

The research on the signal source number estimation algorithm

Wang Peizhi, Raihani Mohamed, Norwati Mustapha, Noridayu Manshor

Faculty of Computer Science and Information Technology, University Putra Malaysia, Serdang, Malaysia

Article Info

Article history:

Received Feb 1, 2024

Revised May 22, 2024

Accepted Jun 5, 2024

Keywords:

Snapshot number

Signal-to-noise ratio

Source number estimation array

Signal processing

Uniform circular array

ULA

ABSTRACT

In array signal processing, Estimating the quantity of signal sources represents a crucial area of investigation. In this paper, a comprehensive introduction and analysis of the estimation methods for determining the number of signal sources are presented, including the background and significance, and the significance of precise estimation of the quantity of signal sources. The influence of factors such as signal-to-noise ratio (SNR), noise background, and number of snapshots on the estimation algorithm is discussed in detail. At the same time, common array models are introduced. Then, different signal source number estimation algorithms are analyzed in detail, and their respective advantages and applicable conditions are pointed out. Finally, the performance of each algorithm in different situations is evaluated by comparing the performance of the algorithms under different SNRs, snapshot numbers, and array elements. The experimental results show that with the increase of the SNR and the number of array elements, the correct estimation probability of the algorithm also increases correspondingly, which provides a reliable experimental basis and performance evaluation for the estimation.

This is an open access article under the [CC BY-SA](#) license.



Corresponding Author:

Wang Peizhi

Faculty of Computer Science and Information Technology, University Putra Malaysia

Serdang 43400, Selangor, Malaysia

Email: wpzupm@163.com

1. INTRODUCTION

Array signal processing is an advanced technique that utilizes multiple received signals for processing and analysis [1]. It has been widely used in many fields such as wireless communication [2], radar [3], sound processing [4] and biomedical engineering [5]. Its main goal is to extract interesting information from signals received by multiple sensors, such as accurate positioning of targets [6], effective separation of sound sources [7], and obvious enhancement of signals [8]. Among them, the estimation of direction of arrival (DOA) has emerged as a focal point in the realm of signal processing, which is used to determine the specific direction or angle of the signal source in space. DOA estimation methods rely on signal processing and beam analysis of signals received by sensor arrays to infer the exact direction of the signal source. Before performing DOA estimation, it is usually necessary to accurately estimate the number of signal sources first, because DOA estimation methods usually rely on the number and location information of signal sources [9]-[15]. Accurately estimating the number of signal sources is critical to achieving accurate DOA estimation.

In the early stages, the estimation relied on empirical rules and heuristic algorithms [16]. These methods often depended on manually selecting and empirically adjusting signal features, lacking theoretical foundations and guarantees of statistical performance. With the development of statistical signal processing theory, researchers began adopting statistical models and estimation criteria to address the challenge of estimating the quantity of signal sources. Classical methods include maximum likelihood estimation [17], Bayesian information criterion [18], and akaike information criterion (AIC) [19]. These methods aim to

select the optimal number of signal sources by maximizing the likelihood function or minimizing the information criterion. Adaptive methods for determining the quantity of signal sources represent further advancements over static statistical methods. These methods leverage adaptive algorithms and model selection techniques to determine the quantity of signal sources based on the dynamic variations in the signal environment. Common adaptive methods include recursive methods based on information criteria [20] and model selection methods based on Bayesian approaches [21]. In recent years, significant progress has been made in the field of determining the quantity of signal sources using sparse representation-based methods. These methods exploit the sparsity assumption of signals, transforming the problem of determining the quantity of signal sources into a model selection problem based on sparse representation [22], [23]. By employing sparse representation algorithms, the quantity of signal sources can be effectively estimated. Additionally, deep learning methods have also found wide applications in the estimation of signal source quantity. These methods train neural networks to learn models for determining the quantity of signal sources, demonstrating high adaptability and generalization capabilities [24].

The following is the structure of this paper: Section 2 analyzes the factors that affect the determining of the quantity of sources. Section 3 introduces commonly used array models. Section 4 introduces different types of signal source number estimation algorithms, and section 5 performs simulation analysis on commonly used algorithms, section 6 summarizes the full paper.

2. THE FACTORS AFFECTING SOURCE NUMBER ESTIMATION

2.1. Signal to noise ration (SNR)

The SNR denotes the relationship between the power of the signal and the power of the noise, which is used to evaluate the relative relationship between signal strength and noise level [25], [26]. The SNR has an important impact on estimation algorithm, mainly in the following aspects: i) A higher SNR improves the resolving power of signal sources. The goal of estimation algorithm is to distinguish different sources. A higher SNR makes the signal more prominent in the background noise, facilitating identification and separation. ii) A higher SNR generally improves the estimated performance of the algorithm. Under high SNR conditions. There is a significant power difference between signal and noise. Estimation algorithms can more easily differentiate the signal from the noise, reducing estimation errors and improving accuracy. iii) A higher SNR yields more reliable signal strength estimates. In low SNR scenarios, noise has a greater impact on signal strength estimation, resulting in less accurate estimation results. iv) A higher SNR enhances the robustness of estimation algorithms. In low SNR conditions, noise has a greater influence, potentially causing estimation bias or instability. A higher SNR mitigates noise interference, thereby improving algorithm stability and robustness. A higher SNR is beneficial for the estimation of signal source numbers, as it improves resolving power, reduces estimation errors, and enhances algorithm robustness. Therefore, in practical applications, measures are often taken to increase signal strength and reduce noise influence, ultimately improving the SNR for accurate and reliable signal source number estimation results.

