

Mathematical Programming Model on Joint and Recovery of Paper Scrap

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Abstract

The joint and recovery of paper scrap was an optimal matching issue. The automatic restoration technology for the broken document was designed according to different cutting ways of the paper shredder. As the fragment data was an one-side print file, a nonlinear programming model with no constraint conditions was established for the straight-cutting broken paper scrap from a given print file of the same page, and the nonlinear programming model with restraint programming model was established for the broken scrap with straight cut and cross cut. For the paper scrap data from a one-page print file in English printed on both sides, the joint was accomplished by the ant colony algorithm.

Keywords: joint, the least square method, ant colony algorithm

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1 . Introduction

Currently, a lot of researches on the automatic restoration technology for the broken document have been done in most of the developed countries internationally. However, in China, the research on this area started too late, and few research accomplishments were reported. The joint of the broken document was widely applied in the judicial physical evidence recovery, historical document restoration and military information acquirement. Traditionally, the joint and recovery work needed to be done manually, which would lead to high accuracy but low efficiency. Especially as the fragment amount was huge, the manual joint would cost a lot of human and material resources.

The automatic joint issue belonged to the computer vision and pattern recognition area, which was accomplished by the computer processing to obtain the information of the paper scrap, like shape and color. Then the paper scrap was restored automatically or semi-automatically. Currently, most of the paper scrap joint works were accomplished manually. Although some researchers have been done abroad, little research findings could be found for the application background specificity of the paper scrap automatic restoration technology. The description on the shape matching algorithm of the similar issue was found in some articles [1-7]. For example, in literature [7], a method to accomplish the paper scrap joint by judging if two profiles matched by boundary and area principles from extracting the contour line of the paper scrap was given to achieve the paper scrap automatic restoration based on the computer assistance.

2. Broken Paper Scrap of One-side Print Document with Only Straight Cut

For a given broken paper scrap from the same print document with only straight cut, the fragment data of the document with 19 items on two pages in Chinese and English respectively was given by the computer processing, and then the joint and recovery were done.

The least square method was a mathematic optimization technology, which minimized the error square and found the optimal function match. The unknown data could be calculated conveniently and the error square between the calculated and actual data was the minimum. Nonlinear least square method could be seen as a special situation for the unrestraint

minimization, which was widely applied in the areas like data fitting, parameter estimation and functional approximation [8]. The optimal matching solution for the paper scrap in the attachment by the least square method and its value was determined.

From the basic knowledge of the image processing, it was known that the pixel value range of each pixel point on the paper scraps to be jointed was 0-255, according to which, the figure could be analyzed. The figure was firstly broken up into the point collection, and the pixel matrix of the point collection of the figure was established for a random figure, which was assumed as j^{th} figure. It was known that the matrix of the j^{th} figure was 1980×72 , and the pixel matrix of the i^{th} column of the j^{th} figure was:

$$P_{ij} = \begin{bmatrix} P_{1ij} \\ P_{2ij} \\ \vdots \\ P_{hij} \end{bmatrix}, 0 \leq i \leq 72, 0 \leq j \leq 19$$

Firstly, for all the paper scrap to be jointed, the first one on the left was determined if all of the left boundary points were white, namely the pixel was 255, from which the first paper scrap to be jointed was found.

Obviously, for a random figure, the corresponding pixel point color difference of the other figure and it was not the same. The screening function $Q_{p,q}$ was defined, and to calculate the square sum of the pixel point color value difference between the right boundary of the p^{th} figure and corresponding left boundary of the q^{th} figure, the expression was as follows:

$$Q_{p,q} = \sum_{h=1}^{1980} (p_{h,72,p} - p_{h,1,q})^2$$

Where, p_{hij} meant the h^{th} row and i^{th} column of the pixel point color value matrix of the j^{th} figure.

