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Variable Step Size Blind Equalization Based on Sign Gradient Algorithm

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Abstract

This work proposes a variable step size sign gradient algorithm to solve the problem of blind equalization under impulse noise environment. This algorithm suppresses the impulse noise interference effectively because of the sign operation on the instantaneous gradient based on constant modulus algorithm (CMA) cost function. Meanwhile, the excess mean square error can be further reduced by use of a variable step size algorithm based on the iterative times and the reliability of the output signal. Simulation results show that, the variable step size blind equalization based on sign gradient algorithm has the fastest convergence rate and the lowest steady state residue error compared with the fractional low order CMA, the nonlinear transform CMA and stop-and-go CMA.

***Keywords**: blind equalization, sign gradient algorithm, CMA, impulse noise, variable step size*

1. Introduction

Blind equalization technology has potential applications both in cooperative and non-cooperative communication systems because it does not require any training sequences [1]. In various blind equalization algorithms, CMA has been widely applied for its simple and robust convergence performance [2]. The theoretical analysis of CMA does not consider the channel noise which is often assumed as Gaussian white noise, but the wireless channel noise often shows properties of impulse noise rather than Gaussian white noise [3]. For impulse noise with no more than two order moments, CMA becomes unstable or even failure is possible. Nonlinear transform CMA [4] and fractional lower order CMA [5] can suppress the impulse noise and show better performance than the traditional CMA. Also stop-and-go algorithm [6] depends on the reliability of the output signal to decide whether the equalizer updating or not, which can avoid the short-time high amplitude impulse noise interference to obtain robust convergence performance under the impulse noise environment.

However, the fractional lower order CMA obtains the robust convergence at the cost of slow convergence rate and the nonlinear transform CMA can only suppress the distinct impulse noise which the robust convergence performance cannot be ensured. The stop-and-go algorithm is often ill-convergence if the impulse noise interference is serious. This paper proposes a variable step size sign gradient algorithm to improve CMA blind equalization, in which the blind equalizer updating based on sign gradient and the step size vary with the iterative times and the reliability of the output signal. Thus a new blind equalization algorithm which has fast convergence rate and robust convergence performance under the impulse noise environment is obtained. Simulation results show that, compared with the fractional low order CMA, the nonlinear transform CMA and stop-and-go CMA, the variable step size sign gradient CMA blind equalization has the fastest convergence rate and the lowest steady state residue error.

2. Proposed Variable Step Size Sign Gradient CMA Blind Equalization

2.1. The Basic Principle of CMA

The basic principle of the baseband model of CMA blind equalization [7] is shown as Figure 1. The send signal $x(n)$ is transmitted on the unknown channel $h(n)$ interfered by noise

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$n(n)$, and the received signal $y(n)$ is obtained. The goal of blind equalization is to implement equalization by equalizer $w(n)$ only rely on the observed signal $y(n)$ without the information of the channel $h(n)$ and the send signal $x(n)$. The output signal $\tilde{x}(n)$ of the equalizer can be detected by the decision device $D(.)$ to recover the transmitted symbol sequence as $\hat{x}(n)$.

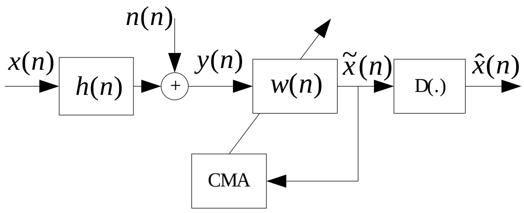

Figure 1. The Basic Principle of CMA Blind Equalization

The cost function of CMA is given by [8]:

$$J_{CMA}(n) = \frac{1}{2} E\left[(|\tilde{x}(n)|^2 - R_2)^2 \right] \tag{1}$$

Where R_2 is the constant modulus which can be calculated by:

$$R_2 = E\left[|\tilde{x}(n)|^4 \right] \Big/ E\left[|\tilde{x}(n)|^2 \right] \tag{2}$$

