QC LDPC Codes for MIMO and Cooperative Networks using Two Way Normalized Min-Sum Decoding

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Abstract

This paper is based on the magnitude overestimation correction of the variable message by using two normalized factors in each iteration for LDPC min-sum decoding algorithm. The variable message is modified with a normalized factor when there is a sign change and with another normalized factor when there is no sign change during any two consecutive iterations. This paper incorporates QC LDPC codes using this new decoding algorithm for flat fading multiple input multiple output (MIMO) channel and single relay cooperative communication networks for improving the bit error performance. MIMO flat fading channel is used with zero forcing (ZF) spatial decoding for noise suppression. The performance is greatly enhanced by using the new min-sum algorithm for medium and short length Cooperative communication network and MIMO LDPC codes.

Keywords: coded MIMO, QC LDPC, min-sum LDPC, channel coding, cooperative communication

1. Introduction

With the advent of wireless communication, efforts have always been made to transmit maximum data with maximum reliability. To achieve the maximum data rate, MIMO wireless systems have gained popularity as its theoretic-capacity increase linearly with increase in the number of antennae [1-3]. The error performance of MIMO system can be greatly improved by error correction codes. Cooperative communication [4] is one of the fastest growing areas of research, and it is likely to be a key enabling technology for efficient spectrum use in future. The key idea in user-cooperation is that of resource-sharing among multiple nodes in a network. Low Density Parity Check (LDPC) codes are one of the most powerful error correction techniques, first proposed by Gallager [5] and were reinvented by Mackay & Neal [6, 7] . LDPC have taken considerable attention recently due to its powerful error correcting capabilities and their near Shannon limit performance [6, 8] with belief propagation (BP) decoding algorithm. The Belief Propagation (BP) or the Sum-Product decoding algorithm (SPA) performs well but at the cost of high hardware, long processing time and has dependency on the noise variance. The LDPC decoding algorithm which offers much lower hardware complexity at the cost of performance degradation is the min-sum algorithm (MSA)[9, 10] It is independent of noise variance as well. Efforts have been made to achieve optimum tradeoff between complexity and bit error performance (BER) of LDPC decoders. Several approaches attempted to keep the performance close to SPA with less hardware complexity for practical applications [11-14]

Different methods are used to bring the simplified form of the algorithm close in performance to the original BP or sum product algorithm. The most popular approaches are the normalized min-sum (Normalized MSA) and the offset min-sum (Offset MSA) algorithms. To reduce the magnitude of overestimation, the check message is modified during the iteration process which brings these min-sum algorithms close in performance to the standard SPA and makes them suitable for practical applications and hardware implementation [15-17]. Moderate length min-sum LDPC decoding algorithm [18, 19], due to its reduced complexity, has gained popularity in the wide area of wireless communication.

Figure 1 below shows the typical flow for LDPC encoded MIMO transmitted data and LDPC decoded data output after correction.

Figure 1. MIMO-LDPC in Typical Communication System

In this paper, the new min-sum decoding algorithm[19] has been embedded with MIMO. Zero forcing (ZF) spatial decoding is used for suppressing the noise in flat fading and additive white Gaussian (AWG) MIMO channels. The proposed algorithm offers better BER performance and can be easily implemented in hardware.

The proposed LDPC method [19] with MIMO corrects overestimated magnitude of the variable message during two consecutive iterations. When the signs of the present and the previous message are the same then the present message is scaled and updated with a normalized factor, however when the signs are different then the two messages are added and scaled with a normalized factor, which is different from the first scaling factor.

2. Introduction To LDPC
2.1. Representation of LDPC Codes

LDPC codes are a type of linear block codes and are represented by parity check matrix. LDCP code can be denoted in general as (N, d_v, d_c); where N is the length of the code equal to the number of columns in the parity check matrix, d_c is the number of ones in a column of the matrix; d_v is the number of ones in a row of the matrix. LDPC codes can be regular or irregular. If the number of ones in each row and column of a parity check matrix are the same then it is a regular code and otherwise it is called irregular code. Following is an example of an irregular LDPC code.

$$\mathbf{H} = \begin{matrix} & \begin{matrix} x_1 & x_2 & & \cdots & & x_7 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \qquad (1)$$

$$\mathbf{H}\,\mathbf{x}^T = \mathbf{0}$$

The code is valid only if H. code^T = 0.

