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# Analysis and Design of Complementary Ring Type Metamaterial Filter in THz Wave Domain

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***Abstract***

*In this paper, the complementary ring type metamaterial unit is theoretically analyzed by using the spectral domain approach. A terahertz bandpass filter is designed, which consists of triple-layer complementary ring metamaterial. The center frequency of the filter is 338GHz, the transmission rate in pass band reaches 97%, the maximum insertion loss in pass band reaches 0.62dB and 3dB bandwidth reaches 75.9GHz. The filter shows stable filtering properties at different polarization wave and the same polarized wave at different incident angles within the scope of 20°. The range of operating frequency matches the first terahertz atmosphere communication window, so it can be used in the field of terahertz atmosphere communication.*

***Keywords**: Terahertz communication, metamateria, bandpass filter*



## 1. Introduction

Terahertz (THz) that refers to frequency from 0.1THz to 10THz lies in the frequency gap between the microwave and infrared. In 1970s, the THz wave attracts many scientists' attentions [1], but the methods of generating and detecting are backward, which causes the so-called "Terahertz Gap". Recently, due to the great progress of THz source such as the quantum dot laser [2], free electron laser [3] and optical rectification [4] appears, the THz technology has been the center of world's extensive research gradually.

Metamaterial (MM) is a kind of sub-wavelength units consisted of periodic or aperiodic arrangement artificial material, which has been applied in many frontier science [5]. As MM can change their structures to make an effective response to the terahertz wave, it can solve the problem of "Terahertz Gap". Now MM is widely applied in terahertz frequency. As one of the important components in terahertz frequency, filters are widely used in imaging, spectrometer, molecular sensing, security and communication.

In this paper, the complementary ring type MM is chosen as the basic unit, which is theoretically analyzed by using the spectral domain approach. The triple-layer complementary ring MM is composed of single layer cascade. By adjusting the related structural parameters, the filtering property can be optimized. The operating frequency matches the first terahertz atmosphere communication window, which can be used in the field of terahertz atmosphere communication.

## 2. The Spectral Domain Approach Analysis

Frequency selective surface (FSS) is widely used in terahertz filters. It consists of periodic metal grid or periodic array aperture on the metal panel [6]. The complementary ring type MM is a sort of periodic array aperture structure.

Using spectral domain approach to analyze FSS [7], Figure 1 (left) shows the patch FSS, the black part is metal. Incident wave is a plane wave $E^{inc}$ with wave vector $K^{i}_{0}$, the incident direction is ($\theta$, $\phi$). According to the different polarization, it can be divided into TE wave and TM wave.

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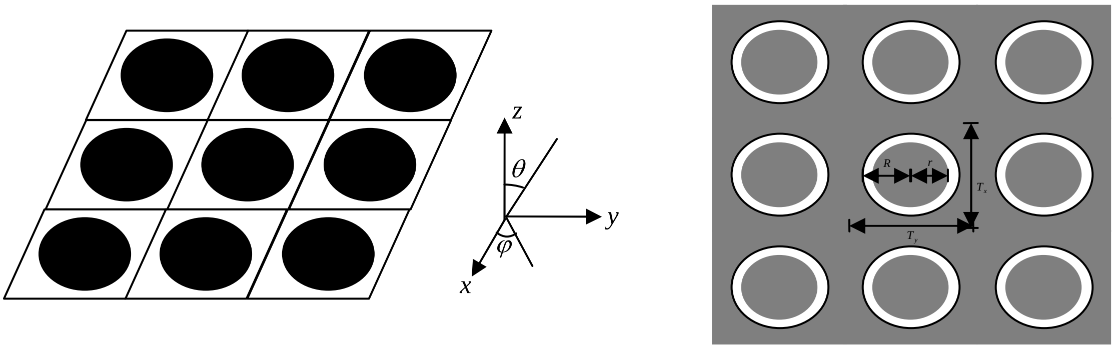


Figure 1. FSS Patch Structural Model (left) and Complementary Ring Type Metamaterial (right)

In the free space, the scattered field produced by the induced current on the patch FSS can be expressed as:

$$E^{s} = -\mathrm{j}\omega\mu_0 A + \frac{1}{\mathrm{j}\omega\varepsilon_0}\nabla\left(\nabla\cdot A\right) \quad (1)$$

