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Reduction on Propositional Logic Set Based on Correlation Analysis

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Abstract

A knowledge base is redundant if it contains parts that can be inferred from the rest of it. In this paper, with no district bound, we study the reduction theory and algorithm on proposition logic set. The propositions of a given proposition set fall into three categories: necessary proposition, useful proposition, and useless proposition. A reduction of a given set is composed of all the necessary propositions and some useful propositions. At the beginning we introduce induced formal context of proposition set, and then propose the method of reduction on proposition set based on correlation analysis.

Keywords*: propositional logic, redundancy, formal concept analysis, association rules*

1. Introduction

More and more available knowledge acquisition ways make knowledge databases become more and more complex, and then a development bottleneck occurs in knowledge database reduction. Unlike truth maintenance system [1] which is due to the ability of these systems to restore consistency, knowledge database reduction is the transformation of information into a corrected, ordered, and simplified form. A knowledge base is redundant if it contains some redundant parts, that is, it is equivalent to one of its proper subsets [2]. Generalized, the term redundancy is defined as by the generation of the same fact more than once during the same problem resolution [3].

The problem of redundancy may be either highly importance of the knowledge it express, or a mistake that has been made in the knowledge base. In particular, database update increases its size exponentially [4], and redundancy makes this problem worse. In any expert system, avoiding redundancy is also of interest in real-time systems for which the inference engine is time consuming [5].

Algorithms for checking redundancy of knowledge bases have been developed for the case of production rules [6]. Complexity analysis on Horn knowledge bases has been given in two papers [7] and [8]; Later, with no other strict bounds, deciding whether formula is minimal is proved coNP-hard [9].

Redundancy elimination is relevant to formula minimization, and can be considered as a weak form of it. Redundancy elimination has two advantages, firstly it seems somehow easier to remove redundant clauses, and secondly it doesn't change the syntactic form of a knowledge base [2].

Whether a system can give an appropriate explanation of inferential as well as reduction process is an important factor which customers will take into account when chose a system, but this problem has gained little concern.

Reduction on fuzzy inference rules is also studied, and neural network [10], implication-based models and conjunction-based models [5] are adopted in these researches, but all these researches give little hint on the problem discussed in this paper.

Formal Concept Analysis (FCA) as a categorization method aims at grouping objects described by common attributes. In this framework, a category is precisely defined as a maximal set of objects sharing a maximal set of attributes. Such groupings are then gathered in a hierarchical, lattice-based structure which straightforwardly exhibits various relationships between categories and their sub- and super-categories. Because of its conceptual structure to

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facilitate the development and discussion, in a sense, the concept lattice has become a means for external recognition decades [11].

FCA has been mentioned in proposition set reduction [12], and formal context is adopeted to depict an information system, but in [12] the technique of correltaion analysis is not used. And for this reason, without of correltaion analysis it is imposible to point out why a formula is redundant.

In this paper, we will use formal context to depict proposition set, and propose a reduction algorithm based on the technique of correlation analysis.

2. Preliminaries

In this section, it is explained some relative concepts of this paper, including two-valued proposition logic and formal concept analyais.

2.1. Two-valued Proposition Logic

The formulas of propositional calculus, also called propositional formulas [13], are expressions such as $(A \wedge (B \vee C))$. Their definition begins with the arbitrary choice of a set Σ of propositional variables. The alphabet consists of the letters in Σ along with the symbols for the propositional connectives and parentheses, all of which are assumed to not be in Σ. The formulas will be certain expressions (that is, strings of symbols) over this alphabet.

Definition 1. Formulas are inductively defined as follows:

(1) Each propositional variable is, on its own, a formula.

(2) If φ is a formula, then $\neg\ \varphi$ is a formula.

(3) If φ and ψ are formulas, and $\cdot$ is any binary connective, then $\varphi \cdot \psi$ is a formula. Here $\cdot$ could be (but is not limited to) the usual operators $\vee$, $\wedge$, $\rightarrow$, or $\leftrightarrow$.

Definition 2. In propositional logic, an atomic formula is a formula that contains no logical connectives.