The sparse parity check matrix is best represented by a bipartite graphs know as Tanner graphs [20]. Each row of the parity check matrix represents the variable node and each column represents the check node. The 1s in each row or column represents the connectivity between variable and check nodes. The set of bit nodes connecting to check node m is denoted by $N(m)=\{n|h_{mn}=1\}$ and the set of check nodes connecting to bit node n is by $M(m)=\{m|h_{mn}=1\}$. A typical Tanner graph is shown in the Figure 2. This graph is for (3, 7) irregular LDPC code.
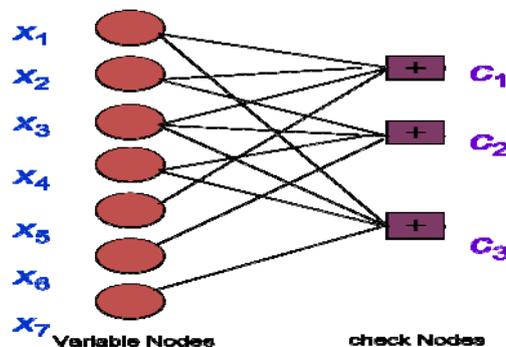

Figure 2. Tanner Graph Representation of Parity Check Matrix

In algebraic form, it can be demonstrated as:

$$c_1 : x_1 + x_2 + x_3 + x_5 = 0$$
$$c_2 : x_2 + x_3 + x_4 + x_6 = 0$$
$$c_3 : x_1 + x_3 + x_4 + x_7 = 0$$

2.2. Two Way Normalized Min-sum Algorithm (MSA)

Let $C = \{c_1, c_2 \ldots c_n\}$ be a transmitted code over an additive white Gaussian (AWGN) channel. $Y = C + n$; where n is an AWGN.

Now LDPC min-sum decoding[19] can be stated in the following steps for a parity check matrix H_{mn}; where m is the number of rows and n is the number of columns.

Step 1: Initialization: Set $R_n = Y$ as initial log *likelihood* ratio (LLR) and for each $(m, n) \in \{(m, n)|h_{mn} = 1\}$

$$V^0_{mn} = R_n \tag{1}$$

Set $i = 0$ to I_{max}, I_{max} is the maximum number of iteration

Step 2: Horizontal process: check node update:
For $m = 0$ to M-1, update C^i_{mn} for each $n \in N(m)$

$$C^i_{mn} = \prod_{\substack{n' \in N(m) \\ n' \neq n}} sign(V^{i-1}_{mn'}) . \min_{\substack{n' \in N(m) \\ n' \neq n}} |V^{i-1}_{mn'}| \tag{2}$$

Step 3: vertical process: bit node update
For $n = 0$ to $N - 1$, update

$$\tilde{R}^i_n = R_n + \sum_{m \in M(n)} C^i_{mn} \tag{3}$$

Now updating V^i_{mn} for each $m \in M(n)$:

$$V^{i,tmp}_{mn} = R^i_n - C^i_{mn} \tag{4}$$

The signs of the present message $v^{i,tmp}_{mn}$ and the previous message v^{i-1}_{mn} are then compared.

If sign $(v^{i,tmp}_{mn}) = = sign(v^{i-1}_{mn})$
Then update the present message as:

$$V^i_{mn} = sf_1 (V^{i,tmp}_{mn}) \tag{5}$$

Else if sign $(v^{i,tmp}_{mn}) \neq sign(v^{i-1}_{mn})$
Then update the present message as:

$$V^i_{mn} = sf_2 (V^{i,tmp}_{mn} + V^{i-1}_{mn}) \tag{6}$$

The scaling factors sf_1 and sf_2 are chosen such that they can be conveniently implemented in hardware and at the same time provide good approximation to the error performance. Now if the signs are different then the change in magnitude is large and is modified with a smaller factor to reduce the overestimation effect. The scaling factors used for the simulations in this paper are $sf_1 = 0.5$ and $sf_2 = 0.25$.