For Figure p, as other figures and it matched, each $Q_{p,q}$ was obtained, and what needed was the optimal match, namely the nonlinear unrestraint mathematic programming model.

$$\text{Min } Q_{p,q}$$

Then Figure p and q could be jointed.

The program was written with the Matlab software in the Acer computer of 5750G configured with i5 processor, the running time was 0.1s, the joint was successful and the figure of the recovery result was as follows:

城上层楼叠嶂，城下清淮古汴。举手揖吴云，人与春天俱远。烧断
魂断。后夜松江月满。脉脉夜中步。寒花。村里村北响辘车。牛衣古柳卖黄
瓜。海棠珠箔一重。清晓近啼鸦。胭脂催与匀淡。偏向晓边凉。小郑非
常强记。二事依旧能持。更有神仙难切盼。几至莫教知。自古相从休务
日。何妨低唱微吟。天曹云笺作春晴。坐中人半影。帘外雪初深。双松绿
壁。娟娟梅波眉黛翠。妙舞幽狂。寒上身轻姿态妍。羞雾轻笼两凤。恁相
淡拂双鸦。为谁流睡不回家。错认门前过马。
我劝瓜张归去好。从来自己忘情。尘心消尽道心平。江南与塞北。何
处不堪行。闲高隐。谁念棠梨覆瓦。何曾梦云雨。旧恨前欢。心事两无踪。
要知欲免无由。痴心犹自。倩人道。一声传语。风卷珠帘自上钩。萧萧乱
叶报新秋。独携纤手上高楼。临水纵横回晚眺。归来转觉情怀动。梅窗烟
中闻几弄。秋阴重。西山青淡云凝冻。凭高眺远。见长空万里。云光留迹
棹飞光射处。冷浸一天秋碧。玉宇琼楼。乘鸾去去。人在清溪国。江山如
画。望中楼阁历历。尚可清言挥玉尘。真须保霜金真。风流何似道家
纯。不应同菊蕊。惟爱卓文君。自惜风流云雨散。关山有限情无限。待君
重见寻芳伴。为说相思。自应西楼燕。莫恨黄花未吐。且教红粉相扶。酒
雨不必看茶瓦。俯仰人间今古。玉骨那知憔悴。冰姿自有仙风。海仙时遣
探芳丛。倒挂绿毛么凤。
粗豉原蚕煮过熟。凭君说与南梁。愿闻吴越推半量。君王如有问。结
袜赖王生。聊唱慷慨曲。宗风嗣阿谁。借君拍板与门槌。我也逐场作戏。
莫相疑。墨墨堪忧印。印犹嫌墨墨。闲时晚妆残。妆残晚照闲。可恨相逢
徒几日。不知重会是何年。茱萸仔细更重看。午夜风翻幔。三更月到床。
翠钿如水玉肌凉。何物与依归去。有残妆。金炉犹暖麝煤残。惜香更把宝
钗煎。重闻处。余熏在。这一番。气味胜从前。霜暗荷枯一夜霜。新苞绿
叶照林光。竹篱茅舍出青黄。霜降水痕收。浅碧触处胜洲。酒力渐消风力
软。寒威。破帽多情却恋头。谁影摇风。一任伤春绪。归不恁。风楼何
处。穿窗透帘。须发云鬓自白。露华花乳轻圆。人间康乐更争妍。斗取
红窗粉面。笑平无人傍屋头。萧萧晚雨敲梧楸。谁冷季子傲貂裘。

Figure 1. In Chinese

fair of face.

The customer is always right. East, west, home's best. Life's not all beer and skittles. The devil looks after his own. Manners maketh man. Many a mickle makes a muckle. A man who is his own lawyer has a fool for his client.

You can't make a silk purse from a sow's ear. As thick as thieves. Clothes make the man. All that glitters is not gold. The pen is mightier than sword. Is fair and wise and good and gay. Make love not war. Devil take the hindmost. The female of the species is more deadly than the male. A place for everything and everything in its place. Hell hath no fury like a woman scorned. When in Rome, do as the Romans do. To err is human; to forgive divine. Enough is as good as a feast. People who live in glass houses shouldn't throw stones. Nature abhors a vacuum. Moderation in all things.