Definition 3. A proof of a formal system is a finite formula series $A_1, A_2, \cdots, A_n$, and every $A_i (i \le n)$ is either a axiom or a result induced by using MP based on A_j and $A_k\left(j,k<i\right)$. Then the series is called a proof of A_n, and A_n is called a theorem and denoted as $\mapsto A_n$.

Notation: Let Γ be a set of formulas and $F(S)$ be the universe of all formulas.

Definition 4. Deduction of Γ is a finite formula series $A_1, A_2, \cdots, A_n$, and every $A_i (i \le n)$ is either a axiom or a result induced by using MP based on A_j and $A_k\left(j,k<i\right)$, then A_n is call Γ deduction, and $\Gamma \mapsto A_n$. A proposition $A \in \Gamma$ is redundant if an only if $(\Gamma - A) \mapsto A$.

Notation: $D(\Gamma) = \{A \in F(S) \mid \Gamma \mapsto A\}$.

Definition 5. Suppose a mapping v: $\Gamma \rightarrow \{0,1\}$, if $v(\neg A) = \neg v(A)$ and $v(A \rightarrow B) = v(A) \rightarrow v(B)$ holds, then v is a homomorphism of type $(\neg, \rightarrow)$ and called an assignment of Γ. Moreover, $v(A)$ is called an assignment of A, and all of the assignments of Γ forms an universe and denoted as Ω.

Definition 6. Suppose $A \in \Gamma$, if for any assignment $v \in \Omega$ and $v(A) = 1$ holds, then A is called a tautology. Otherwise, if any assignment $v \in \Omega$ and $v(A) = 0$ holds, then A is called a contradiction.

Lemma 1. Formula A is an axiom of a formal system, if and only if A is a tautology.

Lemma 2. Suppose Γ is a finite proposition set of $F(S)$, A is a proposition of Γ, $\Gamma \mapsto A$ if and only if A is a tautology respect to Γ.

2.2. Formal Concept Analysis

Before proceeding, we briefly recall the FCA terminology [14]. Given a formal context $K = (G, M, I)$, where G is called a set of objects, M is called a set of attributes, and the binary relation $I \subseteq G \times M$ specifies the relations between objects and attributes, two derivation functions f and g are defined for $A \subseteq G$ and $B \subseteq M$,

$f(A) = \{m \in M \mid \forall g \in A : gIm\}$;
$g(B) = \{g \in G \mid \forall m \in B : gIm\}$.

In words, $f(A)$ is the set of attributes common to all objects of A and $g(B)$ is the set of objects sharing all attributes of B .

Both $g(f(A))$ and $f(g(B))$ are extensive, idempotent and monotonous and therefore said to be closed.

A formal concept of the context $K = (G, M, I)$ is a pair (A, B), where $A \subseteq G$, $B \subseteq M$, $f(A) = B$ and $g(B) = A$. The set A is called the extent and B is called the intent of the concept (A, B) .

A concept (A, B) is a sub-concept of (C, D) if $A \subseteq C$ (equivalently, $D \subseteq B$), then (C, D) is a super-concept of (A, B) . We write $(A, B) \leq (C, D)$ and define the relation $\geq$, $\leq$, $>$ and $<$ as usual. If $(A, B) < (C, D)$ and there is no (E, F) such that $(A, B) < (E, F) < (C, D)$, then (A, B) is a lower neighbor of (C, D) and (C, D) is a upper neighbor of (A, B) ; notation, $(A, B) \prec (C, D)$ and $(C, D) \succ (A, B)$.

The set of all concepts ordered by $\leq$ forms a lattice, which is denoted by $L(K)$ and is called the concept lattice of the context K . The relation $\prec$ defines the edges in the covering graph of $L(K)$.

Definition 7. Let $C = (A, B)$ be a given concept, if $R \subset B$ satisfies $g(R) = g(B) = A$ and for any $T \subset R$ *, we have* $g(T) \supset g(R)$, then R is said to be a intent reduction of C [15, 16].

Remark. Suppose the intent reduction of concept $C = (A, B)$ is R , then we can get an association rule [17] $R \to (B - R)$. The meaning of the rule is: if R can represent concept C , then the other attributes of C can be derived from R .

Example 1. Give a formal context K , and its corresponding concept lattice is shown in Figure 1.