Now Equation (5) and Equation(6) can be re-written as:

$$V_{m\,n}^{i} = 0.5\,(V_{m\,n}^{i,tm\,p})$$ (5a)

$$V_{m\,n}^{i} = 0.25\,(V_{m\,n}^{i,tm\,p} + V_{m\,n}^{i-1})$$ (6a)

Equation (5a) and Equation (6a) gives good performance achievement while the cost for hardware is very low. This brings further improvement to the MSA in both lower and upper region of SNR by using two scaling factors.

The summation in Equation (6a) with the previous message does not affect significantly the decoding performance. Thus the summation with the previous message can be modified as shown below in Equation (6b) which reduces the time latency and still offer better results.

$$V_{m\,n}^{i} = 0.25\,(V_{m\,n}^{i-1})$$ (6b)

Step 4: Hard Decision:

$$\hat{c}_n = \begin{cases} 0, \text{for} \ \tilde{R}_n > 0 \\ 1, \text{for} \ \tilde{R}_n \leq 0 \end{cases}$$ (7)

Step 5: Stop condition: If the parity check equation is satisfied.

$$H.(\hat{c}_1.\hat{c}_2..........\hat{c}_n)^T = 0$$ (8)

Or maximum iteration (I_{max}) is reached then terminate the decoding or otherwise $i=i+1$ and go back to step 2.

3. MIMO Communication Network

The multiple-input and multiple-output (MIMO) is the use of multiple antennas at both the transmitter and receiver to improve communication performance. It is one of several forms of smart antenna technology. MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or transmit power. Because of these properties, MIMO is an important part of modern wireless communication standards such as IEEE 802.11n (Wifi), WiMAX etc.

Consider a flat fading MIMO system model with N_t transmit and N_r receive antennas .The received signal vector at each instant of time is given by:

$R=Hx+n$ (9)

Where R is ithe received signal , H is the channel matrix ($N_t x N_r$) and n is the additive white Gaussian noise (AWGN). MIMO communication system for the Equation (9) is show in Figure (3).

Where r is $N_r \times 1$ received signal vector, H is a $N_r \times N_t$ channel response matrix, x is a $N_t \times 1$ transmitted signal vector and n is the additive white Gaussian noise (AWGN).Typical LDPC coded MIMO communication system is show in Figure 1.
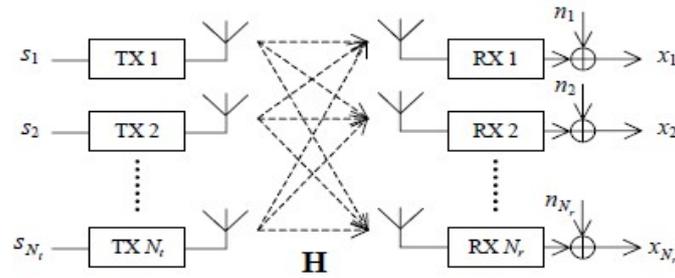
Figure 3. MIMO Communication System

Consider that we have a transmission sequence, for example. $x_1, x_2 \cdots x_n$. In normal transmission, we will be sending x_1 in the first time slot, x_2 in the second time slot and so on. However, as we now have 2 transmit antennae, we may group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna. In second time slot, send x_3 and x_4 from the first and second antenna, send x_5 and x_3 in the third time slot and so on. Notice that as we are grouping two symbols and sending them in one time slot, we need only $n/2$ time slots to complete the transmission – data rate is doubled! This forms the simple explanation of a probable MIMO transmission scheme with 2 transmit antennas and 2 receive antennas. The two transmitted symbols interfered with each other called inter channel interference (ICI). The channel is flat fading – In simple terms, it means that the multipath channel has only one tap. So, the convolution operation reduces to a simple multiplication and the channel experience by each transmit antenna is independent from the channel experienced by other transmit antennas. For the ith transmit antenna to jth receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number h_{ji}. As the channel under consideration is a Rayleigh channel given by:

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} \exp[-(\dfrac{r^2}{2\sigma^2}] & \text{for } r \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(10)

Where r is the envelope amplitude of the received signal, and $2\sigma^2$ is the pre-detection mean power of the multipath signal. The real and imaginary parts of h_{ji} are Gaussian distributed having mean $\mu_{h_{ji}} = 0$ and variance $\sigma^2_{h_{ji}} = 1/2$. The channel experienced between each transmit to the receive antenna is independent and randomly varying in time.

