Modeling and Analyzing for the Friction Torque of a Sliding Bearing Based on Grey System Theory

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Abstract

Based on the grey system theory, the grey relational analysis method is proposed and used in analyzing the influence of various factors on the friction torque of a sliding bearing. On the basis of the grey relational analysis the multidimensional grey model GM(1,N,D) for the friction torque of a sliding bearing is built up. Taking Al-based alloy sliding bearing as an example, the calculation results show that, compared with other influence factors, friction coefficient, load, temperature and rotational speed have more significant influence on the bearing friction torque. Comparing experimental results and the calculated value of the GM(1,N,D) model based on these important influence factors, the maximum relative residuals is 9.09%, the average relative residuals is 7.9% and the accuracy is 92.1%. It verify that GM(1,N,D) model has good accuracy and is applicable for predicting friction torque of a sliding bearing.

Keywords: Grey system theory, sliding bearing, friction torque, influence factors

1. Introduction

The friction torque is an important performance parameter of bearing. It represents an energy loss and causes a retardation of motion,hence it is witnessed as a temperature increase [1-3]. In past several years, the bearing friction torque has received wide attention from many researchers, significant amounts of research haved been devoted to it. In 1959 Palmgren [4] gave a calculation formula to determine the friction torque value of a rolling bearing. Lin Guanyu [5] studied the characteristics of the deep groove ball bearing under different vacuum. In recent years study on the friction torque of a rolling bearing has made great progress in terms of the energy and scuffing failure [6-8]. However, there has been little research on the friction torque of a sliding bearing. So far there has been no systemic analyses and researches on the friction torque of a sliding bearing [9]. The main reason is that the friction torque of a sliding bearing is determined by a numberof factors (such as load, rotational speed, structure, etc), uncertainty factors (such as lubrication, material, lubrication, etc) and other complex factors. These factors were interactional, the friction torque of a sliding bearing shows great randomness and chances [10]. Furthermore, due to structure constrains it is difficult to obtain a mass of experimental data in actual conditions.

Grey system theory was founded by a Chinese scientist, Professor Deng JuLong in 1982 [11], taking the uncertain system of "small sample", "poor information", i.e. "partial information known, partial information unknown" as the research object. This theory holds that for a grey system just some information is known, some information is uncertain, even completely unknown, however there is a necessary intrinsic relation between them.Through exploiting and excavating the limited information, grey system theory can help us to understand the real world and to analyze its inherent characteristics. Because grey prediction model can compute and presume according to the limited information and does not have special requirement and limit to the observational data, it has been widely used in petroleum domain, geological field, medical field, automotive field and management field, and brought remarkable economic and social benefits [12-14].

The friction torque of a sliding bearing has the characteristics of nonlinearity and randomness, in addition, it is unrealistic to anslyze using a mass of original data. This condition

just accords with the grey system theory. Therefore, it is feasible to analyze the friction torque of a sliding bearing by using grey system theory.In this paper the influence of material hardness, rotational speed, load, temperature rise, friction coefficient and themal displacement on the friction torque of a sliding bearing is studied. several key factors are selected to established the multidimensional grey model GM(1,N,D) of the bearing friction torque, this model is verified by an application examples. This paper will provide theoretical basis for analyzing and predicting the friction torque of a sliding bearing.

2. Grey Relational Analysis Model for the Friction Torque of a Sliding Bearing

Affected by many kinds of factors, the friction torque characteristics of a sliding bearing is generally random and nonlinear. Therefore, using conventional methods it is difficult to determined the influence degree of these factors on bearing friction torque with limited original data. Grey relative analysis method is mainly used to analyse the relation between the behavioral variable and influencing factors, it can determine the influence degree of a grey factor on the behavioral variable. Thus, through comparing the influence degree of the material hardness, rotational speed, load, temperature rise, friction coefficient and thermal displacement on the friction torque of a sliding bearing, we can determine key influencing factors.