2.2. The noise background

The presence of noise in the background has a significant impact on the estimation algorithm. It interferes with the detection and separation of signals, thereby increasing the difficulty of accurately determining the quantity of signal sources [27], [28]. The effects of noise background on source number estimation can be summarized as follows: i) A strong background noise can mask low-amplitude signal sources, making them challenging to detect within the noise. This masking effect introduces errors in estimating the number of sources, potentially leading to underestimation or overestimation. ii) Noise interferes with signal sources, resulting in blurred boundaries and overlapping signals. This interference makes it more challenging to separate and locate individual signal sources accurately, consequently affecting the accuracy of source number estimation. iii) Noisy backgrounds can lead to false signal detections, where noise is incorrectly identified as a signal source. This can lead to overestimation of the number of signal sources and generate false alarms. Additionally, the fluctuation of noise levels may result in missed detections, leading to an underestimation of the true number of signal sources.

To mitigate the impact of noise background on source number estimation, the following approaches can be considered: i) Employing signal processing techniques such as filtering and denoising can reduce the impact of noise, enhancing the detectability of signal sources. ii) Utilizing frequency analysis and correlation analysis enables differentiation between signal and noise characteristics, aiding in the localization and separation of signal sources. iii) Leveraging statistical methods and models, such as maximum likelihood estimation and Bayesian inference, can improve the accuracy of estimating the number of signal sources while considering the influence of noise background. iv) Integrating information from different sensors or data sources through data fusion methods can enhance the accuracy and robustness of source number estimation.

2.3. The snapshot number

The number of snapshots refers to the number of signal samples observed in the source number estimation. The number of snapshots has a certain influence on the estimation of the number of sources, especially when using the estimation algorithm of the number of sources based on statistical methods [29], [30]. The specific impact is as follows: i) Accuracy and reliability: A larger number of snapshots generally provides more information for source number estimation, thereby improving the accuracy and reliability of the estimation. More observation samples help to reduce the estimation error and the influence of random noise. ii) Resolution ability: A larger number of snapshots can improve the resolution ability of source number estimation, that is, it can more accurately distinguish different numbers of signal sources. As the number of snapshots increases, the estimation algorithm can better distinguish between signal sources and noise, and thus more accurately determine the number of sources. iii) Computational complexity: The increase in the number of snapshots may lead to an increase in the computational complexity of the source number estimation algorithm. Some algorithms need to process and analyze a large amount of observation data, which may require longer computing time and larger computing resources.

It should be noted that the number of snapshots is not the bigger the better. When the number of snapshots is too small, the accuracy and reliability of source number estimation will be limited, and the estimation result may be less accurate. However, when the number of snapshots is too large, the problem of overfitting may be introduced, causing the estimated results to deviate from the actual situation. Therefore, when choosing the number of snapshots, there is a need to balance considerations between accuracy, computational complexity, and application requirements.

3. THE ARRAY MODEL

Array models have their own advantages and applicability in different application scenarios and signal processing tasks. Selecting an appropriate array model needs to consider specific application requirements, scenario characteristics, and performance requirements. In addition, array configuration, antenna selection, and signal processing algorithm design can also be carried out on a case-by-case basis to achieve the required signal processing functions and performance. Table 1 compares and introduces commonly used array models.

Table 1. The comparison of commonly used array models

Array model	Angle estimate range (azimuth α , elevation β)	Characteristic	Application
ULA [31]	$\alpha \in 0, \pi$ $\beta \in 0, \frac{\pi}{2}$	(1) It has direction selectivity. (2) It can provide sampling of the signal in space. (3) The geometric structure is numerically stable.	(1) Wireless communication system (2) Radar system (3) Sound processing (4) Astronomical research
UCA [32]	$\alpha \in 0, 2\pi$ $\beta \in 0, \frac{\pi}{2}$	(1) omnidirectional (2) Rotation invariance (3) High resolution	(1) Satellite communication system (2) UAV navigation (3) Earthquake monitoring (4) Target tracking
L-shaped array [33]	$\alpha \in 0, 2\pi$ $\beta \in 0, \frac{\pi}{2}$	(1) Directional selectivity (2) It can provide sampling of signals in two-dimensional space. (3) It has smaller physical size and higher integration.	(1) Wireless communication system (2) Radar system (3) Unmanned driving and robot navigation (4) Sound processing
Planar array [34]	$\alpha \in 0, 2\pi$ $\beta \in 0, \frac{\pi}{2}$	(1) Spatial sampling (2) Directional selectivity (3) Flexibility	(1) Communication system (2) Radar system (3) Astronomical research (4) Multi-sensor system