Where *A* is the magnetic vector potential. There is only tangential electric current on the patch FSS surface, *A* can be calculated by Equation (2):

$$\begin{bmatrix} A_{\mathrm{x}}\left(x,y\right) \\ A_{\mathrm{y}}\left(x,y\right) \end{bmatrix} = \bar{\bar{G}}\left(x,y\right)\cdot\begin{bmatrix} J_{\mathrm{x}}\left(x,y\right) \\ J_{\mathrm{y}}\left(x,y\right) \end{bmatrix} \quad (2)$$

Where: $\bar{\bar{G}} = \exp\left(-\mathrm{j}k_0 r\right)\bar{\bar{I}}/4\pi r$ , $r = \sqrt{x^2+y^2}$ , $\bar{\bar{G}}$ express the dyadic Green functions, $\bar{\bar{I}}$ is the dyadic unit. From Equation (1) and Equation (2):

$$\begin{bmatrix} E_{\mathrm{x}}^{\mathrm{s}} \\ E_{\mathrm{y}}^{\mathrm{s}} \end{bmatrix} = \frac{-\mathrm{j}\omega\mu_0}{k_0^2}\begin{bmatrix} \frac{\partial^2}{\partial x^2}+k_0^2 & \frac{\partial^2}{\partial x\partial y} \\ \frac{\partial^2}{\partial x\partial y} & \frac{\partial^2}{\partial y^2}+k_0^2 \end{bmatrix}\begin{bmatrix} A_{\mathrm{x}} \\ A_{\mathrm{y}} \end{bmatrix} \quad (3)$$

The tangential electric current on the unit of FSS is zero. Use Fourier transform on Equation (3).

$$\begin{bmatrix} E_{\mathrm{x}}^{\mathrm{s}}\left(x,y\right) \\ E_{\mathrm{y}}^{\mathrm{s}}\left(x,y\right) \end{bmatrix} = \frac{1}{2\pi}\iint_{\infty}\frac{1}{\mathrm{j}\omega\varepsilon_0}\begin{bmatrix} k_0^2-\alpha^2 & -\alpha\beta \\ -\alpha\beta & k_0^2-\beta^2 \end{bmatrix}\times\tilde{G}\begin{bmatrix} \tilde{J}_{\mathrm{x}} \\ \tilde{J}_{\mathrm{y}} \end{bmatrix}\mathrm{e}^{\mathrm{j}\alpha x}\mathrm{e}^{\mathrm{j}\beta y}d\alpha d\beta \quad (4)$$

The $\tilde{G}$ in Equation (4) is:

$$\tilde{G} = \begin{cases} \dfrac{-\mathrm{j}}{2\sqrt{k_0^2-\alpha^2-\beta^2}} & k_0^2 > \alpha^2+\beta^2 \\ \dfrac{1}{2\sqrt{\alpha^2+\beta^2-k_0^2}} & \text{ot her s} \end{cases} \quad (5)$$

Use Floquet theory:

$$J\left(x\right) = \sum_{m=-\infty}^{\infty}\tilde{J}_{\mathrm{m}}\mathrm{e}^{\mathrm{j}\left(2\mathrm{m}\pi/\mathrm{a}+\mathrm{k}_{\mathrm{x}}^{\mathrm{inc}}\right)\mathrm{x}},\ J\left(y\right) = \sum_{m=-\infty}^{\infty}\tilde{J}_{\mathrm{m}}\mathrm{e}^{\mathrm{j}\left(2\mathrm{m}\pi/\mathrm{b}+\mathrm{k}_{\mathrm{y}}^{\mathrm{inc}}\right)\mathrm{y}} \quad (6)$$

Where *a*,*b* respectively expresses unit period on the direction of *x*,*y*, $k_x^{inc}$,$k_y^{inc}$ respectively expresses incident wave number on the direction of *x*,*y*, From Equation (4), Equation (5) and Equation (6):

$$\begin{bmatrix} E_x^s(x,y) \\ E_y^s(x,y) \end{bmatrix} = \frac{2\pi}{j\omega\varepsilon_0 ab}\sum_m\sum_n \begin{bmatrix} k_0^2-\alpha_m^2 & -\alpha_m\beta_n \\ -\alpha_m\beta_n & k_0^2-\beta_n^2 \end{bmatrix} \times \tilde{G}(\alpha_m,\beta_n)\cdot \begin{bmatrix} \tilde{J}_x(\alpha_m,\beta_n) \\ \tilde{J}_y(\alpha_m,\beta_n) \end{bmatrix} e^{j(\alpha_m x+\beta_n y)} \tag{7}$$

Where $\alpha_m = 2m\pi/a + k_x^{inc}$, $\beta_n = 2n\pi/b + k_y^{inc} - (2m\pi/a)\cos\Omega$, $\Omega$ is the angle between the two period directions of FSS. According to Babinet principle [8], two complementary structures exist complementary frequency responses. The structures of patch type FSS and aperture type FSS are complementary, so the aperture structure of the frequency response can be concluded according to patch structure.