2.1. Determining the Influence Space @$_{INU}$

The influence space @$_{INU}$ is a data set including the behavioral variable and influencing factors, the behavioral variable is the friction torque of a sliding bearing, influencing factors are bearing material hardness, rotational speed, load, temperature rise, friction coefficient, thermal displacement, etc. So the influence space @$_{INU}$ can be expressed as follow.

$$@_{INU} = \{\omega_i \mid i \in I = \{1,2,3,4,5,6,7,8\}\} \tag{1}$$

Where ω_1 is the data series of bearing friction torque, ω_2 is the data series of bearing material hardness, ω_3 is the data series of bearing rotational speed, ω_4 is the data series of bearing load, ω_5 is the data series of bearing temperature rise, ω_6 is the data series of bearing friction coefficient, ω_7 is the data series of bearing inner ring thermal displacement, ω_8 is the data series of bearing outer ring thermal displacement.

2.2. Determining the Grey Relational Factor Space @$_{GRF}$

In order to make these data series comparable in the influence space, we must initial value treat them as follows:

$$@_{GRF} = (X_i(1), X_i(2), X_i(3) \cdots X_i(k)) = (\frac{\omega_i(1)}{\omega_i(1)}, \frac{\omega_i(2)}{\omega_i(1)}, \frac{\omega_i(3)}{\omega_i(1)} \cdots \frac{\omega_i(k)}{\omega_i(1)}) \tag{2}$$

Where k is the total number of series elements, and we call X_i initial value series, a data set $@_{GRF}$ including these initial value series is called the grey relational factor space.

2.3. Determining the Grey Relational Different Information Space Δ$_{GR}$

The grey relational difference information space includes the difference information set Δ, discrimination coefficient ζand environmental parameters set Δ_{1i}.It can be written as follow:

$$\Delta_{GR} = (\Delta, \zeta, \Delta_{1i}(max), \Delta_{1i}(min)) \tag{3}$$

The difference information is the difference series between friction torque series X$_1$and seven influence factors series X$_i$ after intial value treatment. It can be written as follow:

$$\Delta_{1i}(k) = |x_1(k) - x_i(k)|, \text{i=2,3……8} \tag{4}$$

The difference information set Δ include all difference information elements. It can be written as follow:

$$\Delta = \left\{ \Delta_{1i}(k) \middle| i = 2,3,4,5,6,7,8 \right\} \tag{5}$$

The discrimination coefficient ζ is a weighted coefficient of environmental parameter, generally the value of ζ is 0.5.

The environmental parameters set includes maximum value and minmum value in the difference information set Δ. It can be written as follow:

$$\begin{cases} \Delta_{1i}(max) = max_i max_k \Delta_{1i}(k) \\ \Delta_{1i}(min) = min_i min_k \Delta_{1i}(k) \end{cases} \tag{6}$$

2.4. Calculating the Grey Relational Coefficient

The grey relational coefficient provides a criteria to measure the influence degree of influencing factors(material hardness, rotational speed, load, temperature rise, friction coefficient, thermal displacement, etc) on the behavioral variable (the friction torque).It can be written as follow:

$$\gamma(x_1(k), x_i(k)) = \frac{\Delta_{1i}(min) + \zeta \Delta_{1i}(max)}{\Delta_{1i}(k) + \zeta \Delta_{1i}(max)} \tag{7}$$

The grey relational grade $\gamma(x_1, x_i)$ is an average value of the grey relational coefficient. It can be written as follow:

$$\gamma(x_1, x_i) = \frac{1}{n} \sum_{k=1}^{n} \gamma(x_1(k), x_i(k)) \tag{8}$$

2.5. Determining the Grey Relational Order

The grey relational order is the sequential arrangement of the grey relational coefficient. comparing the grey relation grades $\gamma(x_1, x_2), \gamma(x_1, x_3)$, $\gamma(x_1, x_4)$, $\gamma(x_1, x_5)$, $\gamma(x_1, x_6)$, $\gamma(x_1, x_7)$ and $\gamma(x_1, x_8)$,the influencing degree of factors on bearing friction torque can be determined.So, the important influencing factors on the friction torque can be found, meanwhile it is applicable for building GM(1,N,D) model and further analysis.