3.1. The ULA models

The ULA containing M array elements, assuming that the distance between each array element is d , L far-field narrowband signals are incident on the ULA with the distance between array elements d at the incident angle θ_i . Taking the first array element as a reference, the steering vector of this signal is:

$$\begin{aligned} \mathbf{a}(\theta_i) &= [e^{-jw_i\tau_1(\theta_i)}, e^{-jw_i\tau_2(\theta_i)}, \dots, e^{-jw_i\tau_M(\theta_i)}] \\ &= [1, e^{-j\frac{2\pi}{\lambda_i}d \sin \theta_i}, \dots, e^{-j\frac{2\pi}{\lambda_i}(M-1)d \sin \theta_i}] \end{aligned} \quad (1)$$

$$\tau_k(\theta_i) = \frac{d}{c}(k - 1) \sin(\theta_i), k = 1, 2, \dots, M. \tag{2}$$

where τ_k represents the delay between the k th array element and the first array element. Then for the incident L signal sources, the array manifold is:

$$\mathbf{A} = [\mathbf{a}(\theta_1) \quad \mathbf{a}(\theta_2) \quad \dots \quad \mathbf{a}(\theta_L)]_{M \times L} \tag{3}$$

the signal matrix received by ULA is:

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t) \tag{4}$$

where $\mathbf{S}(t) = [\mathbf{s}_1(t), \mathbf{s}_2(t), \dots, \mathbf{s}_L(t)]^T$ is a matrix composed of radiation sources, \mathbf{A} is the array manifold, and $\mathbf{N}(t)$ is a noise matrix. Due to its numerous advantageous characteristics, the ULA is well-suited for a wide range of algorithms.

3.2. The uniform circular array (UCA) model

The UCA model composed of M array elements. Assume that there are K far-field narrowband signals incident on the UCA model with radius r . ϕ is the azimuth angle, and θ is the elevation angle. Then the steering vector of the i th signal source can be expressed as:

$$\mathbf{a}(\theta_i, \phi_i) = [e^{j\phi_{i,1}}, e^{j\phi_{i,2}}, \dots, e^{j\phi_{i,M}}]^T \tag{5}$$

$$\phi_{i,m} = \frac{2\pi r \sin(\theta_i) \cos(\phi_i - \frac{2\pi(m-1)}{M})}{\lambda}, (m = 1, 2, \dots, M) \tag{6}$$

where λ is the carrier wavelength. Then the mathematical model of the array signal of the t th snapshot on the UCA with M array elements is:

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{K}(t) \tag{7}$$

where $\mathbf{X}(t) = [\mathbf{X}_1(\mathbf{t}), \mathbf{X}_2(\mathbf{t}), \dots, \mathbf{X}_M(\mathbf{t})]^T$ is the UCA output with M array elements, $\mathbf{S}(t)$ is the signal source vector, $\mathbf{K}(t)$ is the additive noise, and the array manifold is:

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1), \mathbf{a}(\theta_2, \phi_2), \dots, \mathbf{a}(\theta_k, \phi_k)] \tag{8}$$

4. THE ANALYSIS OF SIGNAL SOURCE NUMBER ESTIMATION ALGORITHM

In signal processing, spatial spectrum estimation is a key problem closely related to the multi-signal environment. However, most spatial spectrum estimation algorithms require advanced knowledge of the signal source quantity is necessary beforehand [35]. The following is an introduction and analysis of various types of signal source number estimation algorithms.

4.1. The estimation methods based on eigenvalues

4.1.1. The information theory method

In information theory, two commonly used methods are the AIC [36] and the minimum description length criterion (MDL) [37]. AIC is a model selection criterion based on the maximum likelihood criterion. It selects the optimal number of signal sources by weighing the fitting ability and complexity of the model. AIC provides a reliable method for estimating the number of signal sources based on the balance between the fitting error of the model and the number of parameters. MDL is a criterion based on coding theory, which transforms the problem of signal source number estimation into a data compression problem. The MDL method selects the model that can minimize the length describing the signal as the optimal model, because less information length indicates better compression of the data. By using MDL, more accurate and effective estimation of the number of signal sources can be obtained. AIC and MDL have different strengths and applicability in signal source number estimation. AIC generally performs well with large sample sizes and simple models, while MDL is more effective with small sample sizes and complex models. Therefore, in practical applications, choosing an appropriate method depends on the characteristics of the data, the complexity of the problem, and the consideration of computational complexity.

4.1.2. The smoothing rank-order method

The smoothing rank-order method is a statistical technique used to handle outliers or extreme observations in ordered data [38]. Its primary purpose is to obtain a smoother distribution of data by mitigating the impact of outliers. The method involves sorting the observed values based on their magnitude and then adjusting the sorted data to minimize the influence of outliers. In particular, the method replaces each observation with its percentile in the sorted data, effectively eliminating the effect of extreme values. Nason and Silverman [39] discusses an important tool in the smoothing order method: the stationary wavelet transform (SWT), and its application in statistics. The principle and properties of SWT are introduced, and how it is applied to statistical applications such as data smoothing and trend estimation is discussed. This paper is of great value for understanding the wavelet method in the smoothing order method and its application in practical problems. Ramsay [40] presents the method and application of locally weighted regression, which can also be viewed as a smoothing technique. This paper proposes a robust locally weighted regression algorithm for smoothing scatterplots and fitting nonlinear relationships.