On the receive antenna, the noise n has the Gaussian probability density function with:

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{\frac{-(n-\mu)^2}{2\sigma^2}}$$

(10a)

Where $\mu = 0$ and $\sigma^2 = \dfrac{N_0}{2}$.

The channel h_{ji} is known at the receiver.

4. Zero Forcing (ZF) Equalizer

For a 2x2 MIMO communication system, in the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{11}x_1 + h_{12}x_2 + n_1 = [h_{11}, h_{12}]\begin{bmatrix} x_1 \\ x_1 \end{bmatrix} + n_1 \tag{11}$$

The received signal on the second receive antenna is:

$$y_2 = h_{21}x_1 + h_{22}x_2 + n_2 = [h_{21}, h_{22}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \tag{12}$$

Where,

y_1, y_2, are the received symbol on the first and second antenna respectively,

h_{11} is the channel from 1st transmit antenna to 1st receive antenna,

h_{12} is the channel from 2nd transmit antenna to 1st receive antenna,

h_{21} is the channel from 1st transmit antenna to 2nd receive antenna,

h_{22} is the channel from 2nd transmit antenna to 2nd receive antenna,

x_1, x_2 are the transmitted symbols and

n_1, n_2 is the noise on 1st and 2nd receive antennas.

We assume that the receiver knows h_{11}, h_{12}, h_{21} and h_{22}. The receiver also knows y_1 & y_2. The unknowns are x_1 & x_2. So we have two equations and two unknowns. For convenience, the above equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \tag{13}$$

Equivalently,

$$r = Hx + n \tag{14}$$

To solve for x, we know that we need to find a matrix W which satisfies $WH = I$. The Zero Forcing (ZF) linear detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \tag{15}$$

This matrix is also known as the pseudo inverse for a general *m x n* matrix.

5. QC LDPC Coded 2x2 MIMO using Two Way Normalized MSA

The performance of the MIMO system can be enhanced by using state of the art error correction technique like Quasi cyclic (QC) LDPC codes as shown in Figure 4.
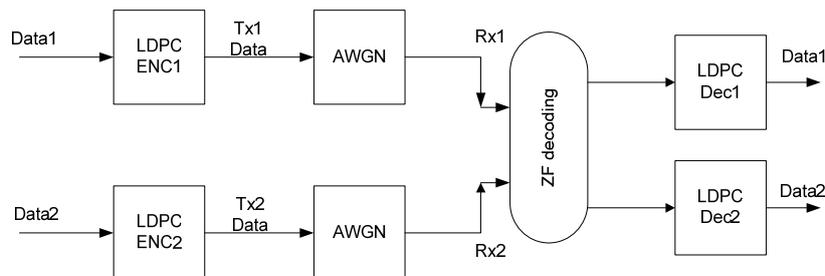
Figure 4. 2x2 LDPC Coded MIMO System with ZF Equalizer

The system has been simulated for a QC LDPC parity check matrix $H_{261 \times 522}$ such that code length=522, and each sub-matrix size is 87. Following QC parity check matrix (H) is designed such that E21= E32=0 and all others are composed of circularly shifted identity sub- matrices, each of size 87.

