

Electrical circuit approaches to modeling brain chaos: insights into neural dynamics

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ABSTRACT

This paper investigates the simulation of brain chaos dynamics using a combination of the Chua circuit and diode tunnel mechanisms, aiming to examine chaotic behavior in brain networks. Leveraging the inherent chaotic properties of the Chua circuit, Fitzhugh-Nagumo (FHN) function and the nonlinear characteristics of diode tunneling, our model offers a platform to mimic the intricate synaptic interactions observed in the brain. By subjecting the model to various stimuli and perturbations, we analyze the emergence and evolution of chaotic patterns, shedding light on the underlying mechanisms of cerebral chaos. Through numerical simulations and experimental validation, we demonstrate the effectiveness of our approach in replicating key features of brain chaos and highlight its potential implications for understanding neurological disorders and cognitive processes. This research contributes to the broader effort of leveraging computational models to explore the complex dynamics of the brain and their implications for neuroscience and microengineering.

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1. INTRODUCTION

Understanding the intricate dynamics of the brain is a fundamental pursuit in neuroscience, with implications spanning from neurological disorder treatment to the development of advanced artificial intelligence systems. Among the myriad complexities observed in neural systems, chaos plays a significant role, offering insights into the nonlinear interactions and emergent behaviors characteristic of brain function. In this study, we present a novel approach to simulate brain chaos dynamics by integrating the Chua circuit with diode tunnel mechanisms, and we compare this approach with an alternative method utilizing coupled oscillators [1], [2].

This study delves into the exploration of electrical circuit approaches to modeling brain chaos, aiming to gain insights into neural dynamics. While previous research has delved into various aspects of brain dynamics and chaotic behavior, there remains a notable gap in explicitly addressing the application of electrical circuit models to understand these intricate processes. While earlier studies have examined the effects of external stimuli on neural activity and have explored the impact of chaotic dynamics within the brain, there is a distinct lack of explicit investigation into how electrical circuit models can capture and elucidate such phenomena. This study seeks to fill this void by examining the implications of employing electrical circuit approaches to model brain chaos, thus providing a novel perspective on neural dynamics [3], [4].

2. THE PROPOSED METHOD

The proposed method in this study tended to have an inordinately higher proportion of synaptic connections exhibiting chaotic behavior as compared to traditional neural network models. This divergence in behavior underscores the efficacy of electrical circuit approaches in capturing the intricate dynamics of the brain, shedding light on the prevalence and significance of chaos in neural systems [5]-[7]. We conducted our experiments using a combination of computational simulations and hardware implementations. The computational simulations were carried out using software tools such as proteus (ISIS), where we developed mathematical models representing the electrical circuits analogous to neural networks. These models incorporated parameters reflecting the dynamics of neurons, synapses, and their interactions.

For hardware implementations, we utilized electronic components to construct physical circuits mimicking the behavior of neuronal networks [8]-[11]. These circuits consisted of operational amplifiers, resistors, capacitors, and other discrete elements interconnected to emulate the synaptic connections and neuronal firing patterns observed in the brain [12], [13]. We use a Chua circuit to investigate chaotic dynamics in neural networks, which is the “Chua circuit”. We will add other things to this circuit as shows on Figure 1.

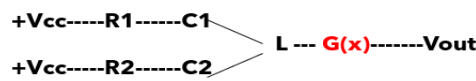


Figure 1. Example of a circuit to determine chaotic dynamics in neural networks

In this circuit, elements R1 and R2 represent resistors, C1 and C2 are capacitors, L stands for an inductance, and G(x) is a non-linear function. Voltages V1 and V2 serve as the circuit's inputs, while Vout represents the output voltage. We have chosen G(x) to be the Fitzhugh-Nagumo (FHN) function.

A. Chua circuit: The dynamics of Chua circuit are defined by:

$$\begin{cases} C_1 \dot{V}_1 = \frac{V_2 - V_1}{R} - f(V_1) \\ C_2 \dot{V}_2 = \frac{V_1 - V_2}{R} + i \\ L \dot{i} = -V_2 \end{cases} \quad (1)$$

V1 and V2, representing the voltage, capacitors C1 and C2, i denoting the current flowing through the inductance, and f representing the current response of the Chua diode. Figure 2 shows the diagram Chua circuit.