According to the stochastic gradient descent algorithm, CMA updates the equalizer weights by [9]:

$$w(n+1) = w(n) + \mu \nabla J_{CMA}(n) \tag{3}$$

Where μ is the step size. Let the iterative error $e(n)$ is:

$$e(n) = R_2 - |\tilde{x}(n)|^2 \tag{4}$$

The equalizer weights updating formula Equation (3) can be rewritten as:

$$w(n+1) = w(n) + \mu e(n)\tilde{x}(n)y^*(n) \tag{5}$$

2.2. The Impulse Noise Model

Although CMA blind equalization does not consider the channel noise interference, it shows robust convergence performance under Gaussian noise condition, and it can obtain better performance if use the fractionally spaced equalizer [10]. However, many communication channels are interference with impulse noisewhich results in instability of CMA. Alpha stationary distribution is often used to describe the impulse noise model and it has been applied to many communication systems [11]. The alpha stationary distribution is commonly described by characteristic function [12] which is given by:

$$\varphi(t) = \begin{cases} \exp\left\{ jat - \gamma |t|^\alpha \left[1 + j\beta \operatorname{sgn}(t) \tan\left(\frac{\alpha\pi}{2} \right) \right] \right\} & \alpha \neq 1 \\ \exp\left\{ jat - \gamma |t|^\alpha \left[1 + j\beta \operatorname{sgn}(t) \frac{2}{\pi} \lg(t) \right] \right\} & \alpha = 1 \end{cases} \tag{6}$$

The characteristic exponent $\alpha \in (0, 2]$ controls the degree of the impulse noise, that is the impulse noise becomes stronger if α becomes smaller. On the contrary, the impulse noise becomes smaller if α becomes lager. If $0 < \alpha < 2$, the alpha stationary distribution is named as fractional low order stationary distribution (FLOA). The symmetry coefficient $\beta \in [-1,1]$ and the alpha distribution is named as symmetrical distributionif $\beta = 0$. If the symmetry coefficient $\beta = 0$ and the characteristic exponent $\alpha = 2$, the alpha stationary distribution is as same as the Gaussian distribution. The dispersion coefficient $\gamma > 0$ is almost as same as the variance of the Gaussian distribution which controls the energy of the alpha distribution. The location parameter $a \in R$ expresses the mean or median value of the stationary distribution. Alpha stationary distribution is called as symmetry alpha stationary distribution ($S\alpha S$) if the symmetry coefficient $\beta = 0$ and the location parameter $a = 0$, $S\alpha S$ has some same properties as Gaussian distribution such as slickness, unimodality and bell type, etc. The study also shows that the alpha stationary distribution with the characteristic exponent $1 < \alpha < 2$ can sufficiently describes the impulse noise in the real world. Therefore the channel noise is assumed to be the FLOA-$S\alpha S$ with $1 < \alpha < 2$. The important difference between the FLOA distribution and the Gaussian distribution is that the FLOA distribution has no more than α-order moments which result in failure of CMA blind equalization based on stochastic gradient descent algorithm.

2.3. The Sign Gradient Algorithm

The alpha stationary noise is different from the White Gaussian noise, and CMA cannot ensure robust convergence performance. Recently, some improved CMA blind equalization algorithms have been proposed for blind equalization under impulse noise environment, and three typical improved algorithms are the fractional low order CMA, nonlinear transform CMA and stop-and-go algorithm. The fractional low order CMA blind equalization redefined the cost function according to the statistic property of the alpha distribution which is given by:

$$J(p,q) = E\left[(|\tilde{x}(n)|^p - R_2)^q \right] \quad (7)$$

Where p/q is a fraction between 0 and α, meanwhile, p/q must meet the condition $pq < \alpha$ to ensure that the cost function of the fractional low order CMA is limited. By using the fractional low order moments of the received signal, the fractional low order CMA obtains robust convergence performance. However, the fractional low order CMA does not use the high order statistic of the received signal and the convergence rate is slow.

The nonlinear transform CMA blind equalization carried nonlinear transform on the received signal to suppress the impulse noise, and the nonlinear transform plays soft limiting effect on the received signal. The nonlinear transform function is often given by:

$$x_f(n) = 2/(1 + \exp(-2\tilde{x}(n))) - 1 \quad (8)$$

Then the cost function of the nonlinear transform CMA is:

$$J_{NCMA}(n) = \frac{1}{2} E\left[(|x_f(n)|^2 - R_2)^2 \right] \quad (9)$$

The blind equalizer weights updating of the nonlinear transform CMA is given by:

$$w(n+1) = w(n) + \mu e(n) x_f(n) y^*(n) \quad (10)$$

However, the soft limiting effect of the nonlinear transform can only suppress the large amplitude impulse noise, so the robust convergence of the nonlinear transform cannot be ensured.