Everything comes to him who waits. Tomorrow is another day. Better to light a candle than to curse the darkness.

Two is company, but three's a crowd. It's the squeaky wheel that gets the grease. Please enjoy the pain which is unable to avoid. Don't teach your Grandma to suck eggs. He who lives by the sword shall die by the sword. Don't meet troubles half-way. Oil and water don't mix. All work and no play makes lack a dull boy.

The best things in life are free. Finders keepers, losers weepers. There's no place like home. Speak softly and carry a big stick. Music has charms to soothe the savage breast. Ne'er cast a clout till May be out. There's no such thing as a free lunch. Nothing venture, nothing gain. He who can does, he who cannot, teases. A stitch in time saves nine. The child is the father of the man. And a child that's born on the Sub-

Figure 2. In English

3. Broken Paper Scrap of One-side Print Document with Straight and Cross Cut

For a given broken paper scrap from the same print document with straight and cross cut, the fragment data of the document with 209 items on two pages in Chinese and English respectively were given by the computer processing, and then the joint and recovery was done.

3.1. Establishment of Nonlinear Programming Model

In the situation of the cross cut, the method above was improved. Firstly, each paper scrap in the attachment was analyzed, and the minimum vertical distance between the white pixel point on the top of each paper scrap and the first non-white pixel point was utilized and analyzed in the computer to roughly judge the shape match of the font. According to the literatures, the geometrical characteristic of the row where the paper scrap lay could be obtained by existed technology, like the row height of the character and the space between the character lines [9].

Random paper scraps were picked as the i^{th} and j^{th} page, and if the minimum vertical distances between the white pixel point on the top of the i^{th} and j^{th} paper scrap and the first non-white pixel point were d_{mi} and d_{mj} , $\Delta d_{ij} = d_{mi} - d_{mj}$.

According to the figures in this paper, the boundary value of Δd was 0 and 5 respectively. If the value of Δd was between 0 and 5, it was considered that the i^{th} and j^{th} paper scrap could be jointed in the same line, or else the two could not be jointed in the same line for the big difference of the character structure, which meant that the two paper scrap could not be jointed. Then the distances of the peaks of i^{th} paper scrap on the left and right and the first non-white pixel point horizontally were calculated and analyzed to find the minimum distance x_{si} and x_{ti} , the boundary value of which was chosen as 12, 16 and 6 to 28. By the same way, the maximum character space $y_{\max,i}$ between the top and bottom point in another restraint condition was determined.

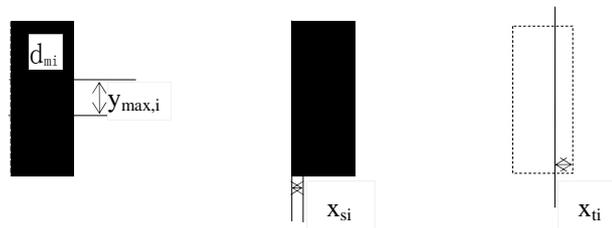


Figure 3. The Character of the i^{th} Paper Scraps

In the process above, the binaryzation method was adopted. By setting the threshold value, the conversion from the gray level image to the binaryzation image was accomplished [10]. The gray level value of the white pixel point was marked as 255, while the gray values of the other pixel points were marked as 0. In this process, the point, whose pixel value was close to that of the white pixel, was considered as the white point. The Form could be expressed as follows:

$$u = \begin{cases} 255, & \text{the point is white} \\ 0, & \text{else} \end{cases}$$

Then, based on the theory above, for all the paper scrap to be jointed, the program was firstly compiled to rank the paper scraps in the descending way according to the minimum principle for the sum of d_{mi} and x_{si} , the paper scrap with the minimum value was chosen as the first element on the left top. Then each of the paper scraps was jointed by the recurrence

algorithm based on the principle of from left to right and from top to bottom. In the joint process the least square method in the first model was used to screen the paper scraps to be jointed with the objective paper scrap, the $Min Q_{p,q}$ was then calculated, which meant the difference of paper scrap p and q was the minimum, namely paper scraps p and q were the best match.