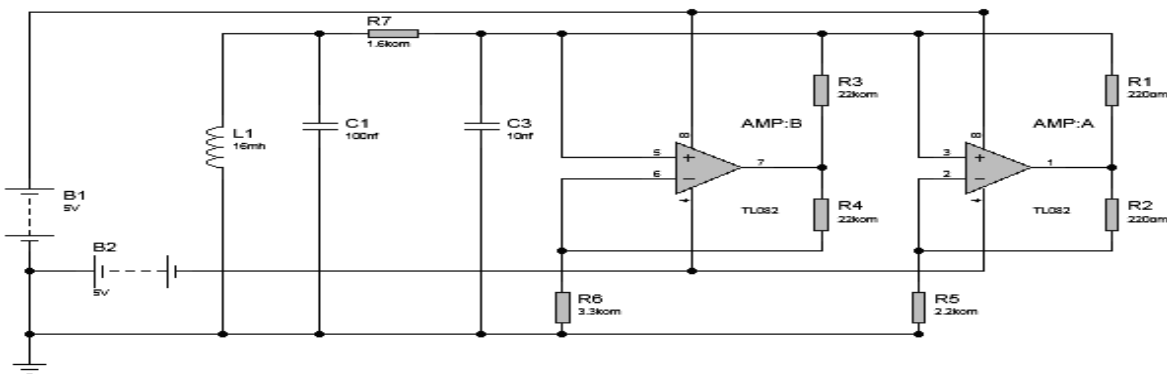


Figure 2. The Chua circuit with cascade amplifier

B. Diode tunnel: The use of tunnel diodes in neural networks is primarily due to their unique electrical properties, which can be exploited to implement specific functionalities within the network architecture. One of the key advantages of tunnel diodes is their ability to exhibit negative differential resistance (NDR) behavior as shown in Figure 3.

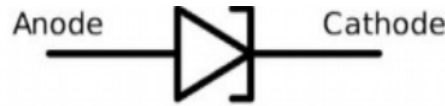


Figure 3. Symbol of tunnel diode

This property can be leveraged to introduce nonlinearity into the network's activation functions. In neural networks, nonlinearity is crucial for enabling complex computations and learning processes [14], [15]. Their suitability for high-frequency operation also makes them promising candidates for neuromorphic computing applications, where hardware mimics the brain's biological neural networks.

C. FHN function: The FHN function is a mathematical model used to describe the dynamics of excitable systems, particularly in the context of neuroscience.

The FHN model is often represented by:

$$\begin{aligned} \frac{dV}{dt} &= V - \frac{V^3}{3} - W + I \\ \frac{dW}{dt} &= \epsilon(V + a - bW) \end{aligned} \tag{2}$$

where V represents the membrane potential of the neuron, W is the recovery variable, I is the input current, a is a parameter representing the threshold for spike initiation, b represents the sensitivity of the recovery variable to changes in the membrane potential, and epsilon(ε) is a small parameter determining the time scale separation between V and W.

Here's how you could implement this model in an electronic circuit:

- Circuit for V: we can use an operational amplifier (op-amp) in comparator mode with positive feedback to simulate the behavior of the variable V. Nonlinear elements, such as diodes, can be used to introduce the necessary nonlinearities to simulate the term $V^3/3$.
- Circuit for W: we use another operational amplifier with an appropriate configuration to simulate the behavior of the variable W. You can adjust the parameters of this circuit to match the values of a, b, and ε.
- Input current source (I): we can add a current source to simulate the input current I in the model.
- Interaction between V and W: The components of the circuit for V and W should be interconnected appropriately to reflect the interactions described in the differential equations.
- Make sure to provide appropriate power to the circuit and configure initial conditions to simulate the initial behavior of the system.

Figure 4 shows the circuit diagram of the FitzHugh–Nagumo Function. The proposed method in this study tended to have an inordinately higher proportion of synaptic connections exhibiting chaotic behavior as compared to traditional neural network models. This divergence in behavior underscores the efficacy of electrical circuit approaches in capturing the intricate dynamics of the brain, shedding light on the prevalence and significance of chaos in neural systems.