4.1.3. The Gerschgorin disk estimation (GDE) method

The GDE method is used to estimate the range of eigenvalues of a matrix. It is based on Gerschgorin's theorem [41], which states that each eigenvalue of a complex matrix lies within a circle in the complex plane. The GDE method calculates a series of disks, where each disk's center corresponds to a diagonal element of the matrix, and its radius is determined by the sum of the absolute values of the diagonal element and other off-diagonal elements. These disks cover the possible range of all eigenvalues of the matrix. By examining the coverage of these disks, the range of eigenvalues can be estimated. If all disks are fully contained within the real or complex plane, the exact range of eigenvalues can be obtained. If some disks intersect the boundary of the plane, the range of eigenvalues will fall within the corresponding region. The GDE method is a simple and effective approach for preliminary eigenvalue estimation, eigenvalue condition number estimation, and analysis of matrix eigenvalue distribution. However, it is important to note that the GDE method provides an estimate of the range of eigenvalues and does not yield specific eigenvalues. For precise eigenvalue calculations, more advanced numerical methods are required. Later, researchers made improvements to the GDE method. One improvement is based on the standard Gerschgorin disk theorem [42], where the matrix elements are appropriately scaled and offset, resulting in Gerschgorin disks that more accurately contain the eigenvalues of the matrix. The validity and accuracy of this improved method have been demonstrated through theoretical analysis and numerical experiments. Wu and Chen [43] proposed an enhanced Gerschgorin disk estimation method specifically designed for estimating eigenvalues of Hermitian matrices. This method improves the accuracy of eigenvalue estimation by taking into account the intersecting relationship between disks and employing a more precise radius estimation technique. The enhanced method has been validated in numerical linear algebra applications and yields more accurate eigenvalue estimation results.

4.2. The estimation methods based on eigenvectors

4.2.1. The estimation method based on matrix factorization

Matrix decomposition is a widely employed technique in estimating the quantity of sources. It decomposes the matrix of observed data and uses the decomposed structural information to infer the number of sources. Singular value decomposition (SVD) [25] is a commonly used matrix decomposition method, which decomposes the observation data matrix into the product of three matrices. By analyzing the distribution of singular values, the number of sources can be estimated. When there are large singular values, it may indicate that the number of sources is large. Principal component analysis (PCA) [44] is a commonly used data dimensionality reduction technique, which extracts the principal components by decomposing the eigenvectors of the observed data. In the estimation of the number of information sources, the number of principal components extracted by PCA can be used to infer the number of information sources. Factor Analysis is a statistical model used to infer the underlying factor structure underlying observed data [45]. The number of factors in the factor analysis model to be able to infer the number of signal sources. Independent component analysis (ICA) [46] can be used to solve the problem of blind source separation, which assumes that the observation data is linearly mixed from multiple independent signal sources. In the estimation of the number of sources, the number of independent components estimated by ICA can be used to infer the number of sources.

4.2.2. Signal parameter estimation using rotational invariance techniques (ESPRIT)

The ESPRIT is a widely used method for signal parameter estimation. It is particularly suitable for estimating signals with a linear structure, such as sinusoidal signals or chirped signals. The ESPRIT method exploits the principle of rotation invariance of the signal subspace to estimate various parameters of the

signal, including frequency and incident angle. The ESPRIT method has been extensively studied and applied in various fields. For example, in the domain of array signal processing, ESPRIT has been employed for DOA estimation in sensor arrays [47]. Additionally, in the context of radar systems, ESPRIT has been employed for target localization and tracking [48]. The effectiveness and accuracy of the ESPRIT method have been validated through theoretical analysis and practical experiments. Later scholars improved the ESPRIT method. Zhang *et al.* [49] proposed an enhanced ESPRIT-like algorithm that addresses the issue of misestimation in the presence of coherent signals. The traditional ESPRIT algorithm is known to have limitations when coherent signals are present, leading to inaccurate estimations. To overcome this challenge, the paper introduces an improved algorithm that leverages the direction information of the known signal and employs an iterative optimization approach to estimate the signal subspace. The enhanced algorithm incorporates new calculation steps and an iterative process, which contribute to improved accuracy and robustness in estimating the DOA for coherent signals. By utilizing the known signal's direction information and refining the estimation through iterative optimization, the algorithm effectively mitigates the misestimation problem. In this study, an improved ESPRIT algorithm is proposed for the case where the source signals have close spacing [50]. By introducing spatial smoothing techniques and optimized estimation of signal subspace, the method shows better performance in high-resolution DOA estimation.

5. EXPERIMENT AND ANALYSIS

This paper uses Gaussian white noise background to conduct simulation experiments, and evaluates the performance of AIC, MDL and GDE under different SNRs, snapshot numbers and array element numbers. The experimental setup utilized for this study consisted of an Intel(R) Core(TM) i7-10700H CPU @ 2.60 GHz, 16.00 GB RAM, and a 1 TB solid-state drive. The software environment is Windows 10 operating system and MATLAB 2021b.

In the experiment, an 8-element UCA is employed to receive a signal comprising three far-field narrowband uncoherent sources. The radius of the signal sources is twice that of the UCA. The study investigates the performance of three algorithms, namely AIC, MDL, and GDE, under various conditions of SNR, snapshot number (L), and array elements (Figure 1). Each experiment is repeated 100 times using Monte Carlo simulations to ensure accurate results.