The optimized model was a nonlinear restraint optimal problem:

$$\begin{array}{l} Min \quad Q_{p,q} \\ s.t. \quad \left\{ \begin{array}{l} 0 \leq d_{mi} - d_{mj} \leq 5 \\ 12 \leq x_{si} \leq 16 \\ 6 \leq x_{ti} \leq 28 \\ 38 \leq y_{\max,i} \leq 42 \end{array} \right. \end{array}$$

As the paper scrap p and q were jointed crosswise, the objective function was:

$$\min Q_{p,q} = \sum_{h=1}^{1980} (p_{h,72,p} - p_{h,1,q})^2$$

As the paper scrap p and q were jointed lengthways, the objective function was:

$$\min \sum_{i=1}^{72} (p_{180,i,p} - p_{1,i,q})^2$$

According to the model establishment and analysis, the program was compiled with Matlab software and the initial joint was obtained.

3.2. Artificial Interference Solution

In many situations, as two paper scraps were both considered as the suitable one to be jointed with the objective paper scrap by the computer, the computer would automatically chose one of them which was considered the better to joint without the logical analysis. At this time, it was easy to produce a large-scale and extensive joint error. Hence the artificial interference was of significance as the solution error of the computer happened. As shown in Figure 4 and 5, obviously the two figures could be totally matched with the objective figure, and in this situation, if the wrong figure was picked by the computer (as shown in Figure 2), the other paper scrap would be jointed with other paper scraps wrongly either, which would result in the domino effect. Meanwhile, if the artificial interference was used with the logical thinking and analysis of human, it could be realized that the word "catens" in the figure to be jointed in Figure 5 didn't actually exist, hence the joint error would be found and corrected, which would lead to a high accuracy and right restoration of the literatures.

The ea the ea
catches th catens be
hatched. haumme

Figure 4. Jointed Correct Figure 5. Jointed Error

Hence, the artificial interference needed to be added after the initial joint was obtained, and the interference way was: the joint couple of the paper scraps in the running result of the program was firstly found, and then the serial number and the joint order were then recorded.

Another section of the program was then added in the initial joint program to achieve the descending order of the sum of d_{mi} and x_{si} for each paper scrap, the second sequencing value obtained was chosen as the first element on the left top to find the paper scrap to be jointed again, the serial number and joint order were recorded, then the third, the fourth etc. were found until no single paper scrap was left or after the program was run for three times or more, no new paper scraps were found to be jointed with other paper scraps. Then the artificial interference was finished, and the final right result was achieved by the artificial joint with the logical analysis on the character and language.

In addition, for English documents, the variable value range of the restraint condition could be changed appropriately because of the difference between English and Chinese character, the following was used:

$$\begin{cases} 0 \leq d_{mi} - d_{mj} \leq 16 \\ 12 \leq x_s \leq 16 \\ 6 \leq x_t \leq 28 \\ 23 \leq y_{\max} \leq 42 \end{cases}$$

The right joint result obtained was as Figure 6. and Figure 7:

便邮。温香熟美。醉慢云鬟垂两耳。多谢春工。不是花红是玉红。一颗樱
桃糝素口。不爇黄金，只爱人长久。学画鸦儿犹未就。眉尖已作伤春皱。
清泪斑斑，斩断柔肠寸。嗔人问。背灯偷拭尽残妆粉。春事阑珊芳草
歇。客里风光，又过清明节。小院黄昏人忆别。落红处处闻啼鸟。岁云暮，
须早计，要褐裘。故乡归去千里，佳处辄迟留。我醉歌时君和，醉倒须君
扶我，惟酒可忘忧。一任刘玄德，相对卧高楼。记取西湖西畔，正暮山好
处，空翠烟霏。算诗人相得，如我与君稀。约他年、东还海道，愿谢公、雅
志莫相违。西州路，不应回首，为我沾衣。料峭春风吹酒醒。微冷。山头
斜照却相迎。回首向来萧瑟处。归去。也无风雨也无晴。紫陌寻春去，红
尘拂面来。无人不道看花回。惟见石榴新蕊，一枝开。