Table1. Formal Context

	a	*b*	*c*	*d*	*e*	*f*	*g*
1	×		×				
2	×	×	×		×		×
3	×	×	×		×		×
4		×			×		
5	×	×	×		×		×
6		×	×	×	×	×	
7			×				×

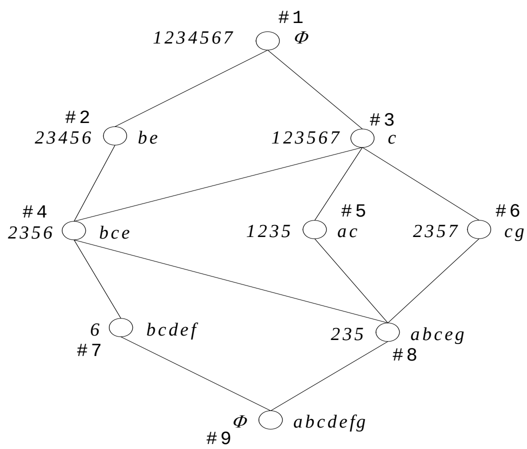

Figure 1. $L(K)$

For example, the intent reduction of concept $(\{2,3,5,6\},\{b,c,e\})$ is $\{b,c\}$ and $\{c,e\}$, so by Definition 7 and its remark we get association rules $bc \to e$ and $ce \to b$.

3. Proposition Logic Set Reduction Based on FCA

A knowledge base is redundant if it contains parts that can be inferred from the rest of it. We assume that the set Γ discussed in this paper is a set without tautology, for tautologies can be easily checked and removed, and these don't change the complexity of the problems considered here.

Not all the element of a given proposition set plays the same role, some are necessary, while some are unwanted. The reduction of a proposition set must ensure the same reasoning ability after deleting unwanted formula.

Definition 8. Suppose $\Gamma_0 \subset \Gamma$ and $D(\Gamma_0) = D(\Gamma)$, for $\forall A \in \Gamma_0$, if $D(\Gamma_0 - A) \neq D(\Gamma)$, then Γ_0 is called a reduction of Γ and denoted as Γ_{red}.

Definition 9. Given a proposition set Γ, and its reduction set is $\{(\Gamma_{red})_t \mid t \in T\}$ (T is the index set), then the proposition of Γ falls into three categories:

a) Necessary proposition A: $A \in \bigcap_{t\in T}(\Gamma_{red})_t$;

b) Useful proposition B: $B \in (\bigcup_{t\in T}(\Gamma_{red})_t - \bigcap_{t\in T}(\Gamma_{red})_t)$;

c) Useless proposition C: $C \in (\Gamma - \bigcup_{t\in T}(\Gamma_{red})_t)$.

Notation: all the necessary propositions form necessary proposition set denoted by Γ_A, and all the useful propositions form useful proposition set denoted by Γ_B, and all the useless propositions form useless proposition set denoted by Γ_C.

Example 1. Given a proposition set $\Gamma = \{a, b, a \to b\}$, it is easy to verify that $\Gamma_{red} = \{a, a \to b\}$ or $\{a, b\}$, and therefore by Definition 8 we have $\Gamma_A = \{a\}$, $\Gamma_B = \{b, a \to b\}$, and $\Gamma_C = \phi$ (ϕ denotes an empty set).

Let $\Gamma = \{A_1, A_2, \cdots, A_n\}$ be an two-valued proposition set and assume it has m atomic formulas, namely, $p_1, p_2, \cdots, p_m$. Let Ω_Γ be the universe of all assignment of Γ, then Ω_Γ can be depicted by a series of m-dimensional vectors and Ω_Γ has 2^m such vectors.

Define a binary relation I on $\Omega_\Gamma \times \Gamma$, for any $v \in \Omega_\Gamma$ and $A_i \in \Gamma$, we have vIA_i if and only if $v(A_i) = 1$. Then $(\Omega_\Gamma, \Gamma, I)$ forms a formal context, and is called Γ-induced formal context. In this formal context, objects are the elements of Ω_Γ, attributes are the elements of Γ.