3. GM(1,N,D) Model for the Friction Torque of a Sliding Bearing

GM(1,N,D) model is multivariable grey model, Its form includes one behavioral variable and N-1 factor variables. In this paper the behavioral variable is the friction torque and factor variables are material hardness, rotational speed, load, temperature, friction coefficient, thermal displacement ,etc. Based on the grey relational analysis of the friction torque of a sliding bearing, some key factors influencing friction torque are selected to build GM(1,N,D) model and predict the friction torque of a sliding bearing. The GM(1,N,D) model is built up as follow.

One-accumulating the grey relational factor space $@_{GRF}$, for instance, $x_i^{(1)}$ is the one-accumulate series of x_i , i=1,2,3….

$$\begin{cases} x_i^{(1)}(1) = x_i(1) \\ x_i^{(1)}(k) = x_i^{(1)}(k-1) + x_i(k) \\ k = 2,3,4...n \end{cases} \tag{9}$$

$z_1^{(1)}$ is the neighbor mean series of $x_1^{(1)}$.

$$z_1^{(1)}(k) = 0.5x_1^{(1)}(k) + 0.5x_1^{(1)}(k-1) \tag{10}$$

Then, the GM(1,N,D) model can be expressed as follow:

$$x_1^{\hat{(0)}}(k) = \sum_{i=2}^{N} b_i x_i^{(1)}(k) - az_1^{(1)}(k) \tag{11}$$

Where, a is development coefficient , b_i is driving coefficient.

Suppose a parameter packet P_N is:

$$P_N = [a, b_2, b_3, ..., b_N]^T \tag{12}$$

Using least square method, P_N can be got.

$$P_N = \begin{bmatrix} a \\ b_1 \\ \vdots \\ b_N \end{bmatrix} = (B^T B)^{-1} B^T y_N \tag{13}$$

Where,

$$B = \begin{bmatrix} -z_1^{(1)}(2) & x_2^{(1)}(2) & \cdots & x_N^{(1)}(2) \\ -z_1^{(1)}(3) & x_2^{(1)}(3) & \cdots & x_N^{(1)}(3) \\ \vdots & \vdots & & \vdots \\ -z_1^{(1)}(n) & x_2^{(1)}(n) & \cdots & x_N^{(1)}(n) \end{bmatrix} \tag{14}$$

$$y_N = \begin{bmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \vdots \\ x_1^{(0)}(n) \end{bmatrix} \tag{15}$$

There are three test methods to test the precision of the gray prediction model, that is residual error examination, posterior difference examination and relative degree examination. In this paper, we take the residual test. Note residuals and the percentage of residuals respectively as $e(k)$ and $e(avg)$. Then,

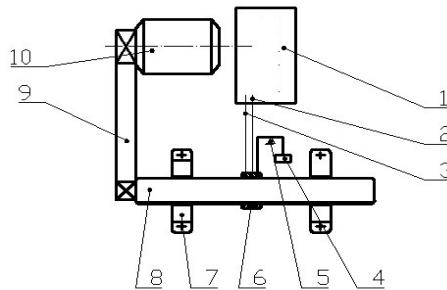$$e(k) = \frac{x_i^{(0)}(k) - x_i^{\hat{(0)}}(k)}{x_i^{(0)}(k)} \times 100\% \tag{16}$$

$$e(avg) = \frac{1}{N} \sum_{k=1}^{N} |e(k)| \tag{17}$$

The precision of the model p° is:

$$p^{\circ} = (100 - e(avg))\% \tag{18}$$

4. Example Analysis

Taking an Al-based alloy sliding bearing as an example, its friction torque is analysed using the grey system theory. The experimental data of a sliding bearing friction torque is obtained on FALEX-5 test-bed producted by FELAX from England. The inner and outer ring diamete of tested bearing is 19mm and 23mm respectively, bearing sleeve height is 19mm. According to the experiment, the testing rotational speed is from 24rpm to 3000rpm, the test maximum temperature is 70℃, the radial load is from 0 to 3600N. The structure diagram of experimental installation is shown in Figure 1.