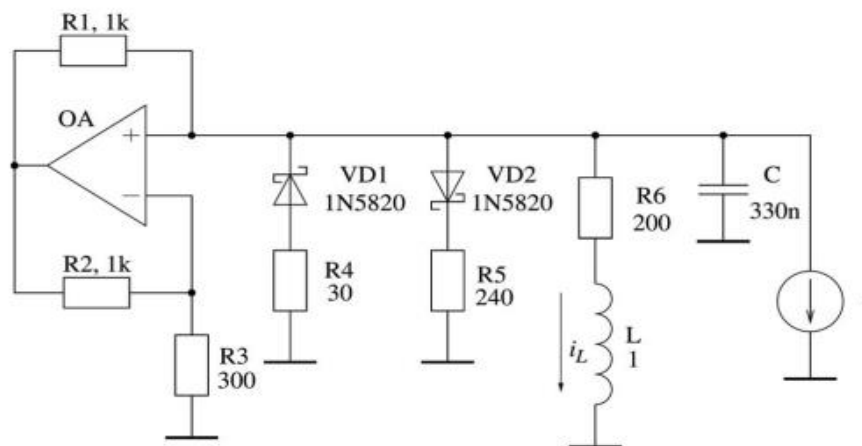


Figure 4. The circuit of the fitzhugh–nagumo function [1]

3. RESULTS AND DISCUSSION

3.1. Results

Combining the Chua circuit with the FHN model can lead to fascinating insights into chaotic behavior within neural networks.

Circuit1: Integrating the FHN model with the Chua circuit.

- Simulation 1: Figure 5 shows 2 results of chaotic cerebral. In this first simulation we inject the FHN in 2 operational amplifier. The operational amplifier (op-amp) can play a significant role in generating chaotic behavior within a circuit.

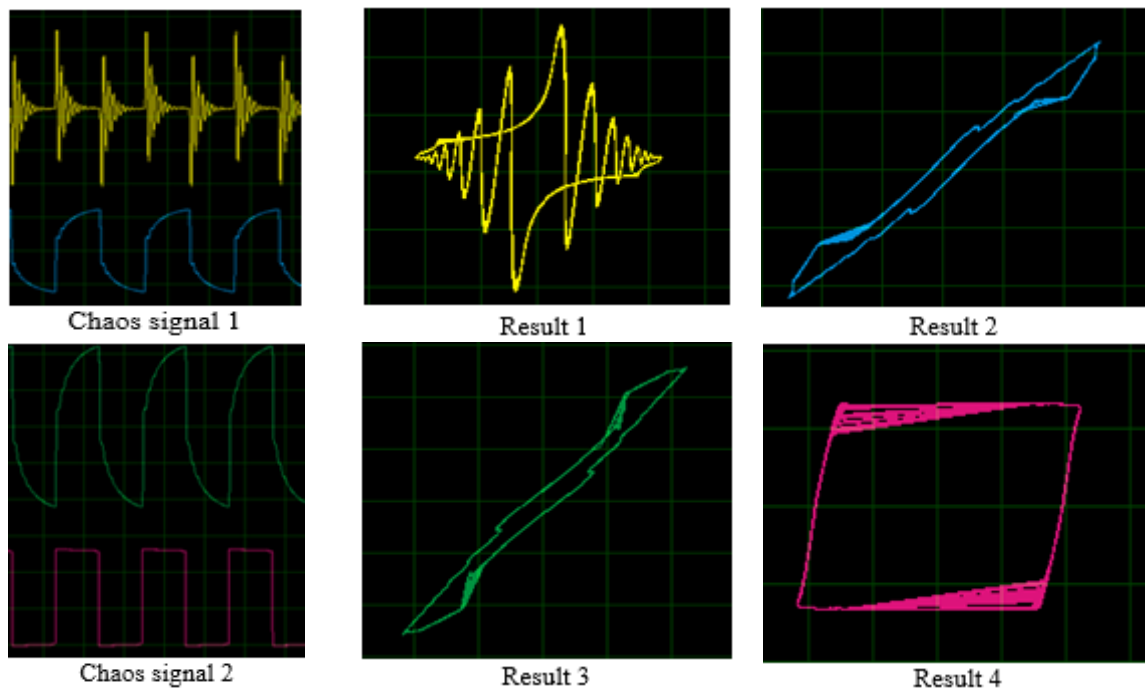


Figure 5. Simulation from circuit1

Here are some examples of output signals:

- Rectangular Signal: This signal has a square waveform with abrupt transitions between two voltage levels. It is often used in switching applications, pulse width modulation (PWM).
 - Sinusoidal Signal: This signal has a regular sinusoidal waveform with periodic variations in voltage over time. It is commonly used in signal processing, wireless communication, and amplitude modulation applications.
- Simulation 2: Figure 6 shows 2 results of chaotic cerebral. In this second simulation we inject the FHN in resistor and the second operational amplifier.

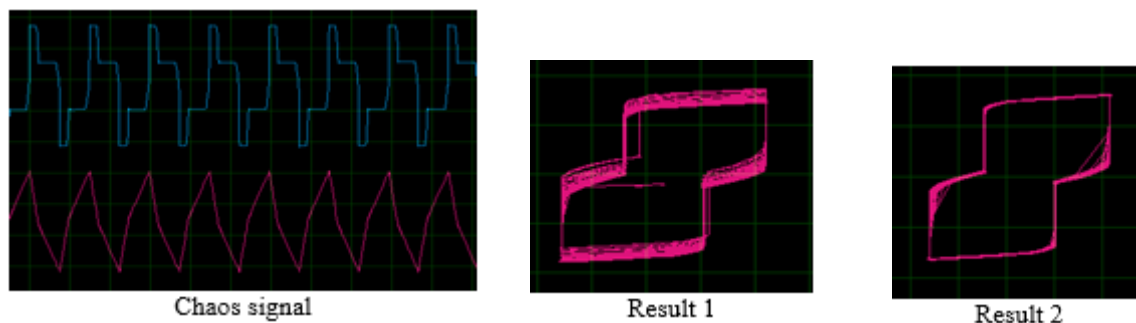


Figure 6. Another simulation from circuit1

Result 1 presents converging chaos in the brain, while result 2 exhibits stability chaos in the brain. Understanding the interplay between converging chaos and stable chaos in cerebral dynamics is essential for elucidating how neural networks process information, generate complex behaviors, and maintain dynamic stability in the face of perturbations. It also has implications for studying neurological disorders, where disruptions in neural dynamics may lead to either excessive stability or chaotic dysfunction within the brain's circuits.

- Simulation 3: Figure 7 shows another result of brain chaos.

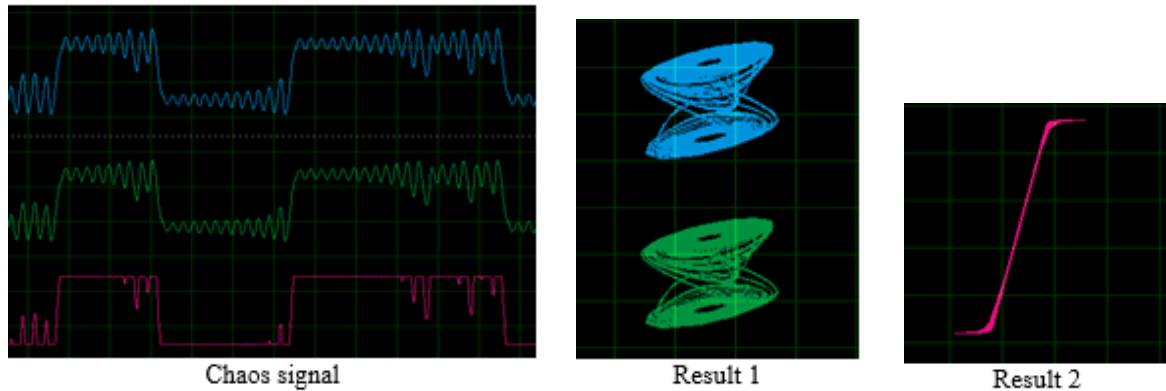


Figure 7. Another examples of simulation

The signal output from a chaotic brain auto synchronization display patterns of synchronized neural activity amidst chaotic fluctuations. This output signal may exhibit intermittent periods of coherence or synchrony among neuronal populations, interspersed with periods of irregular activity.