Definition 10. Suppose $\Gamma \subset F(S)$, and Γ-induced formal context is $K_\Gamma = (\Omega_\Gamma, \Gamma, I)$, and define two derivation function f and g on $P \subseteq \Omega_\Gamma$ and $Q \subseteq \Gamma$ as follows:

$$f(P) := \{A_i \in \Gamma \mid \forall v \in P, v(A_i) = 1\};$$
$$g(Q) := \{v \in \Omega_\Gamma \mid \forall A_i \in Q, v(A_i) = 1\}.$$

It is easy to prove that functions f and g form a Galois connection.

By computing intent reduction of every concept of formal context $(\Omega_\Gamma, \Gamma, I)$, we can get all the association rules which tell the correlations between propositions contained in Γ, and denote all these association rules by set R_Γ.

Theorem 1. In formal context $(\Omega_\Gamma, \Gamma, I)$, $Q_1, Q_2 \subseteq \Gamma$ and $Q_2 \subset Q_1$, $D(Q_2) = D(Q_1)$ if and only if Q_2 is an intent reduction of concept $(g(Q_1), Q_1)$.

Proof. $D(Q_2) = D(Q_1) \Leftrightarrow$ for any $A \in F(S)$, $Q_1 \mapsto A, Q_2 \mapsto A \Leftrightarrow Q_2 \mapsto Q_1 \Leftrightarrow Q_2 \mapsto Q_1 - Q_2$. According to Definition 7 and its remark, we can get that Q_2 is an intent reduction of concept $(g(Q_1), Q_1)$.

Corollary 1.1. In formal context $(\Omega_\Gamma, \Gamma, I)$, $Q_1, Q_2 \subseteq \Gamma$, $D(Q_2) = D(Q_1)$ if and only if $g(Q_1) = g(Q_2)$.

Theorem 2. In formal context $(\Omega_\Gamma, \Gamma, I)$, $A \in \Gamma$, if A doesn't lie at the consequent of any rules of R_Γ, then A is a necessary proposition of Γ.

Proof. Since A doesn't lie at the consequent of any rules of R_Γ, it implies for any $Q \in 2^{\Gamma - A}$ ($2^{\Gamma - A}$ denote the power set of $\Gamma - A$), $Q \to A$ doesn't hold, and therefore $D(\Gamma - A) \neq D(\Gamma)$, and this means A must be contained in Γ_{red}. By definition 9, we know A is a necessary proposition of Γ.

Corollary 2.1. In formal context $(\Omega_\Gamma, \Gamma, I)$, $A \in \Gamma$, A is a necessary proposition if $v(A) = 0$ and $v(B) = 1$ for any $B \in \Gamma - A$.

Corollary 2.2. If A is a useful proposition of Γ, if and only if it lies at the antecedent of some rules of R_Γ.

Theorem 3 In formal context $(\Omega_\Gamma, \Gamma, I)$, $A \in \Gamma$, Γ_A is the set of all necessary proposition of Γ, A is a useless proposition if and only if there is a subset $Q \subseteq \Gamma_A$ that makes $Q \mapsto A$ holds.

Proof. A is a useless proposition $\Leftrightarrow A \notin \Gamma_{red} \Leftrightarrow \exists Q \subseteq \Gamma_{red} \subseteq \Gamma_A$, that makes $Q \mapsto A$ holds.

Corollary 3.1. In formal context $(\Omega_\Gamma, \Gamma, I)$, $A \in \Gamma$, Γ_A is the set of all necessary proposition of Γ, if there exists a subset $Q \subseteq \Gamma_A$, and $v(Q) = 1$ implies $v(A) = 1$, then A is a useless proposition.

Theorem 4. In formal context $(\Omega_\Gamma, \Gamma, I)$, Q is a reduction of Γ, if and only if for any $P \in 2^Q$ (2^Q denote the power set of Q), there must exist a concept whose intent is equal to P.

Proof. (proof by contradiction) Suppose in formal context $(\Omega_\Gamma, \Gamma, I)$, there doesn't exist a concept whose intent is equal to P.