Stop-and-go CMA updates the blind equalizer according to the reliability of the output signal, which can avoid the impulse noise effect on the instantaneous gradient. However, the reliability of the output signal is difficult to be judged, which results in performance degradation of stop-and-go CMA.

Based on the above discussion, blind equalization under the impulse noise environment still doesn't have satisfactory algorithms. From the signal transmission in the communication system, the impulse noise has influence on the received signal $y(n)$, the output signal $\tilde{x}(n)$ and the error function $e(n)$ of the gradient based on CMA for the blind equalizer weights updating. The gradient of CMA for blind equalization weights updating is given by:

$$\nabla J_{CMA}(n) = -e(n)\tilde{x}(n)y^{*}(n) \tag{11}$$

Sign error gradient descent algorithm [13] has been proved to be an effective algorithm to reduce the computational complexity and improve the performance of the stochastic gradient descent algorithm. Signum operation can suppress the impulse noise effectively. However the sign error gradient descent algorithm carries signum operation only on the error $e(n)$, the effect of impulse noise on the received signal $y(n)$ and the output signal $\tilde{x}(n)$ still hinder the robust convergence performance. Hereby we carry the signum operation on the error $e(n)$, the received signal $y(n)$ and the output signal $\tilde{x}(n)$ at the same time, a new algorithm called sign gradient algorithm is obtained. The sign gradient is given by:

$$\nabla J_{SCMA}(n) = -sign(e(n))sign(\tilde{x}(n))sign(y^{*}(n)) \tag{12}$$

Where $sign(.)$ denotes signum operation and for the complex x the signumoperation is given by:

$$sign(x) = sign(real(x)) + j * sign(imag(x)) \tag{13}$$

Then the blind equalizer weights updating formula of the sign gradient CMA is given by:

$$w(n+1) = w(n) + \mu\nabla J_{SCMA}(n) \tag{14}$$

2.4. The Variable Step Size Algorithm

Sign gradient CMA can obtain robust convergence performance under the impulse noise environment. However, the signum operation causes the quantization error for calculating the gradient which results in large steady state residual error. Although the excess mean square error of CMA still has no effective quantitative analysis method [14]. the step size can control the excess mean square error after the algorithm convergence [15]. As long as the step size is small enough, the excess mean square error can be reduced to a desired degree.

The step size controls the convergence rate and the convergence precision [16] and variable step size algorithm is in a compromise between them [17]. The basic principle of variable step size algorithm is that faster convergence rate is obtained with a big step size at the initial stage, and the step size decreases gradually along with the iteration process to obtain higher convergence precision. Therefore, variable step size algorithm can reduce the excess mean square error, as a result, the variable step size sign gradient CMA can obtain better performance under impulse noise environment. The blind equalizer updating formula of variable step size sign gradient CMA is given by:

$$w(n+1) = w(n) + \mu\phi(n)\nabla J_{SCMA}(n) \tag{15}$$

Where $\phi(n)$ is the step size gain control function and it meets the condition $0 < \phi(n) \leq 1$, which controls the change scale of the step size in the iterative process. The ideal step size gain control function should meet the conditions that $\phi(n) \to 1$ at the initial and $\phi(n) \to 0$ [18] after convergence. In the most variable step size algorithm, the step size gain control function is set according to a nonlinear transform of the output error of the blind equalizer and some parameters need to be set. For there is limited received data samples at one iterative time, to obtain the accurate successive estimation error and the other information of the output with the channel noise interference is very difficult. So the parameters setting of the nonlinear transform are not universal. We hereby proposed a new variable step size algorithm for sign gradient CMA. In this algorithm, the step size gain control function $\phi(n)$ does not rely on the information of the output signal or output error, and also it needs no man-made setting parameters. The step size gain control function is given by:

$$\phi(n) = 1 - \rho(2/(1+\exp(\eta)) - 1) \tag{16}$$

Where ρ is the order of magnitude of the maximum step size which can be calculated according to the input signal of the blind equalizer as:

$$\mu_{\max} = 1/\lambda_{\max} \tag{17}$$

Where $\lambda_{\max}$ is the maximum eigenvalue of the autocorrelation matrix of the input signal. η is the ratio of the number of the sign consistency and the iterative times, and the sign consistency refers to that the error sign based on CMA and decision decided (DD) algorithm is same. The cost function of DD algorithm is given by:

$$J_{DD}(n) = \frac{1}{2}(sign(\tilde{x}(n)) - \tilde{x}(n))^2 \tag{18}$$

Let the error function of DD algorithm is:

$$e_D(n) = sign(\tilde{x}(n)) - \tilde{x}(n) \tag{19}$$

If the blind equalizer obtains rightconvergence, the sign of the error $e_D(n)$ according to DD algorithm and the error $e(n)$ according to CMA will be the same. Therefore, we can use the sign consistency to judge the constellation open state. Based on this idea, η can be calculated by:

$$\eta = k/N \tag{20}$$

Where N is the iterative times and k is calculated by:

$$\begin{cases} k = k+1 & if \quad sign(e(n)) = sign(e_{DD}(n)) \\ k = k & if \quad sign(e(n)) \neq sign(e_{DD}(n)) \end{cases} \tag{21}$$

Thus, the new variable step size algorithm is given by:

$$\mu = \begin{cases} \mu_{\max} & sign(e_{DD}(n)) \neq sign(e(n)) \\ \mu\phi(n) & else \end{cases} \tag{22}$$

According to Equation (22), the step size is diminished when the error sign of the CMA and DD algorithm is same, otherwise, the blind equalizer weights is updated with the maximum step size. CMA blind equalization based on sign gradient algorithm with the variable step size method can obtain faster convergence rate at the initial stage and further lower the excess mean square error. Meanwhile, the parameters ρ and η of the step size gain control function both can be calculated in the program without man-made setting.

3. Simulations and Discussion

In the simulations, equivalent probability binary sequence is adopted to act as sending signal and QPSK modulation is utilized. The typical shallow sea and deep sea underwater acoustic channel with impulse noise interference are used in our simulations. The channel models have verified by the sea experiment. For the shallow sea channel model, the parameters set as follow: the carrier frequency is 10kHz, the channel bandwidth is 2kHz, the transmit baud rate is 1000 *symbol/s*, the wind speed is 20kn, the sender and the receiver are located in underwater 10 meters and the distance is 5000 meters. For the deep sea channel model, the parameters set as follow: the depth of sea is 5000 meters, the sound source is located in 1000 meters underwater, the receiver is located in the 900 meters underwater, the distance between the sound source and the receiver is 56 kilometers, the carrier frequency is 1kHz, the transmit baud rate is 100*symbol/s*. The parameters of eight rays of the channel models [19] can be shown as Table 1.

Table 1. The Parameters of Eight Rays of the Channel Models

Ray number	Shallow sea channel		Deep sea channel	
	Time delay *t/ms*	Amplitude	Time delay *t/ms*	Amplitude
1	0.000	1.0000	0.0000000	0.4954
2	0.026	-1.0000	0.0265385	-0.1464
3	0.026	-0.3286	0.0319367	0.5079
4	0.100	0.3286	0.0647739	-0.1555
5	0.100	0.3286	0.2056037	0.8399
6	0.240	-0.3286	0.2320864	1.0000
7	0.420	-0.1080	0.2359591	0.6914
8	0.420	0.1080	0.3671784	0.2187

The length of the blind equalizer *N*=21 for the shallow sea channel and *N*=34 for the deep sea channel.In order to verify the performance of the variable step size blind equalization based on sign gradient algorithm (VSSG-CMA) proposed in this paper, the fractional low order CMA (FL-CMA), the nonlinear transform CMA (NT-CMA) and stop-and-go CMA (SAG-CMA) are done in the simulations for comparison. The comparison is in terms of the residual inter-symbol interference (ISI) [20] which is given by:

$$ISI = \frac{\sum_i |C_i|^2 - |C_i|^2_{\max}}{|C_i|^2_{\max}} \tag{23}$$

Where*C* is the combined impulse response of the channel and the equalizer.