## 3. Complementary Ring type MM Filter

### 3.1. Structure Parameter Estimation

Before simulation, the unit structure parameters of complementary ring type MM are needed to be estimated. As the terahertz atmosphere communication window is 300~376GHz, the center frequency of the filter is set as 338GHz. Figure 1 (right) shows the 2×2 periodic MM, the unit is aluminium sheet with ring slot whose thickness is 2μm. The substrate is high-resistance Si, the relative permittivity $\varepsilon_r$=11.9.

Preliminary determining the geometry size of unit, particularly the inner radius *r*, the size of unit cell $T_x$ and $T_y$, the thickness $t_{Si}$ of the silicon substrate have to be set.

The inner radius *r* can be determined by using the resonance condition [9]:

$$r = \frac{\lambda_r}{2\pi} = \frac{\lambda_0}{2\pi\sqrt{\varepsilon_{eff}}} = \frac{c}{2\pi f_r\sqrt{\varepsilon_{eff}}}, \quad \varepsilon_{eff} = \frac{\varepsilon_r+1}{2} \tag{8}$$

Where *c* is velocity of light, $\varepsilon_{eff}$ is effective permittivity, $\varepsilon_r$ is relative permittivity of substrate. The center frequency $f_r$=338GHz, according to Equation (8) *r*=55.65μm, and the ring slot width *s* is set as 20μm, so the outer radius *R*=75.65μm.

To avoid the grating lobe, $T_x$ and $T_y$ must be less than the minimum of unit size. The condition of grating lobe appearing is [10]:

$$\beta T_x(\sin\eta + \sin\eta_g) = 2\pi n, \quad \beta = \frac{2\pi}{\lambda_g} \tag{9}$$

Where $\eta$ is angle of incidence, $\eta_g$ is the angle of grating lobe. When the grating lobe skims over the unit($\eta_g$=90°), the minimum frequency $f_{g0}$ occurs.

$$T_x = T_y = \frac{nc}{f_{g0}(\sin\eta + 1)} \tag{10}$$

In Equation (10), *c* is velocity of light, setting sin$\eta$=1, n=1, the minimum size of unit cell can be determined. Choosing the maximum scanning frequency $f_{g0}$=500GHz, the minimum size of unit cell $T_x$= $T_y$=300μm.

As the filter works in terahertz frequency band, the thickness of the substrate is in the same order of magnitude as passband center wavelength, which will cause the Fabry-Perot resonance occers. The Fabry-Perot resonance condition [11] is:

$$t_{Si} = k\cdot\frac{c}{2f\sqrt{\varepsilon_r}}\cdot\cos\left[\arcsin\left(\frac{1}{\sqrt{\varepsilon_r}}\sin\theta\right)\right] \tag{11}$$

Where $\theta$ is angle of incidence, $k$=1、2、3…, $f$ is the resonant frequency, $\varepsilon_r$ is relative permittivity of substrate. Setting $f$=338GHz, $k$=1, $\theta$=90°, the minimum thickness of the substrate can be determined $t_{Simin}$=123.126μm when the Fabry-Perot resonance occurs.

To avoid the Fabry-Perot resonance, the thickness of the substrate must be less than 123.126μm. The thickness is less, the transmission rate can get higher. Nevertheless, it also cause poorer mechanical strength. Taking all the factors into account, the thickness of the substrate is set as $0.02\lambda_0$ (18.38μm).

The discontinuous boundary will affect the filtering properties. So for the THz wave beam, the size must be approximate infinite. In the practical application, the size of array structure must be approximate infinite as compared to various of THz sources.

### 3.2. Filtering Property

In this paper, Ansoft HFSS is used to simulate the MM structure. Due to the large size of complementary ring array, the actual size of the model will cause huge calculated amount, which will affects the accuracy of the simulation. So the single unit is set up and simulated by using Floquet port and Master-slave boundary conditions.