H = [E11 E12 E13 E14 E15 E16
        E21 E22 E23 E24 E25 E26
        E31 E32 E33 E34 E35 E36]

And the channel matrix for flat fading MIMO is: C = [h11 h12 ; h21 h22]
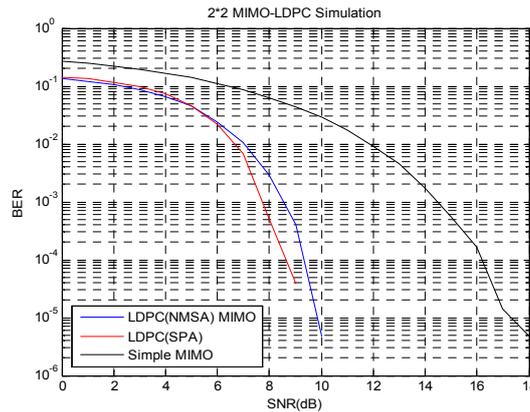

Figure 6. Performance Comparison of LDPC Coded MIMO and Simple MIMO with ZF

The graph in Figure 6 shows improved performance for LDPC coded MIMO in comparison to simple MIMO with ZF decoding. The LDPC decoding algorithm used are the standard SPA and the new improved MSA [19] mentioned in section 2.2 which shows better performance for medium and short length codes and are suitable for the MIMO channels to split data into small packets and transmit independently.

6. Cooperative Communication Using Joint Layered Decoding
We consider a one relay cooperative communication systems [21, 22] as shown in Figure 7. The typical distance for the coded copperative system are mentioned in [23]. Here the distances between source, relay and destination are such that the distance between source to destination is normaliezed to 1 as shown in equation below.

$$\gamma_1 = 1, \quad \gamma_0 = \frac{1}{d^\alpha} \quad , \text{ and } \gamma_2 = \frac{1}{(1-d)^\alpha}$$
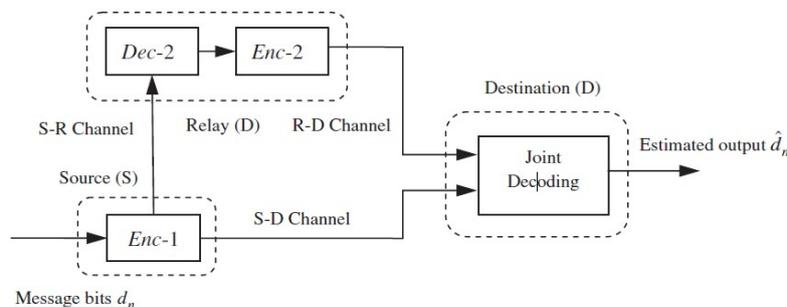
Figure 7. Single Relay Cooperative Communication System

Where α is the path loss and is usually taken in the range 2~3.The graph in the figures 9 & 10 are simulated for ideal and non-ideal cooperative communication. The distance between R-D in a non-ideal cooperation is such that it receives 4db more power. The distance between R-D is such that it receives power 1db more than S-D for both ideal and non-ideal situation. A parity check matrix for the source encoder is H^1 & H^2 is selected with 250 rows and 500 columns such that:

$$H_S = \begin{bmatrix} H^1_{250 \times 500} & I_{250 \times 500} \end{bmatrix} \& H_R = \begin{bmatrix} H^2_{250 \times 500} & I_{250 \times 500} \end{bmatrix} \tag{16}$$

Where $H^1 \neq H^2$ and H^2 is the row permutation matix obtained from H^1 and both are regular matrices. The number of ones in row and columns are equal in both the matrices $d_v = 4$ and $d_c = 8$. I is the identity matrix, H_R is the irregular systematic pairty check matrix at the relay encoder, H_S is the irregular systematic pairty check matrix at thesource encoder. The final matrix at destination D is H_D and is given by:

$$H_D = \begin{bmatrix} H_1 & I_{250 \times 250} & 0_{250 \times 250} \\ H_2 & 0_{250 \times 250} & I_{250 \times 250} \end{bmatrix} \tag{17}$$
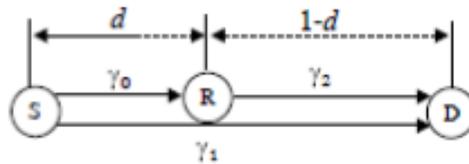
Figure 8. Typical Way for Showing the Distances between Sources(S), Relay(R) and Destination (D)