According to the property of formal context, we have $P \subseteq f(g(P))$. As there doesn't exist a concept whose intent is equal to P, so in $P \subseteq f(g(P))$ equivalent relation doesn't hold, and thus $P \subset f(g(P))$. Moreover $P \mapsto f(g(P)) - P$ can be derived from $P \subset f(g(P))$, and this implies that there must be a proposition $q \in f(g(P)) - P$ which makes $D(Q) = D(Q - p)$, and this is contradict to the claim Q is a reduction of Γ.

A reduction of a proposition set is composed of necessary propositions and useful propositions. In order to find a reduction of a given proposition set, the first step is to find all the necessary propositions, and the next step is to find minimal number of useful propositions from which all the other propositions can be deduced.

The following algorithm is to find a reduction of a proposition set.

Algorithm 1. Find a reduction of a given proposition set:

Input: formal context $(\Omega_\Gamma, \Gamma, I)$.

Output: Γ_{red} (a reduction of Γ)

Step1: Delete redundant rows and columns (whose elements are entirely composed of 1 or 0);

Step2: Find all the necessary propositions based on Corollary 2.1;

Step3: Find all the useless propositions based on Corollary 3.1;

Step4: Delete the column labeled with useless propositions form the formal context;

Step5: Compute useful proposition set;

*Step6:*Construct concept lattice of the clarified formal context;

*Step7:*Mining association rules by using intent reduction of every concept;

*Step8:*Compute reduction of Γ:

8-1: initialize $\Gamma_{red} = \Gamma_A$;

8-2: while Γ_B is not empty do the following

(1) randomly select a proposition form Γ_B and move it to Γ_{red} ;

(2) if there exist $Q \in 2^{\Gamma_{red}}$ and association rule $Q \to A_i$, then delete A_i from Γ_B .

8-3: return result Γ_{red} .

The rest of this section is a simple example, which is presented to vividly illustrate our methods once more.

Example 2. $\Gamma = \{A_1, A_2, A_3, A_4, A_5, A_6\}$, $A_1 = p_1$, $A_2 = p_2 \to p_3$, $A_3 = (p_1 \to p_2) \to p_3$, $A_4 = \neg p_2$, $A_5 = (\neg p_2 \to p_3) \wedge (p_3 \to \neg p_2)$, $A_6 = p_2 \to p_1 \wedge \neg p_3$. Formal context $(\Omega_\Gamma, \Gamma, I)$ deduced by Γ is shown in Table 1.

Table 1. Formal Context $(\Omega_\Gamma, \Gamma, I)$

	A_1	A_2	A_3	A_4	A_5	A_6
v_1 (0,0,0)		*		*		*
v_2 (1,0,0)	*	*	*	*		*
v_3 (0,1,0)					*	
v_4 (0,0,1)		*	*	*	*	*
v_5 (1,1,0)	*				*	*
v_6 (1,0,1)	*	*	*	*	*	*
v_7 (0,1,1)		*	*			
v_8 (1,1,1)	*	*	*			

Step1: Delete the 6th row form the formal context.
Step2: Find all the necessary propositions based on Corollary 2.1:
$\because v_4(A_1) = 0, \forall i \in \{2,3,4,5,6\}, v_4(A_i) = 0 \therefore$ A_1 is a necessary proposition.
$\because v_2(A_5) = 0, \forall i \in \{1,2,3,4,6\}, v_2(A_i) = 0 \therefore$ A_5 is a necessary proposition.
So $\Gamma_A = \{A_1, A_5\}$.
Step3: Find all the useless propositions based on Corollary 3.1:
$\because v_5(A_1) = 1, v_5(A_5) = 1$ implies $v_5(A_6) = 1$ $\therefore$ A_6 is an unnecessary proposition.
So $\Gamma_C = \{A_6\}$.
Step4: Delete the column labeled with A_6 form the formal context.
Step5: Compute useful proposition set:
$\Gamma_B = \Gamma - \Gamma_A - \Gamma_C = \{A_2, A_3, A_4\}$.
Step6: Construct concept lattice of the clarified formal context (Table 2):