5.1. The algorithm comparison under different SNRs

In the conducted experiment, the number of snapshots (L) is set to 500, and the SNR is gradually increased from -20 dB to 20 dB in 1 dB steps. The Figure 1(a) shows the correct detection results of the algorithms AIC, MDL and GDE. Figure 1(a) clearly illustrates in the Gaussian white noise environment, AIC, MDL, and GDE can effectively estimate the number of sources, and as the SNR increases, the correct estimation probability of the three algorithms also increases. Since AIC does not have consistent estimability, but MDL and GDE have consistent estimability, so as the SNR continues to increase, the correct estimation rate of AIC can never reach 1. As the SNR decreases, the correct detection probability of the three methods decrease rapidly until reduced to 0.

5.2. The algorithm comparison under the different numbers of snapshots

In the experiment, the SNR is set to -5 dB, and the number of snapshots (L) is gradually increased from 0 to 1,000 in steps of 50. The Figure 1(b) shows the correct detection results of the algorithms AIC, MDL and GDE. According to Figure 1(b), the following observations can be drawn: With a growing number of snapshots, the amount of information carried by the array signal increases, leading to an increase in the overall correct detection probability of the three algorithms. However, due to the lack of consistent estimation properties of the AIC algorithm, the correct detection probability of the AIC algorithm cannot reach 100% compared with the MDL and GDE algorithms when the number of snapshots reaches a sufficient magnitude. However, it is worth noting that the AIC algorithm exhibits better estimation performance than the MDL and GDE algorithms at a smaller number of snapshots.

5.3. The algorithm comparison under the different numbers of array elements

In the experiment, the number of array elements of UCA is set, from 4 to 15 in steps of 1. The received signal contains five far-field narrowband uncoherent sources. The radius of the UCA is set to be twice the wavelength of the signal source. Gaussian white noise is used as the noise background, and the SNR is set to -5dB. In each experiment, the Monte Carlo experiment was repeated 100 times. The correct detection results of the algorithms AIC, MDL, and GDE.

Through the detailed analysis of Figure 1(c), we can draw the following conclusions: as the number of array elements increases, the overall correct detection probability of the three algorithms also shows an increasing trend. This means that as the number of array elements increases, the detection ability of the

algorithm for signal sources is also improved. However, when the number of array elements is not greater than 5, the correct detection probability of the three algorithms are all 0. This is because the number of signal sources is 5, and the number of array elements is too small compared with the number of signal sources, resulting in insufficient degrees of freedom of the system, which cannot effectively separate the signal sources. As the degree of freedom of the system increases, that is, the number of array elements increases, the probability of correct detection increases gradually. When the number of array elements is 9, the correct detection probability of the GDE algorithm reaches more than 90%, the correct detection probability of the AIC algorithm reaches 97%, and the correct detection probability of the MDL algorithm reaches 99%. This indicates that as the number of array elements increases, the detection ability of the algorithm for the signal source is gradually improved. However, as the number of array elements continues to increase, the correct detection probability of the AIC algorithm cannot always reach 1 due to the lack of estimation consistency. Although the AIC algorithm can provide reliable signal source detection results to a certain extent, its accuracy is still limited. Compared with this, the GDE algorithm loses at least one degree of freedom when estimating the number of sources, that is, it uses at least one array element less than the MDL criterion. Therefore, the correct detection probability of the GDE algorithm is always not greater than that of the MDL algorithm. Although the GDE algorithm can provide accurate signal source detection results to a certain extent, its performance is still limited by itself.

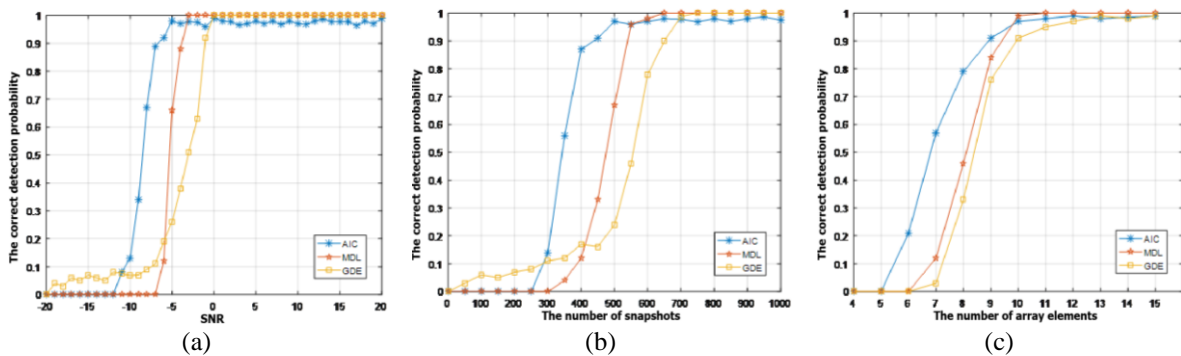


Figure 1. The performance comparison of three algorithms; (a) based on different SNR, (b) based on different snapshot, and (c) based on different elements