1.Measurement and control holder   2. Friction coefficient instrument   3. Temperature sensor
4.Load weights   5.Fulcrum   6. Tested bearing   7. Clamping support   8. Spindle   9. Belt
10. Motor

Figure 1. The Structure Diagram of Experimental Installation

In this experiment, the experimental datas of the friction torque, temperature rise, friction coefficient, thermal displacement of bearing inner and outside ring are tested, tested bearings have four different materials, temperature rise and friction coefficient of tested bearing is real-time measurement value, the thermal displacement value of inner diameter and thermal displacement are tested after shutdown. the experimental data is shown in Table 1. As the original data, the experimental value is used to build the grey relational analysis model and GM(1,N,D) model for the friction torque of a sliding bearing.

Table 1. The Data of GM(1,N,) Model

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
/N ·m	/ HV	/rpm	/ N	/℃		/μm	/ um
2.520	180	100	88.96	66	0.297	1.83	11.49
3.447	180	100	88.96	101	0.407	3.16	19.87
2.94	180	100	88.96	103	0.347	3.24	20.35
0.407	700	100	22.24	18	0.192	0.04	0.24
0.554	700	100	44.48	23	0.131	0.19	1.20
0.870	700	100	66.72	31	0.137	0.50	3.11
1.209	700	100	88.96	40	0.143	0.84	5.27
1.503	700	100	111.21	45	0.142	1.03	6.46
1.887	700	100	133.45	54	0.148	1.37	8.62
2.294	700	100	155.69	64	0.155	1.75	11.01
2.644	700	100	177.93	75	0.156	2.17	13.65
2.836	700	100	200.17	84	0.149	2.52	15.80
3.379	700	100	222.4	94	0.159	2.90	18.19
3.899	700	100	244.65	107	0.167	3.39	21.31
1.684	250	200	111.21	100	0.16	3.13	19.63
1.571	250	200	111.21	88	0.148	2.67	16.76
2.102	250	200	111.21	109	0.198	3.47	21.79
1.379	300	400	111.21	142	0.13	4.73	29.69
2.158	300	400	111.21	178	0.204	6.10	38.30
2.339	300	400	111.21	226	0.221	7.93	49.80

4.1. Results and Discussions of Grey Relational Analysis

Based on the grey theory, the grey relational analysis method is used in analyzing the influence degree of the material hardness, rotational speed, load, temperature, friction coefficient and thermal displacement on the friction torque of a sliding bearing.Taking the experimental data shown in Table 1 as the original data, the grey relational coefficient of various factors can be obtained by analyzing the grey relational order and solving the Equation (1)-(8). The calculation result of grey relational grade is represented in Table 2.

Table 2. The Grey Relational Grade of the Friction Torque

Influence factors	Material hardness	Rotational speed	Load	Temperature	Friction coefficient	Hermal displacement of inner diameter	Hermal displacement of outter diameter
Grey relational grade	0.65	0.83	0.85	0.87	0.95	0.71	0.68

As shown in Table 2, compared with other influence factors, friction coefficient, rotational speed, load and temperature rise have more significant influence on the friction torque of a sliding bearing, among them friction coefficient has most significant influence. The reason is that, the increase of rotational speed, load, temperature and the friction coefficient reduce the film thickness of lubricantion oil and hinder the formation of elastohydrodynamic lubrication, leading to the increase in friction torque.