The filter which consists of triple-layer complementary ring is shown in Figure 2 (left). According to the analysis in Part 2, the single-layer FSS can be regarded as a LC resonant circuit, so multi-layer FSS is a filtering network that is composed of multiple LC resonant circuits cascaded. Due to the theory of transmission lines [12], the interlayer spacing must be $\lambda_0/4$ to make sure the impedance match between the layers, and $\lambda_0$ is the resonance wavelength in free space. The shape and size of every FSS must be identical so as to make the coincidence of reflection coefficient.

When simulating, the structure parameters are set according to Part 3.1 and the interlayer spacing is $\lambda_0/4$, the S parameters show in Figure 2 (middle) N1. The center frequency is 332GHz, 3dB bandwidth is 52.7GHz, the in-band transmission coefficient ($S_{21}$) is not flat. By reducing the layer spacing, it can strengthens the interlayer coupling in the electromagnetic field, so it can improve the filter performance and expand the bandwidth. Compared with model N1, the interlayer spacing of N2 reduces 10μm, and the in-band transmission coefficient is flatter, and the Bandwidth is broader.

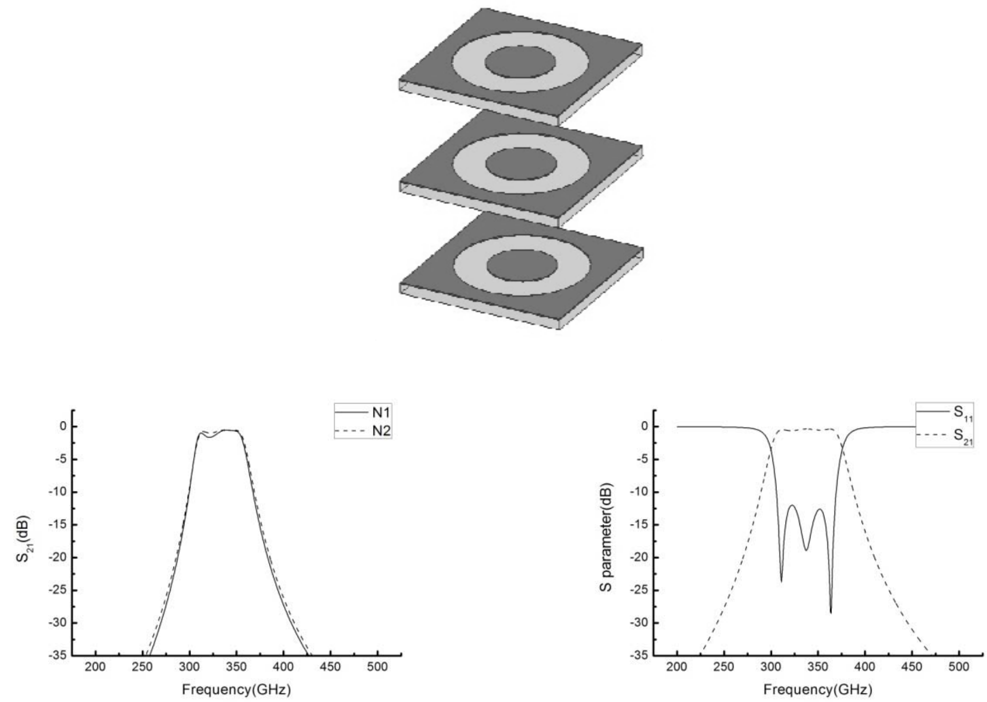


Figure 2. The Triple-layer Complementary Ring Metamaterials Model (left) S Parameter of Different Interlayer Spacing (middle) S Parameter of Improved Model (right)

According to Equation (8), by increasing the inner radius *r*, it can make the center frequency redshift. And combining with interlayer spacing, it can make further improvement of the filtering properties. Ultimately, the inner radius *r*=59.3μm, ring slot width *s*=56μm, interlayer spacing *d*=190μm, the size of unit cell $T_x$= $T_y$=300μm, the thickness of the silicon substrate $t_{Si}$= 18.38μm, the S parameter shows in Figure 2 (right). The center frequency is 338GHz, the transmissivity reaches 97%, the max in-band insertion loss is 0.62dB and 3dB bandwidth is 75.9GHz, which matches the first terahertz atmosphere communication window.