Because the α-stationary distribution with the characteristic exponent α has no statistical moments above α order, the signal-to-noise (*SNR*) defined based on the two order statistics cannot describe the degree of the impulse noise interference. Therefore, the generalized *SNR* (*GSNR*) which can measure the impulse noise in the signal [21] is defined as:

$$GSNR = 10\lg |x(n)|^2 / \gamma \tag{24}$$

Where γ is the dispersion coefficient of the α-stationary distribution impulse noise and $|x(n)|^2$ is the signal energy. Figure 2 and Figure 3 show the *ISI* comparison results under the shallow sea channel and the deep sea channel with*GSNR*=15dB respectively. The step size is set to $\mu = 0.0015$ in the shallow sea channel and $\mu = 0.0024$ in the deep sea channel simulation. From Figure 2 and Figure 3 can see that VSSG-CMA has the fastest convergence rate and the lowest steady state residual error, which proved the effectiveness of VSSG-CMA blind equalization under the impulse noise environment.

In order to further prove the effectiveness of VSSG-CMA blind equalization, the steady state residual errors of VSSG-CMA, FL-CMA, NT-CMA and SAG-CMA are compared by 500 times Monte Carlo simulation under different *GSNR* conditions and the results are shown in Table 2. From Table 2 can see that, VSSG-CMA blind equalization has the lowest steady state residual error in the four blind equalization algorithms.

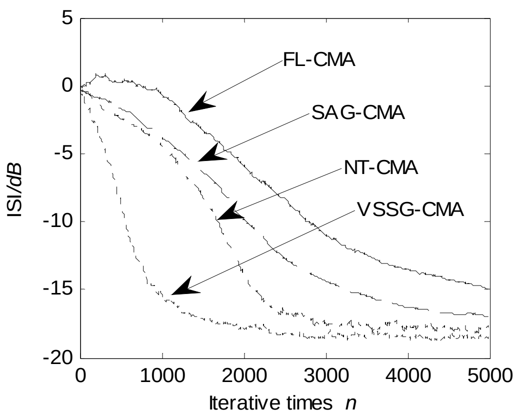

Figure 2. *ISI* in Shallow Sea Channel

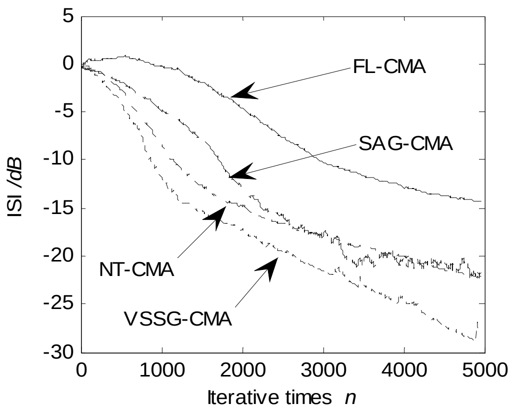

Figure 3. *ISI* in Deep Sea Channel

Table 2. The Comparision of Steady State Residual Errors (dB)

Channel	GSNR(dB) Algorithm	0	5	10	15	20	25
The shallow sea channel	FL-CMA	-8.5	-9.7	-10.3	-13.2	-15.6	-18.3
	SAG-CMA	-8.8	-12.2	-14.5	-16.1	-18.4	-19.6
	NT-CMA	-9.4	-13.5	-15.2	-17.3	-18.9	-21.2
	VSSG-CMA	-10.6	-16.8	-17.4	-18.0	-22.6	-28.6
The deep sea channel	FL-CMA	-9.0	-10.4	-12.2	-14.2	-16.4	-18.8
	SAG-CMA	-9.8	-12.6	-16.5	-21.2	-24.5	-24.5
	NT-CMA	-11.2	-12.8	-17.2	-21.3	-24.6	-24.8
	VSSG-CMA	-14.8	-18.5	-24.9	-28.4	-32.8	-36.5

4. Conclusion

In this work, we proposed a sign gradient algorithm for CMA to solve the problem of blind equalization under impulse noise environment. The signum operation on the iterative gradient can suppress the impulse noise effectively, which ensures the blind equalization algorithm to obtain robust convergence performance. Furthermore, a variable step size algorithm is designed according to the iterative times and the reliability of the output signal without man-made parameters setting to improve the performance of sign gradient algorithm. The simulation results show the effectiveness of the variable step size sign gradient CMA blind equalization under the impulse noise environment.

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