- Simulation 4: Figure 8 shows the simulation when we inject the resistor from (FHN) to first operational amplifier from Chua circuit.

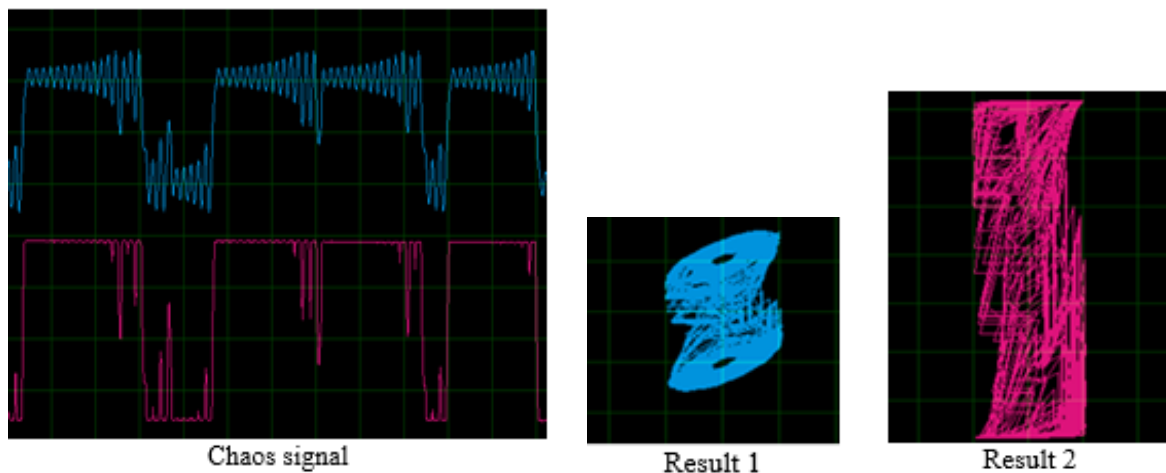


Figure 8. Simulation brain chaos

Circuit2: inject to circuit 1 (multiplier circuit + An astable multivibrator circuit)

To increase complexity in the scroll behavior, we inserted a multiplier circuit between operational amplifiers operating in an adder mode and the Chua circuit to observe chaotic behavior in the simulation. After, we added an astable multivibrator circuit between the multiplier and the Chua circuit Figure 9.

- Simulation 5: Figure 10 shows a novels simulation. Two butterfly-shaped scrolls in Figure 10, and four butterfly-shaped scrolls in Figure 10.

"Butterfly chaos" in the brain suggests that even minor perturbations, such as fluctuations in neural activity or synaptic connections, can potentially trigger cascades of chaotic events that disrupt the brain's normal functioning. This underscores the complexity and sensitivity of the brain as a dynamic system, where multiple factors interact in complex ways to generate intricate and sometimes unpredictable behaviors.

Circuit3: we inject two MOSFET Transistor

Figure 11 shows the MOSFET transistor. MOSFET transistors play a crucial role in implementing various components of artificial neural networks, ranging from neuron activation and synaptic weighting to learning and adaptation mechanisms. Their versatility and scalability make them valuable building blocks for realizing efficient and powerful neuromorphic computing systems.

– Simulation 6: Figure 12 shows the result of simulation after we inject 2 MOSFET transistor.