There are 12 concepts in the concept lattice, i.e. *#1* $(\{v_1,v_2,v_3,v_4,v_5,v_7,v_8\},\{\})$, *#2* $(\{v_1,v_2,v_4,v_7,v_8\},\{A_2\})$, *#3* $(\{v_2,v_4,v_7,v_8\},\{A_2,A_3\})$, *#4* $(\{v_3,v_4,v_5\},\{A_5\})$, *#5* $(\{v_2,v_5,v_8\},\{A_1\})$, *#6* $(\{v_1,v_2,v_4\},\{A_2,A_4\})$, *#7* $(\{v_2,v_4\},\{A_2,A_3,A_4\})$, *#8* $(\{v_2,v_8\},\{A_1,A_2,A_3\})$, *#9* $(\{v_2\},\{A_1,A_2,A_3,A_4\})$, *#10* $(\{v_4\},\{A_2,A_3,A_4,A_5\})$, *#11* $(\{v_5\},\{A_1,A_5\})$, and *#12* $(\{\},\{A_1,A_2,A_3,A_4,A_5\})$.

Table 2. Clarified Formal Context

	A_1	A_2	A_3	A_4	A_5
v_1 (0,0,0)		*		*	
v_2 (1,0,0)	*	*	*	*	
v_3 (0,1,0)					*
v_4 (0,0,1)		*	*	*	*
v_5 (1,1,0)	*				*
v_7 (0,1,1)		*	*		
v_8 (1,1,1)	*	*	*		

Step7: Mining association rules by using intent reduction:

There are 5 concepts can be used to derive association rules, while form the other 7 concepts no association rules can be derived.

Intent reduction of *#3* $(\{v_2,v_4,v_7,v_8\},\{A_2,A_3\})$ is A_3 and we get association rule $r_1: A_3 \rightarrow A_2$;

Intent reduction of *#6* $(\{v_1,v_2,v_4\},\{A_2,A_4\})$ is A_4 and we get association rule $r_2: A_4 \rightarrow A_2$;

Intent reduction of *#7* $(\{v_2,v_4\},\{A_2,A_3,A_4\})$ is *23* and we get association rule $r_3: A_2A_3 \rightarrow A_4$;

Intent reduction of *#8* $(\{v_2,v_8\},\{A_1,A_2,A_3\})$ is A_1A_2 or *13* and we get association rules $r_4: A_1A_2 \rightarrow A_3$ and $r_5: A_1A_3 \rightarrow A_2$;

Intent reduction of *#9* $(\{v_2\},\{A_1,A_2,A_3,A_4\})$ is *14* and we get association rule $r_6: A_1A_4 \rightarrow A_2A_3$.

So the rule set $RS=\{\ r_1,\ r_2,\ r_3,\ r_4,\ r_5,r_6\}$.

Step8: Compute reduction of Γ

We first initialize $\Gamma_{red}=\Gamma_A$, and randomly select a proposition form Γ_B and move it to Γ_{red} . For example, we select A_2 , and remove it from Γ_B to Γ_{red} . Then by using $r_4: A_1A_2 \rightarrow A_3$, we delete A_3 from Γ_B ; by using $r_3: A_2A_3 \rightarrow A_4$, we delete A_4 from Γ_B . Then Γ_B is empty and we get the result $\Gamma_{red}=\{A_1,A_2,A_5\}$.

Change the selection order, we can get the other two reductions: $\{A_1,A_3,A_5\}$ and $\{A_1,A_4,A_5\}$.

4. Conclusion

In this paper, we present a new approach, which is based on FCA, for the efficient reduction of two-valued proposition set. A basis is a set of non-redundant propositions from which all propositions can be derived, thus it captures all useful information. The approach is realized by using correlation analysis, and the main idea is to find the relationship between propositions, and discover the feature of different kinds of proposition. In our research, the theory of formal concept analysis is adopted to mining association rules which is used to ascertain redundant propositions.

Our approach has a twofold advantage: on one hand, it is correlation analysis that makes it different from other method, as it can explain the reduction process by pointing out the association rules which are used in deleting non-necessary propositions form the proposition set. When applying this method in computer-aided system, the users will have confidence when using this system just because they know the running mechanics of the system. On the other hand, it has reduced complexity, because of deleting redundant rows and columns form the context before computing a reduction.

But there are still problems left in this paper. If the established association rule set is extremely large, the organization of set will become a problem, which will be left for further research. Moreover, how to make our method to be more sophisticated to accommodate n-valued proposition is another important problem for further research.

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