6. SUMMARY

The subject of this paper is the research on the estimation algorithm of the number of signal sources. Based on the analysis of different factors and the discussion of different array models, the estimation algorithm of the number of signal sources is analyzed and summarized. First, the background and significance of the research are introduced, as well as the purpose of the research on the signal source number estimation algorithm. Next, three important factors are discussed in detail: SNR, noise background and number of snapshots. These factors have an important impact on the accuracy and stability of the signal source number estimation algorithm. Then, two common array models are introduced: ULA model and UCA model. These models provide the basis for the analysis and experiment of the signal source number estimation algorithm. Eigenvalue-based and eigenvector-based estimation methods are introduced in detail. Among them, the methods based on eigenvalues include information theory method, smoothing rank-order Method and Gerschgorin disk estimation method; the methods based on eigenvectors include smoothing rank-order method and estimating signal parameters by rotation invariance technique. These methods have different advantages and disadvantages in the estimation of the number of signal sources, and the appropriate method can be selected according to the specific application scenario. Finally, in the part of experiment and analysis, the algorithm's performance is evaluated and analyzed through the experimental comparison of different SNR, different number of snapshots and different numbers of array elements. Based on the experimental findings, the comparison results of the algorithms under different factors can be obtained, which provides a basis for choosing the appropriate algorithm.

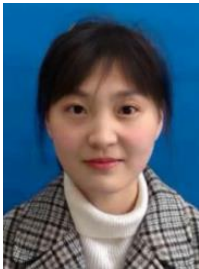
REFERENCES




- [1] M. Arrays, "Signal processing techniques and applications," *Microphone Arrays: Signal Processing Techniques and Applications*. Springer-Verlag, 2021.