A decode /re-encode/forward strategy has been adopted at the relay channel. At the relay a message is deocoded and then re-encode the parity only and then transmit the pairty bits to the destination where it is combined with the message from the sources such that the p_r is sent by relay R:

$$code = \begin{bmatrix} s & p_s & p_r \end{bmatrix} \tag{18}$$

This code is decoded by parity check matrix in Equation (17) by LDPC normalized layered min-sum decoding method. The channels are simulated for the rayleigh fading coefficients such that:

$$y = hx + n \tag{19}$$

Where h is the rayleigh fading channel coefficient, n is the additive white guassian noise, x is the information bits. The simulation results shows the comparison for the coded cooperation with ideal, non-ideal and non-cooperative communication (deirect source to destination) under the same channel conditions. This same code has been simulated under the same channel conditions for layered min sum decoding algorithm which has fast convergence and better results as this is free of noise variance. So prior channel information is not required to initialize the information bits. The graph in Figure 10 shows the performance comparison for this pratical type of LDPC decoder.
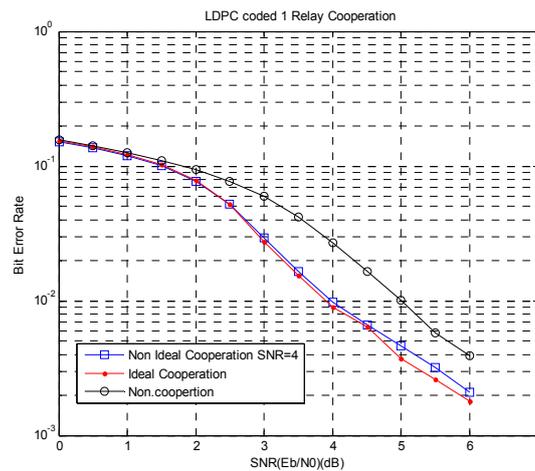
Figure 9. BER Performance for One Relay-cooperation and Non-cooperation Systems for Code Length=750, rate (at S and R)=2/3, iterations=10
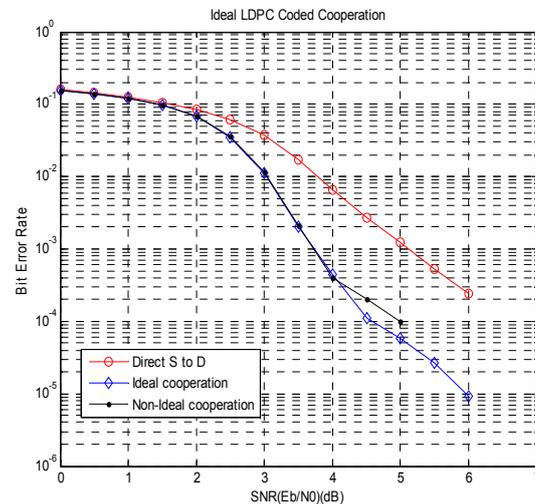

Figure 10. BER Performance for One Relay-cooperation and Non-cooperation Systems for Code Length=750, rate (at S and R)=2/3, iterations=10

7. Conclusion

In this paper, two way normalized min-sum and joint layered LDPC decoding algorithm has been simulated for MIMO and cooperative communication respectively as it is free of noise variance and give significantly low complexity. For MIMO communication, an improved two way normalized min-sum decoding has been used to show the performance comparison with standard LDPC sum-product algorithm. The bit error graph clearly shows an improved performance for the new scheme. A joint layered min-sum LDPC decoding approach is used for cooperative communication for achieving fast decoding convergence and better performance. New dimensions can be explored for multiple relay scheme and cooperative MIMO communication using this algorithm for bit error performance improvement.

References
[1] David Tse PV. Fundamental of Wireless Communication. 2005.
[2] JR Barry EAL, D.G. Messserschmitt. Digital Communication. 3rd Edition.
[3] Vahid Tarokh NS, AR Calderbank. Space-time codes for high data rate wireless communication: performance criterion and code construction. *IEEE Tran. on Infor. Theory.* 1998; 44(2): 744–765.