## 4. Stability of Complementary Ring Type MM Filter

### 4.1. Stability of Polarization

Figure 3 shows the S parameter of filter, when the incident wave is different polarized. As shown in Figure 3 (left), transmission coefficients ($S_{21}$) of TE wave and TM wave have an overlap, both of the center frequencies are 338GHz. When the incident wave is TE wave, the 3dB bandwidth is 75.9GHz. when the incident wave is TE wave, 3dB bandwidth is 76GHz. So under the different polarized incident wave, filter shows good stability. Meanwhile, TM wave has the less reflection coefficient ($S_{11}$).

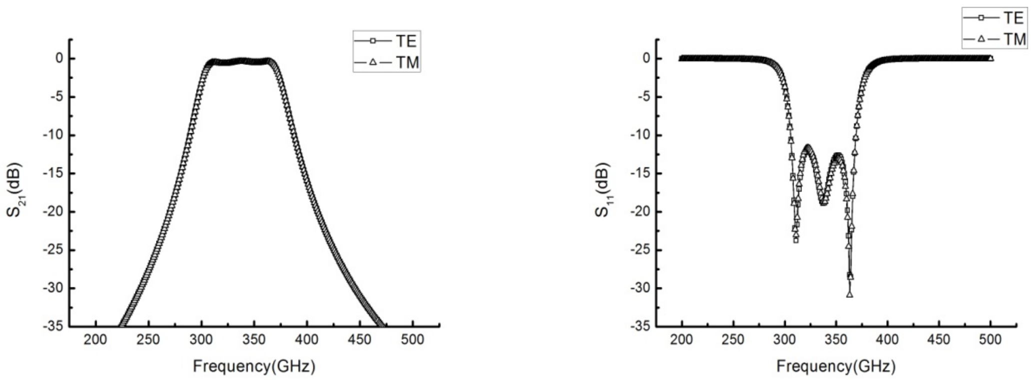


Figure 3. The S Parameter of Different Polarized Waves $S_{21}$ (left) $S_{11}$ (right)

### 4.2. Stability of Incident Angle

Figure 4 shows the frequency response of the filter when the incident wave has the different angle. When incident wave is TE wave, the center frequency is 338GHz with the incident angle *θ*=0°; the center frequency is 339GHz with *θ*=10°; the center frequency is 340GHz with *θ*=20°. It can be seen that when the incident angle *θ* increases, the center frequency slightly drifts, and the 3dB bandwidth gets a little narrower from 75.9GHz to 74.6GHz.

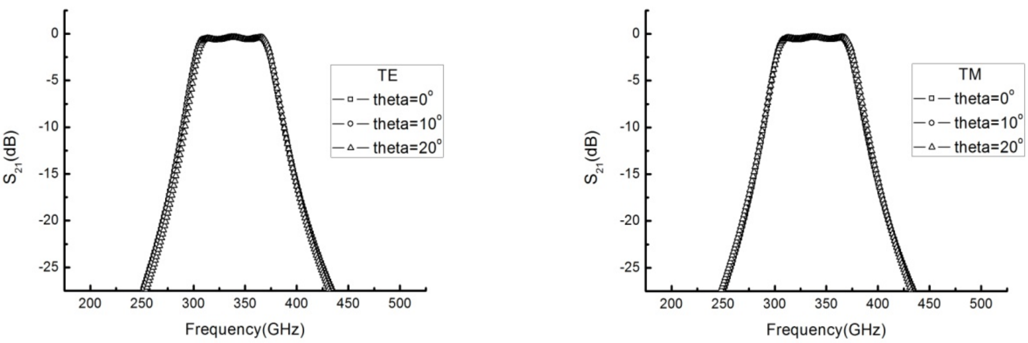


Figure 4. The S Parameter of Different Incedent Angle Waves TE (left) TM (right)

When incident wave is TM wave, with the increasing of incident angle, the center frequency has a blueshift in small range, the 3dB bandwidth gets slightly broader. In conclusion, within the 20° scope of the incident angle, the filter has a good stability.

## 5. Conclusion

In this paper, the complementary ring type MM unit is theoretically analyzed by using the spectral domain approach. And the bandpass filter that works in terahertz is designed, which consists of triple-layer complementary ring type MM. The center frequency is 338GHz, the transmissivity reaches 97%, the max in-band insertion loss is 0.62dB and the 3dB bandwidth is 75.9GHz, which matches the first terahertz atmosphere communication window. The filter offers a superior steepness of skirts and out-of-band rejection. When the incident wave is different polarized, it shows a stable frequency response. And within 20° scope of incident angle, it has a stable filtering property.

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