- [2] P. S. Naidu, *Sensor array signal processing, second edition*. CRC Press, 2009.
- [3] A. Lee Swindlehurst and P. Stoica, "Maximum likelihood methods in radar array signal processing," *Proceedings of the IEEE*, vol. 86, no. 2, pp. 421–441, 1998, doi: 10.1109/5.659495.
- [4] W. Herboldt and SpringerLink, *Sound capture for human machine interfaces*, vol. 315. Springer Berlin Heidelberg, 2005.
- [5] B. Liao, A. Madanayake, and P. Agathoklis, "Array signal processing and systems," *Multidimensional Systems and Signal Processing*, vol. 29, no. 2, pp. 467–473, Feb. 2018, doi: 10.1007/s11045-018-0555-7.
- [6] S. Ge, K. Li, and S. N. B. M. Rum, "Deep learning approach in DOA estimation: a systematic literature review," *Mobile Information Systems*, vol. 2021, pp. 1–14, Sep. 2021, doi: 10.1155/2021/6392875.
- [7] H. Saruwatari, T. Kawamura, T. Nishikawa, and K. Shikano, "Fast-convergence algorithm for blind source separation based on array signal processing," in *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, 2003, vol. E86-A, no. 3, pp. 634–639, doi: 10.1109/ssp.2001.955323.
- [8] J. Benesty, I. Cohen, and J. Chen, *Fundamentals of signal enhancement and array signal processing*. Wiley, 2017.
- [9] P. Vallet, X. Mestre, and P. Loubaton, "Performance analysis of an improved MUSIC DoA estimator," *IEEE Transactions on Signal Processing*, vol. 63, no. 23, pp. 6407–6422, Dec. 2015, doi: 10.1109/TSP.2015.2465302.
- [10] X. H. Cao, H. W. Liu, and S. J. Wu, "DOA estimation based on online music algorithm," *Dianzi Yu Xinxi Xuebao/Journal of Electronics and Information Technology*, vol. 30, no. 11, pp. 2658–2661, Apr. 2008, doi: 10.3724/sp.j.1146.2007.00565.
- [11] B. Tang, J. Tang, Y. Zhang, and Z. Zheng, "Maximum likelihood estimation of DOD and DOA for bistatic MIMO radar," *Signal Processing*, vol. 93, no. 5, pp. 1349–1357, May 2013, doi: 10.1016/j.sigpro.2012.11.011.
- [12] F. Yan, L. Jin, and X. Qiao, "Low-complexity DOA estimation based on compressed MUSIC and its performance analysis," *IEEE Transactions on Signal Processing*, vol. 61, no. 8, pp. 1915–1930, Apr. 2013, doi: 10.1109/TSP.2013.2243442.
- [13] F. M. Han and X. Da Zhang, "An ESPRIT-like algorithm for coherent DOA estimation," *IEEE Antennas and Wireless Propagation Letters*, vol. 4, no. 1, pp. 443–446, 2005, doi: 10.1109/LAWP.2005.860194.
- [14] Y. Wu, L. Amir, J. R. Jensen, and G. Liao, "Joint pitch and DOA estimation using the ESPRIT method," *IEEE/ACM Transactions on Audio Speech and Language Processing*, vol. 23, no. 1, pp. 32–45, Jan. 2015, doi: 10.1109/TASLP.2014.2367817.
- [15] J. Y. Lee, R. E. Hudson, and K. Yao, "Acoustic DOA estimation: an approximate maximum likelihood approach," *IEEE Systems Journal*, vol. 8, no. 1, pp. 131–141, Mar. 2014, doi: 10.1109/JSYST.2013.2260630.
- [16] H. L. Van Trees, *Detection, estimation, and modulation theory, Part I*. John Wiley & Sons, Inc., 2001.
- [17] I. J. Myung, "Tutorial on maximum likelihood estimation," *Journal of Mathematical Psychology*, vol. 47, no. 1, pp. 90–100, Feb. 2003, doi: 10.1016/S0022-2496(02)00028-7.
- [18] A. A. Neath and J. E. Cavanaugh, "The Bayesian information criterion: background, derivation, and applications," *Wiley Interdisciplinary Reviews: Computational Statistics*, vol. 4, no. 2, pp. 199–203, Dec. 2012, doi: 10.1002/wics.199.
- [19] E. Parzen, Y. Sakamoto, M. Ishiguro, G. Kitagawa, and D. Reidel, "Akaike information criterion statistics," *Journal of the American Statistical Association*, vol. 83, no. 403, p. 907, Sep. 1988, doi: 10.2307/2289329.
- [20] X. Wang and H. V. Poor, "Blind multiuser detection: a subspace approach," *IEEE Transactions on Information Theory*, vol. 44, no. 2, pp. 677–690, Mar. 1998, doi: 10.1109/18.661512.
- [21] D. Posada and T. R. Buckley, "Model selection and model averaging in phylogenetics: advantages of akaike information criterion and bayesian approaches over likelihood ratio tests," *Systematic Biology*, vol. 53, no. 5, pp. 793–808, Oct. 2004, doi: 10.1080/10635150490522304.
- [22] P. Boffill and M. Zibulevsky, "Underdetermined blind source separation using sparse representations," *Signal Processing*, vol. 81, no. 11, pp. 2353–2362, Nov. 2001, doi: 10.1016/S0165-1684(01)00120-7.
- [23] Y. Li, S. I. Amari, A. Cichocki, D. W. C. Ho, and S. Xie, "Underdetermined blind source separation based on sparse representation," *IEEE Transactions on Signal Processing*, vol. 54, no. 2, pp. 423–437, Feb. 2006, doi: 10.1109/TSP.2005.861743.
- [24] S. Ge, S. N. B. M. Rum, H. Ibrahim, E. Marsilah, and T. Perumal, "An effective source number enumeration approach based on SEMD," *IEEE Access*, vol. 10, pp. 96066–96078, 2022, doi: 10.1109/ACCESS.2022.3204998.
- [25] H. Sun, J. Guo, and L. Fang, "Improved singular value decomposition (TopSVD) for source number estimation of low SNR in blind source separation," *IEEE Access*, vol. 5, pp. 26460–26465, 2017, doi: 10.1109/ACCESS.2017.2754487.
- [26] J. F. Gu, P. Wei, and H. M. Tai, "Detection of the number of sources at low signal-to-noise ratio," *IET Signal Processing*, vol. 1, no. 1, pp. 2–8, Mar. 2007, doi: 10.1049/iet-spr:20060019.
- [27] S. Li, X. Yang, W. Li, X. Hu, and T. Long, "Estimation of source number based on power-inversion and adaptive threshold in colored noise," Oct. 2017, doi: 10.1109/RADAR.2016.8059544.
- [28] J. Wang, J. Huang, J. Han, and Z. Xu, "New targets number estimation method under colored noise background," *Journal of Systems Engineering and Electronics*, vol. 23, no. 6, pp. 831–837, Dec. 2012, doi: 10.1109/JSEE.2012.00101.
- [29] Z. Yang, B. Tan, G. Zhou, and J. Zhang, "Source number estimation and separation algorithms of underdetermined blind separation," *Science in China, Series F: Information Sciences*, vol. 51, no. 10, pp. 1623–1632, Sep. 2008, doi: 10.1007/s11432-008-0138-6.
- [30] K. Han and A. Nehorai, "Improved source number detection and direction estimation with nested arrays and ULAs using jackknifing," *IEEE Transactions on Signal Processing*, vol. 61, no. 23, pp. 6118–6128, Dec. 2013, doi: 10.1109/TSP.2013.2283462.
- [31] W. Hu and Q. Wang, "DOA estimation for UCA in the presence of mutual coupling via error model equivalence," *IEEE Wireless Communications Letters*, vol. 9, no. 1, pp. 121–124, Jan. 2020, doi: 10.1109/LWC.2019.2944816.
- [32] R. Cao, B. Liu, F. Gao, and X. Zhang, "A low-complex one-snapshot DOA estimation algorithm with massive ULA," *IEEE Communications Letters*, vol. 21, no. 5, pp. 1071–1074, May 2017, doi: 10.1109/LCOMM.2017.2652442.
- [33] J. F. Gu, W. P. Zhu, and M. N. S. Swamy, "Joint 2-D DOA estimation via sparse L-shaped array," *IEEE Transactions on Signal Processing*, vol. 63, no. 5, pp. 1171–1182, Mar. 2015, doi: 10.1109/TSP.2015.2389762.
- [34] W. Zheng, X. Zhang, and H. Zhai, "Generalized coprime planar array geometry for 2-D DOA estimation," *IEEE Communications Letters*, vol. 21, no. 5, pp. 1075–1078, May 2017, doi: 10.1109/LCOMM.2017.2664809.
- [35] W. J. Zeng and X. L. Li, "High-resolution multiple wideband and nonstationary source localization with unknown number of sources," *IEEE Transactions on Signal Processing*, vol. 58, no. 6, pp. 3125–3136, Jun. 2010, doi: 10.1109/TSP.2010.2046041.
- [36] H. Bozdogan, "Model selection and akaike's information criterion (AIC): the general theory and its analytical extensions," *Psychometrika*, vol. 52, no. 3, pp. 345–370, Sep. 1987, doi: 10.1007/BF02294361.
- [37] E. Y. Hamid and Z. I. Kawasaki, "Wavelet-based data compression of power system disturbances using the minimum description length criterion," *IEEE Transactions on Power Delivery*, vol. 17, no. 2, pp. 460–466, Apr. 2002, doi: 10.1109/61.997918.
- [38] R. R. Wilcox, "Introduction to robust estimation and hypothesis testing," *Academic press*, 2012.
- [39] G. P. Nason and B. W. Silverman, "The stationary wavelet transform and some statistical applications," in *Lecture Notes in Statistics*, Springer New York, 1995, pp. 281–299.




