299 | | |
| 10800 | 1177733 | 1177733 | 1177733 |
| 11550 | 1405435 | | |
| 11400 | 2419069 | | |
| 12000 | 362400 | 362400 | 362400 |
| 10250 | 1966169 | | |
| 10500 | 1848084 | | 2444609 |
| 10400 | 1449570 | 2158206 | 2158206 |
| | | 2158206 | 2158206 |
| | | 2158206 | 2158206 |
| 11800 | 923049 | | |
| 10350 | 2119598 | | |
| 10000 | 3138679 | 3138679 | 3138679 |
| 10.200 | | | 2628443 |
| | | | 2628443 |
| | | | 2628443 |
| 10.600 | | | 1667970 |
| | | | 1667970 |
| | | | 1667970 |
| 10.700 | | | 1954373 |
| | | | 1954373 |
| 10.800 | | | 1177733 |
| 10.900 | | | 1464137 |
| | | | 1464137 |
| | | | 1464137 |
| | | | 1464137 |
| 11.000 | | 1750539 | 1750539 |
| 11.100 | | | 973900 |
| | | | 973900 |
| 11.200 | | | 1260303 |
| 11.400 | | 770067 | 770067 |
| | | 770067 | |
| 11.700 | | | 566234 |
| | | | 566234 |

In Table 2, the interpolation results are presented in the form of a rice demand value approach. In iteration 1, the demand approach values are obtained for prices of 10800, 12000, 10400, 10000, 1100, and 11400. In iteration 2, approximate values for the rice prices were obtained: 10500, 10800, 11400, 10400, 10000, 11000, 11200, 11700, 10200, 10600, 10700, and 10900. Up to iteration 2, MAPE cannot be calculated because the approach for several prices in Table 1 is not yet available. Therefore, the iteration continues until the MAPE calculation can be carried out and the expected level of accuracy is achieved. Based on the data, a better level of accuracy was achieved in iteration 9 compared to iterations 6, 7, and 8. Figure 4, visualizing iteration 9.

Figure 2 depicts the result of the 1st iteration of interpolation, where 3 similar triangles and 9 pairs of points are obtained. Figure 3 displays the Sierpinski triangle acquired from the 2nd iteration of interpolation, yielding 9 new triangles with 27 pairs of points. In the third iteration, 27 new triangles and 81 point pairs will be obtained, and so on. For the n-th iteration, it can be expressed as $3^n$ triangles, where n represents the number of iterations. The number of pairs of interpolation points generated is $3^{n+1}$. The level of accuracy is determined using MAPE, and the calculation of fractal interpolation and visualization is conducted using Python programming. The following are the values of the demand parameter approach based on fractal interpolation for iterations 6, 7, 8, and 9.

Figure 4. Sierpinski triangle iteration 9

In Table 3, the interpolation results are provided for the rice prices listed in Table 1. Based on the data in Table 1, MAPE values can be calculated starting from iteration 6. In this study, interpolation calculations were carried out up to iteration 9. The MAPE values obtained for iterations 6 to 9 are 24.603%, 24.603%, 23.858%, and 23.772%, respectively. Using the FIF constructed from the Sierpinski triangle, the MAPE values meet the sufficient criteria. There is a decrease in the MAPE value, indicating an increase in the accuracy value for the interpolation stages with higher iterations. However, the decrease in the MAPE value from iteration 6 to iteration 9 is not significant and remains within the sufficient criteria.

This paper introduces an interpolation technique with the development of FIF using affine mapping to construct Sierpinski triangles. In short, fractal interpolation with FIF, built from the affine mappings $f_1$, $f_2$ and $f_3$, forms a new triangle half the size of the previous one, ignoring the middle part. The pair of points forming the triangle results from interpolation. Calculatively, this interpolation technique is relatively easy to apply, yet the level of accuracy obtained still meets the sufficient criteria. The developed interpolation technique is applied to determine the approximate value of the demand data, which can be used to ascertain the uncertainty of demand parameter values in the inventory model.

Table 3. Demand fractal interpolation results

| Demand (kilogram) | Demand interpolation of iterations 6 and 7 (kilogram) | Demand interpolation of iterations 8 (kilogram) | Demand interpolation of iterations 9 (kilogram) |
|---|---|---|---|
| 1877819 | 1913087.6 | 1913087.6 | 1913087.6 |
| 1512299 | 2771001.2 | 2771001.2 | 2771001.2 |
| 1177733 | 1177733.0 | 1177733 | 1177733 |
| 1405435 | 933910.5 | 983740.7 | 983740.7 |
| 2419069 | 1168707.5 | 1168707.5 | 1193622.5 |
| 362400 | 362400.0 | 362400 | 362400 |
| 1966169 | 2525883.0 | 2525883.0 | 2525883.0 |
| 1848084 | 1913087.6 | 1913087.6 | 1913087.6 |
| 1449570 | 2158205.8 | 2158205.8 | 2158205.8 |
| 923049 | 586875.7 | 636705.8 | 636705.8 |
| 2119598 | 2280764.8 | 2280764.8 | 2280764.8 |
| 3138679 | 3138678.5 | 3138678.5 | 3138678.5 |
| **MAPE** | **24.603** | **23.858** | **23.772** |

## 4. CONCLUSION

Fractal interpolation calculations, applying the concept of building Sierpinski triangles, are generally conducted by transforming the initial triangle into three new triangles. The size of each new triangle is half that of the previous one, with a translation of the horizontal and vertical axes by half a unit distance. The interpolation results manifest as pairs of points. In the discussion of this paper, the interpolation results represent demand approach values based on rice prices. From the results and discussion, it can be inferred that the selection of initial conditions impacts the interpolation outcomes. The number of triangles generated for each iteration is $3^n$, while the number of interpolated values is $3^{n+1}$ where $n$ is the number of iterations. In this study, the level of accuracy was assessed using MAPE. According to the data presented in the results and discussion section, the MAPE values for Iterations 6 to 9 were 24.603%, 24.603%, 23.858%, and 23.772%, respectively. A decrease in the MAPE value indicates an improvement in the accuracy of the interpolation results. However, the level of accuracy obtained still requires enhancement. Utilizing the Sierpinski triangle ensures that the formed triangle consistently disregards the middle part, possibly neglecting values in that omitted section. To address this issue, further research could explore the application of the generalized Sierpinski triangle concept.

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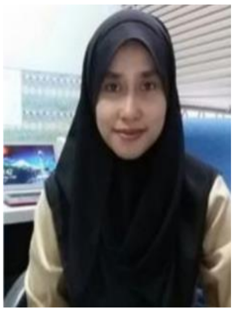

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