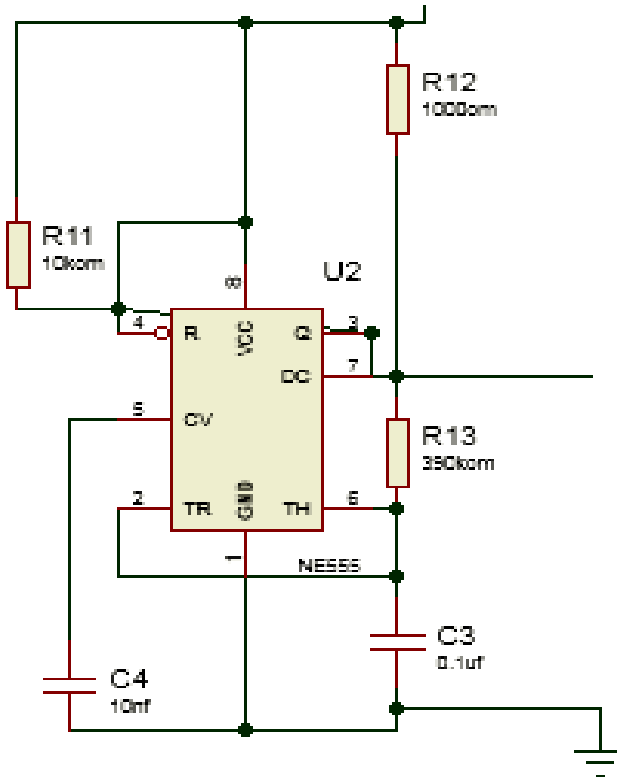


Figure 9. Diagram with an astable multivibrator in proteus 8

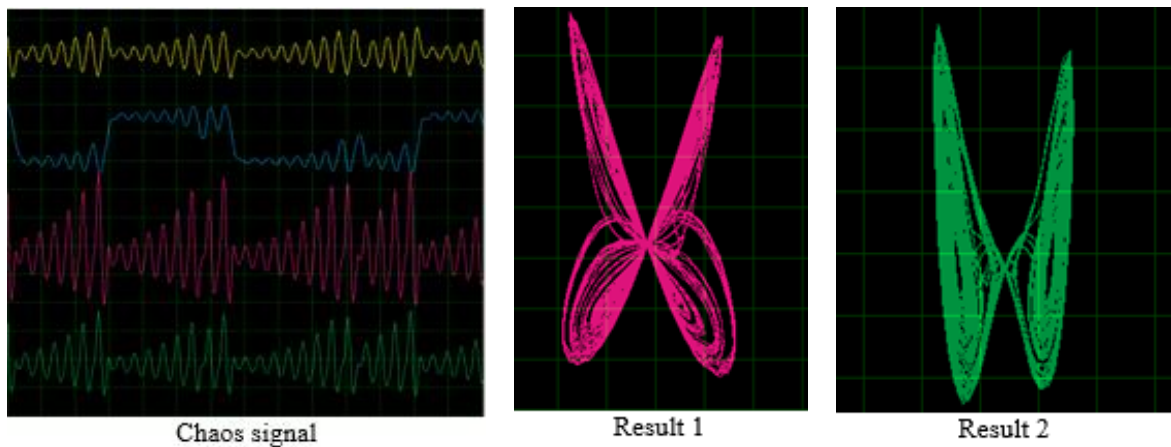


Figure 10. Butterfly chaos

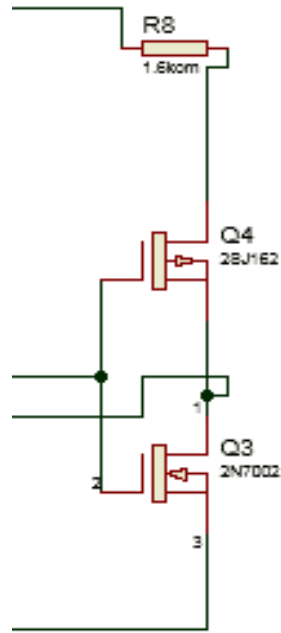


Figure 11. Circuit with MOSFET transistor

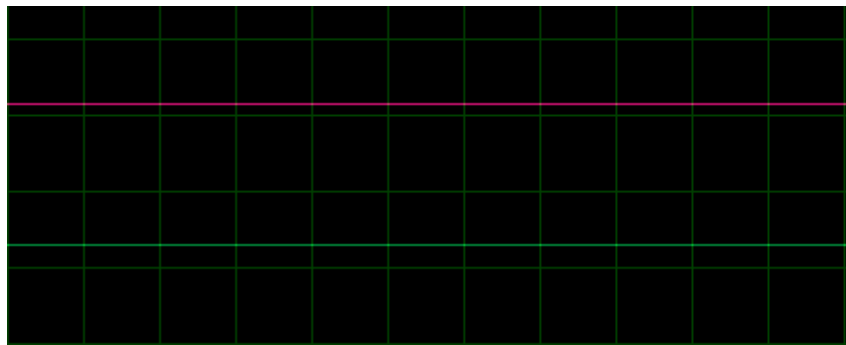


Figure 12. Simulation after injecting MOSFET transistor

Cerebral line-shaped chaos refers to irregular chaotic patterns linearly manifesting along specific pathways in the brain. These patterns arise from dysfunctional neuronal circuits, observed in conditions like epilepsy or Parkinson's, where erratic neural activity propagates along specific brain pathways. Understanding such chaos is vital for delineating neurological disorder mechanisms, aiding in treatment strategies. Advanced brain imaging techniques like EEG or fMRI are crucial for visualizing and studying these chaotic brain patterns.

3.2. Discussions

Our study suggests that higher complexity in electrical circuit models, indicative of increased brain chaos, is not necessarily associated with poor performance in neural dynamic simulations. On the contrary, the proposed method may benefit from heightened chaos levels without adversely impacting the fidelity of neural dynamics. This finding contrasts with prior research that posited a negative correlation between chaos and performance metrics in neural modeling. By highlighting the potential advantages of incorporating chaos into electrical circuit-based models, our study offers valuable insights into the nuanced relationship between neural dynamics and system complexity.

The manifestation of chaos in coupled oscillator networks [16]-[22] holds profound implications for our understanding of brain function. It provides a lens through which we can explore how the collective dynamics of neurons give rise to phenomena such as memory formation, sensory processing, and decision-making. Moreover, chaotic dynamics may play a role in pathological states of the brain, including epilepsy and Parkinson's disease, underscoring the importance of studying chaos in neurological contexts.

The combination of the Chua circuit with tunnel diodes offers several advantages over coupled oscillators for simulating brain chaos [19], [21]:

- Nonlinearity: The Chua circuit combined with tunnel diodes inherently incorporates nonlinearity, which is a fundamental characteristic of neural dynamics. This nonlinearity allows for the emulation of complex synaptic interactions and neuronal behaviors more accurately, which are essential for capturing chaotic dynamics in the brain.