- [40] J. O. Ramsay, "Fitting differential equations to functional data: principal differential analysis," in *Functional Data Analysis*, Springer New York, 2005, pp. 327–348.
- [41] C. Carstensen, "Inclusion of the roots of a polynomial based on Gerschgorin's theorem," *Numerische Mathematik*, vol. 59, no. 1, pp. 349–360, Dec. 1991, doi: 10.1007/BF01385785.
- [42] H. T. Wu, J. F. Yang, and F. K. Chen, "Source number estimators using transformed gerschgorin radii," *IEEE Transactions on Signal Processing*, vol. 43, no. 6, pp. 1325–1333, Jun. 1995, doi: 10.1109/78.388844.
- [43] H. T. Wu and C. L. Chen, "New Gerschgorin radii based method for source number detection," in *IEEE Signal Processing Workshop on Statistical Signal and Array Processing, SSAP, 2000*, pp. 104–107, doi: 10.1109/ssap.2000.870091.
- [44] Lambert, R. H., M. Joho, and H. Mathis, "Polynomial singular values for number of wideband source estimation and principal components analysis," *Independent Component Analysis*, pp. 379–383, 2001.
- [45] T. A. Brown and M. T. Moore, *Confirmatory factor analysis*. 2021.
- [46] J. V. Stone, "Independent component analysis: an introduction," *Trends in Cognitive Sciences*, vol. 6, no. 2, pp. 59–64, Feb. 2002, doi: 10.1016/S1364-6613(00)01813-1.
- [47] R. Roy and T. Kailath, "ESPRIT—estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 7, pp. 984–995, Jul. 1989, doi: 10.1109/29.32276.
- [48] F. Gao and A. B. Gershman, "A generalized ESPRIT approach to direction-of-arrival estimation," *IEEE Signal Processing Letters*, vol. 12, no. 3, pp. 254–257, Mar. 2005, doi: 10.1109/LSP.2004.842276.
- [49] W. Zhang, Y. Han, M. Jin, and X. S. Li, "An improved ESPRIT-Like algorithm for coherent signals DOA estimation," *IEEE Communications Letters*, vol. 24, no. 2, pp. 339–343, Feb. 2020, doi: 10.1109/LCOMM.2019.2953851.
- [50] N. Karmous, M. O. El Hassan, and F. Choubeni, "An improved esprit algorithm for DOA estimation of coherent signals," Nov. 2018, doi: 10.1109/SMARTNETS.2018.8707432.

BIOGRAPHIES OF AUTHORS






Wang Peizhi    is from Zhejiang, China. Received a master's degree from the University of Leeds, UK. She is currently studying for a Ph.D. degree from Universiti Putra Malaysia. Research interests include data mining, computer vision, and deep learning. She can be contacted at email: wpezupm@163.com.






Raihani Mohamed    received her bachelor from International Islamic University Malaysia (UIAM) and obtained her M.Sc. (IT) from Universiti Teknologi MARA, Malaysia (UiTM). She has 12 years working experience and is familiar with system floor, Logistics, SCMS and RFID technology in automation company and research in the geospatial industry. Her Ph.D. obtained from University Putra Malaysia (UPM) in Intelligent Computing. She is currently a senior lecturer at the Department of Computer Science, Universiti Putra Malaysia. Her research interests are multi label classification with deep learning, image recognition, federated learning, and precision biodiversity. She can be contacted at email: raihanimohamed@upm.edu.my.



Norwati Mustapha    from Malaysia. Received a master's degree from the University of Leeds, UK. Ph.D. degree from Universiti Putra Malaysia. She is currently a associate professor at the Department of Computer Science, Universiti Putra Malaysia. Research interests include data mining and intelligent computing. She can be contacted at email: norwati@upm.edu.my.



Noridayu Manshor    received the Ph.D. degree in 2014 from Universiti Sains Malaysia (USM). Currently, she is a lecturer at the Faculty of Computer Science and Information Technology, Universiti Putra Malaysia (UPM). Her main research includes image processing, computer vision, and pattern recognition. She can be contacted at email: ayu@upm.edu.my.