4.2. Results and Discussions of GM(1,N,D) Dodel

Based on the grey relational analysis, friction coefficient, rotational speed, load, temperature have more significant effect upon the friction torque of a sliding bearing. Therefore, the four key factors are selected as factor variables and the friction torque is the behavioral variables. Taking the experimental data shown in Table 1 as the original data, n is 20, and P_N can be obtained by solving the Equation (9)-(18).

$$P_N = \begin{bmatrix} 0.0433 & , -0.2979 & , 0.2877 & , 0.4981 & , 0.4618 \end{bmatrix}^T \tag{19}$$

The prediction model of controlling friction torque of a sliding bearing can be built. The model is:

$$\hat{x}_1^{(0)}(k) = 0.1280 x_2^{(1)}(k) - 0.0152 x_3^{(1)} + 0.2437 x_4^{(1)}(k) - 0.6432 z_1^{(1)}(k) \tag{20}$$

Where $\hat{x}_1^{(0)}$ is the predictive value of bearing friction torque, $x_2^{(1)}$ is one-accumulate series of rotation speed, $x_3^{(1)}$ is one-accumulate series of load, $x_4^{(1)}$ is one-accumulate series of temperature, $x_5^{(1)}$ is one-accumulate series of friction coefficient, $z_1^{(1)}$ is one-accumulate series of bearing friction torque.

The residual test is represented as follow. According to Equation (13)-(18), the average residuals e(avg) is 7.9%, the precision P is 92.1%.

As shown in the Equation (19), the bearing rotational speed has the negative influence on the friction torque, that is, the friction torque of a sliding bearing decreases with the increasing of rotation speed. However, load, temperature and friction coefficient have positive influence on bearing friction torque, namely, the friction torque increase with the increasing of load, temperature and friction coefficient. Moreover, among the positive influence factors, friction coefficient has most significant influence. This conclusion is consistent with the grey relational analysis.

Table 3. The Residual Test of of GM(1,N,D) Model

k	$\hat{x}_1^{(0)}$	$x_1^{(0)}$	e(k)
2	2.4076	2.3677	-1.68%
3	3.7691	3.5336	-6.66%
4	4.0063	3.6951	-8.42%
5	4.2378	3.9148	-8.25%
6	4.6150	4.2601	-8.33%
7	5.1469	4.7399	-8.59%
8	5.7924	5.3363	-8.55%
9	6.5929	6.0852	-8.34%
10	7.5584	6.9955	-8.05%
11	8.6870	8.0448	-7.98%
12	9.9492	9.1704	-8.49%
13	11.3807	10.5112	-8.27%
14	13.0037	12.0583	-7.84%
15	13.8190	12.7265	-8.58%
16	14.5051	13.3498	-8.65%
17	15.4310	14.1839	-8.79%
18	15.9028	14.7309	-7.96%
19	16.7618	15.5874	-7.53%
20	18.0178	16.5157	-9.09%

As shown in Table 3, the maximum residual is 9.09%, the average relative residuals is 7.9%, the accuracy is 92.1%.It verify that GM(1,N,D) model has good accuracy and is applicable for predicting friction torque of a sliding bearing.

4. Conclusion

(1) In this paper, the grey relational analysis method is used in analyzing the friction torque of a sliding bearing and its influence factor. Based on the grey relational analysis,four main influence factor were selected, GM(1,N,D) model for the friction torque of a sliding bearing was built.

(2) Analysis show that compared with other influence factors, friction coefficient, rotational speed, load and temperature rise have more significant influence on the friction torque of a sliding bearing; Furthermore, the bearing rotational speed has the negative influence on the friction torque, load, temperature and friction coefficient have positive influence on the friction torque; Among the positive influence factors, friction coefficient has most significant influence on sliding bearing torque.

(3) Comparing experimental results and the calculated value of the GM(1,N,D) model based on these important influencing factors, the maximum residual of GM(1,N,D) is 9.09%, the average relative residuals is 7.9%, the accuracy is 92.1%. It verify that GM(1,N,D) model has good accuracy and is applicable for predicting the friction torque of a sliding bearing.

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