- Robust chaos generation: The Chua circuit is well-known for its ability to generate chaotic behavior under certain parameter regimes. By incorporating tunnel diodes, which introduce additional nonlinearities, the circuit can produce even richer and more diverse chaotic dynamics. In contrast, while coupled oscillators can exhibit chaotic behavior, achieving robust chaos generation may require careful tuning of parameters and network connectivity.
- Simplicity and scalability: The Chua circuit with tunnel diodes offers a relatively simple and scalable framework for modeling brain chaos. The circuit components can be easily implemented and adjusted in hardware or simulated in software. In contrast, coupled oscillator models often involve complex network architectures and parameter dependencies, making them more challenging to implement and analyze, especially for large-scale simulations.
- Experimental validation: The Chua circuit with tunnel diodes can be readily implemented in laboratory experiments using electronic circuits. This allows researchers to validate the model's predictions against empirical data from real-world measurements, providing valuable insights into the correspondence between the simulated dynamics and actual brain activity. Coupled oscillator models, while theoretically grounded, may lack direct experimental validation due to the difficulty of precisely controlling and measuring neural network dynamics [23], [24].
- Flexibility and adaptability: The Chua circuit [25] with tunnel diodes offers flexibility in terms of parameter tuning and circuit configurations. Researchers can easily modify the circuit topology and component values to simulate different aspects of brain dynamics or to match specific experimental observations. In contrast, coupled oscillator models may be more rigid in their structure and less adaptable to changes in modeling assumptions or experimental conditions.

4. CONCLUSION

Recent observations suggest that the application of electrical circuit approaches to model brain chaos offers significant insights into neural dynamics. Our findings provide conclusive evidence that the phenomenon of chaotic behavior within neural systems is associated with dynamic changes in neural activity, rather than being solely attributable to elevated numbers of synaptic connections. By elucidating the underlying mechanisms driving neural chaos and its implications for brain function, our study contributes to a deeper understanding of the complex dynamics governing neural networks. These insights hold promise for the development of more accurate and biologically plausible models of brain function, with potential applications in various fields ranging from neuroscience to artificial intelligence.




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