九十日春都过了，贪忙何处追游。三分春色一分愁。雨翻榆荚阵，风
转柳花球。白雪清词出坐间。爱君才器两俱全。异乡风景却依然。团扇只
堪题往事，新丝那解系行人。酒阑滋味似残春。

缺月向人舒窈窕，三星当户照绸缪。香生雾縠见纤柔。搔首赋归欤。
自觉功名懒更疏。若问使君才与术，何如。占得人间一味愚。海东头，山
尽处。自古空槎来去。槎有信，赴秋期。使君行不归。别酒劝君君一醉。
清润潘郎，又是何郎婿。记取钗头新利市。莫将分付东邻子。西塞山边白
鹭飞。散花洲外片帆微。桃花流水鳜鱼肥。主人眼小。欲向东风先醉倒。
已属君家。且更从容容等待。愿我已无当世望，似君须向古人求。岁寒松
柏肯惊秋。

水涵空，山照市。西汉二疏乡里。新白发，旧黄金。故人恩义深。谁
道东阳都瘦损，凝然点漆精神。瑶林终自隔风尘。试看披鹤氅，仍是谪仙
人。三过平山堂下，半生弹指声中。十年不见老仙翁。壁上龙蛇飞动。暖
风不解留花住。片片著人无数。楼上望春归去。芳草迷归路。犀钱玉果。
利市平分沾四坐。多谢无功。此事如何到得依。元宵似是欢游好。何况公
庭民讼少。万家游赏上春台，十里神仙迷海岛。

虽抱文章，开口谁亲。且陶陶、乐尽天真。几时归去，作个闲人。对
一张琴，一壶酒，一溪云。相如未老。梁苑犹能陪俊少。莫惹闲愁。且折

Figure 6. In Chinese

bath day. No news is good news.

Procrastination is the thief of time. Genius is an infinite capacity for taking pains. Nothing succeeds like success. If you can't beat em, join em. After a storm comes a calm. A good beginning makes a good ending.

One hand washes the other. Talk of the Devil, and he is bound to appear. Tuesday's child is full of grace. You can't judge a book by its cover. Now drips the saliva, will become tomorrow the tear. All that glitters is not gold. Discretion is the better part of valour. Little things please little minds. Time flies. Practice what you preach. Cheats never prosper.

The early bird catches the worm. It's the early bird that catches the worm. Don't count your chickens before they are hatched. One swallow does not make a summer. Every picture tells a story. Softly, softly, catchee monkey. Thought is already late, exactly is the earliest time. Less is more.

A picture paints a thousand words. There's a time and a place for everything. History repeats itself. The more the merrier. Fair exchange is no robbery. A woman's work is never done. Time is money.

Nobody can casually succeed, it comes from the thorough self-control and the will. Not matter of the today will drag tomorrow. They that sow the wind, shall reap the whirlwind. Rob Peter to pay Paul. Every little helps. In for a penny, in for a pound. Never put off until tomorrow what you can do today. There's many a slip twixt cup and lip. The law is an ass. If you can't stand the heat get out of the kitchen. The boy is father to the man. A nod's as good as a wink to a blind horse. Practice makes perfect. Hard work never did anyone any harm. Only has compared to the others early, diligently

Figure 7. In English

4. Broken Paper Scrap of both Sides Print Document

For a given both sides print document of English version from the same page with both straight and cross cut by the shredder, the joint and restoration of the paper scrap were accomplished with the help of the both sides fragment data of 209 documents given by the computer.