[4]   Aria Nosratinia TEH, Ahmadreza Hedayat. Cooperative Communication in Wireless Networks. *IEEE Communications Magazine*. 2004.

[5]   RG Gallager. Low-Density Parity-Check Code. Cambridge. MA: MIT Press. 1963.

[6]   DJ Mackay, RN. Near Shannon Limit Performance of Low Density Parity Check Codes. Electronic Letters. 1996; 32(18): 1645-1646.

[7]   DJ MacKay. Good error-correcting codes based on very sparse matrices. I*EEE Trans. Infor. Theory*. 1999; 45: 399-431.

[8]   S Chung GDF, JJ Richardson, R Urbanke. On the design of low-density parity-check codes within 0.0045db of the Shannon limit. *IEEE Communication Letters*. 2001; 5: 58-60.

[9]   M Fossorier MM. Reduced Complexity Iterative Decoding of Low-Density Parity Check Codes Based on Belief Propagation. *IEEE Transactions on communications*. 1999; 47.

[10] J Chen MPCF. Density evolution for two improved BP-Based decoding algorithms of LDPC codes. *IEEE Communications Letters*. 2002; 6: 208 - 210.

[11] J Chen MPCF. Near optimum universal belief propagation based decoding of low-density parity check codes. *IEEE Transactions on Communication*. 2002; 50: 406-414.

[12] LIU Hai-yang QWz, LIU Bin, LI Jiang-peng, LUO Shi-dong, CHEN Jie. Novel modified min-sum decoding algorithm for low-density parity-check codes. *The Journal of China Universities of Posts and Telecommunications*. 2010; 17: 1-5.

[13] Meng Xu JW, Meng Zhang. *A modified Offset Min-Sum decoding algorithm for LDPC codes*. 3rd IEEE International Conference on Computer Science and Information Technology (ICCSIT2010). 2010; 3: 19 - 22.

[14] Xiaofu Wu YS, Ming Jiang, Chunming Zhao. Adaptive-Normalized/Offset Min-Sum Algorithm. *IEEE Communications Letters*. 2010; 14: 667-669.

[15] Nan Jiang KP, Jian Song, Chanyong Pan, Zhixing Yang. High-Throughput QC-LDPC Decoders. *IEEE Transactions On Broadcasting*. 2009; 55(2): 251-259.

[16] Kai He JS, Li Li, Zhongfeng Wang. *Low Power Decoder Design for QC LDPC Codes*. Proceedings of IEEE International Symposinm on Circuirts and Systems (ISCAS). 2010: 3937-3940.

[17] C Roth AC, C Studer, Y Leblebiciz, ABurgz. *Area, throughput, and energy-efficiency trade-offs in the VLSI implementation of LDPC decoders*. IEEE International Symposium on Circuits and Systems (ISCAS). 2011: 1772 - 1775.

[18] Waheed Ullah J, Fengfan Yang, SM Aziz. *Two-Way Normalization of Min-Sum Decoding Algorithm for Medium and Short Length Low Density Parity Check Codes*. 7th International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM). 2011: 1-5.

[19] Waheed ullah J, Yang Fengfang. *Improved min-sum decoding algorithm for moderate length low density parity check codes*. International conference on Computer, Informatics, Cybernetics and Application (ICIA-2011), LNEE Springer (ISSN: 1876-1100). 2011: 939-944.

[20] M Tanner R. A recursive approach to low complexity codes. *IEEE Transactions on Information Theory* 1981; IT-27: 533-547.

[21] Fengfan Yang JC, Peng Zong, Shunwai Zhang, Qiuxia Zhang. Joint iterative decoding for pragmatic irregular LDPC-coded multi-relay cooperations. *International Journal of Electronics*. 2011; 98(10): 1383-1397.

[22] Marjan Karkooti JRC. *Cooperative Communications Using Scalable, Medium Block-length LDPC Codes*. IEEE Wireless Communications and Networking Conference, (WCNC 2008), 2008: 88 - 93.

[23] MA Khojastepour NA, B Aazhang. *Code design for the relay channel and factor graph decoding*. Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers. 2004: 2000 - 2004.