The paper scrap collection of the first line & column and the last line & column after being jointed was firstly jointed. For which, a new variable d_{ni} was introduced to indicate the minimum distance between the white pixel point of all the peaks at the bottom of the i^{th} paper and the first non-white pixel point vertically. From the computer program, the d_{ni} , d_{mi} , x_{si} and x_{ti} could be obtained and the four matrixes were established for the paper scrap given.

According to the principle that d_{mi} of the first line elements was bigger than other elements, and the difference between d_{mi} and the elements in each line should not be too big, and by the artificial interference to observe the value of each element in the matrix, the 19 paper scraps in the first line, the last line, the first column and the last column could be found soon. Then according to the principle that only one common element existed in a random line or column, the first element in the first line could be determined soon, and the optimal match for each paper scrap could be found according to the principle of from left to right and from top to bottom. In the same way, the last element of the first line, the first element of the last line and the last element of the last line were determined, and each match result could be obtained by taking the three elements as the initial objective paper scrap to be matched.

For the restriction of the both sides print, the algorithm accuracy was lowered. Hence, in the matching process of the objective paper scrap and other paper scraps, the ant colony algorithm was adopted.

The ant colony algorithm was a probabilistic and overall situation search method, and the uncertainty property in this algorithm would lead to more opportunities to find the optimal solution in the overall situation [11-13]. The ant colony algorithm could increase the match accuracy for the paper scrap, its optimization process did not rely on the rigorous mathematic property of itself, and it had the potential parallelism, which increased the efficiency and instant reaction ability of the algorithm. In the matching process, the paper scrap number became larger, and the pixel of the figures was smaller, hence the ant colony algorithm was used in this paper to process optimally and increase the match accuracy.

The four matching results from the process above were observed, and the artificial interference was added at this time, which was to record the right joint results, adjust the wrong jointed paper scraps which was artificially recognized, and re-match the wrong jointed paper scraps which could not be artificially recognized from the paper edge. The process should be repeated until the right matching results were obtained:

What can't be cured must be endured. Bad money drives out good. Hard cases make bad law. Talk is cheap. See a pin and pick it up, all the day you'll have good luck; see a pin and let it lie, bad luck you'll have all day. If you pay peanuts, you get monkeys. If you can't be good, be careful. Share and share alike. All's well that ends well. Better late than never. Fish always stink from the head down. A new broom sweeps clean. April showers bring forth May flowers. It never rains but it pours. Never let the sun go down on your anger.

Pearls of wisdom. The proof of the pudding is in the eating. Parsley seed goes nine times to the Devil. Judge not, that ye be not judged. The longest journey starts with a single step. Big fish eat little fish. Great minds think alike. The end justifies the means. Cowards may die many times before their death. You can't win them all. Do as I say, not as I do. Don't upset the apple-cart. Behind every great man there's a great woman. Pride goes before a fall.

You can lead a horse to water, but you can't make it drink. Two heads are better than one. March winds and April showers bring forth May flowers. A swarm in May is worth a load of hay; a swarm in June is worth a silver spoon; but a swarm in July is not worth a fly. Might is right. Let bygones be bygones. It takes all sorts to make a world. A change is as good as a rest. Into every life a little rain must fall. A chain is only as strong as its weakest link.

Don't look a gift horse in the mouth. Old soldiers never die, they just fade away. Seeing is believing. The opera ain't over till the fat lady sings. Silence is golden. Variety is the spice of life. Tomorrow never comes. If it ain't broke, don't fix it. Look before you leap. The road to hell is paved with good

Figure 8. Joint Results

5. Conclusion

The technology of joint and recovery of paper scrap are considered in shape before. This paper give a method combined ant colony algorithm and the least square method according to the words message in the paper scrap. However, for the paper scrap with big information, a great amount of artificial interferences was needed in this model.

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