

Image Segmentation Based on Chaotic Quantum Ant Colony Algorithm

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Abstract

Ant colony algorithm is a new type of biomimetic evolutionary algorithm, which has some outstanding characteristics like good robustness, parallelism and positive feedback. It has been widely used in many fields but it still has some weaknesses, such as slow-convergence and easily-falling into local extreme value. Therefore we propose a new algorithm which combines the quantum evolutionary algorithm and ant colony algorithm together based on these shortcomings, this new type of algorithm considers the two quantum bit probability amplitudes as ants' current location information. The searching space will be doubled when ants' numbers remain the same. Both introducing the pixel point gradient into quantum revolving door, and dynamically changing rotation angle are to achieve local-searching. Searching by chaotic quanta near candidates with optimal solution is to improve the global optimization. Meanwhile, adopting a NAND gate is to achieve mutating operation, and to avoid algorithm's precocity. Compared with the traditional algorithm, the new algorithm has a better population diversity and an outstanding ability to overcome the precocity and stagnation in the optimization process. It has been proved that this improved algorithm is effectively to the problems like slow-convergence and easily-falling into local extreme value, and vividly increase the speed and accuracy of the image segmentation.

Keywords: railway signaling, computer interlocking, all-electronic, reliability, availability, maintainability

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1. Introduction

Image segmentation is a way of extracting the region of interest from the background, or dividing the image into several different species¹ according to its relevance. It is a complex problem in the field of image processing. Many researchers have thought up various theoretical approaches from different areas of image segmentation. The traditional method of image segmentation is based on numerous methods like, threshold, edge detection, region growing, fuzzy theory, mathematical morphology and so on. However, because of the different application purposes and image features, these methods still have some limitations.

With the constant development of the intelligent optimization algorithms, many researchers adopt the intelligent algorithms into image segmentations because their common advantages like high convergence speeds and global optimizations. For example, Documents [1, 2] which are relevant to Ant Colony Algorithm, Document [3] related with Genetic Algorithm and Documents [4, 5] relevant to Particle Swarm Optimization have proposed different image segmentation methods. Because these intelligent algorithms also have some weaknesses like long-searching, slow- convergence, they are likely to be premature when the image data are large enough.

The quantum computing was first proposed by the American physicist, R.P.Feynman in 1982. Recently, some researchers have combined the quantum computing with pattern recognition, and come up with the recognition algorithm based on quantum theory, such as quantum template matching algorithm, quantum image recognition algorithm and so on. In Document [6], quantum ant colony algorithm is used to solve QoS unicast routing, however, Document [7] quantum adopts ant colony algorithm to diagnose faults in naphtha cracker. This paper is inspired by combination of ant colony algorithm and quantum theory, its aim is to solve the problem of data clustering, after which the image will be segmented.

2. Quantum Ant Colony Algorithm

The quantum ant colony algorithm (QACA) was first proposed by A.Narayanan & M.Moore "in his paper "Quantum-ant Algorithm". The algorithm was used for solving problem of combinatorial optimization in order to achieve good results [9]. The quantum bit introduced by each ant indicates the position of ant; the ant's forward moving target is selected by pheromone strength and the probability of selection of the visible structure; the algorithm allows quantum gate to update quantum bit, then determines the ants' movement. The pheromone strength is updated by the moved position. The algorithm considers the two probability amplitude of quantum bit as the ants' current location. When the ants' numbers are the same, the searching-space will be doubled; the mutation operation is achieved by quantum gate. Compared with traditional algorithm, this algorithm has a better population diversity, and effectively overcome the precocity and the stagnation of the ant colony algorithm in optimization process.

2.1. Quantifying the Ant Colony Space

Assume that each pixel (i, j) in the image is a point, one 3D space will be built after considering the grayscale, gradient, and 3×3 realms information. Randomly distribute m ants in this space. So a quantum bit state is conveyed as the following according to the quantum theories.

$$\varphi \Rightarrow \alpha |0\rangle + \beta |1\rangle \tag{1}$$

The α, β respectively represent probability amplitude of $|0\rangle, |1\rangle$, and $|\alpha|^2 + |\beta|^2 = 1$, the ants' current locations are indicated by probability amplitude just as shown in Figure 1.

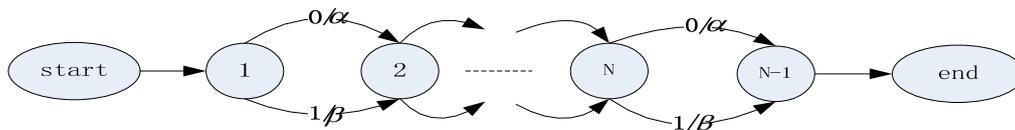


Figure 1. Quantum Individual Transferring Diagram

The n bits quanta are endowed to the No. i Ant.

$$p_i = \begin{vmatrix} \cos \theta_{i1} & \cos \theta_{i2} & \dots & \cos \theta_{in} \\ \sin \theta_{i1} & \sin \theta_{i2} & \dots & \sin \theta_{in} \end{vmatrix} \tag{2}$$

So each ant occupies two positions in space, Position $[\cos(\theta_{i1}) \cos(\theta_{i2}) \dots \cos(\theta_{in})]$ and Position $[\sin(\theta_{i1}) \sin(\theta_{i2}) \dots \sin(\theta_{in})]$.

Compared with the ordinary ant colony algorithm when the population numbers are the same, QACA has doubled the quantum population searching space and vastly increased convergence speed.

2.2. Transferring the Quantum Ant' Position

The transferring-state probability is calculated by the pheromone intensity and the heuristic information on ant's moving process in Ant Colony Algorithm (ACA), and the transferring path is determined also. In QACA, the following Formula is adopted to identify ant's position from point i to point j.

$$p_k(i, s) = \begin{cases} 0 & \text{Others} \\ \frac{[\tau(i, s)]^\alpha [\eta(i, s)]^\beta}{\sum [\tau(i, s)]^\alpha [\eta(i, s)]^\beta} & j \in S \end{cases} \quad (3)$$

$$j = \arg \max p_k(i, s) \quad (4)$$

Its Heuristic function is:

$$\eta_{ij} = 1 / \sqrt{\sum_{k=1}^m q_k (x_{ik} - x_{jk})^2} \quad (5)$$

q is the weighting coefficient about the image's Grayscale, Gradient and the realm information, τ is the pheromone density.

In quantum space, the ant's movement can be achieved by its own quantum bits, while phase changing is usually determined by quantum revolving door, as defined following:

$$R = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (6)$$

Suppose that P_r is the current position, P_d is the target position, P_d attempts to get close to P_r by the force of R , and to realize position updating. Generally, the constructor function $S(\alpha, \beta)$, the polling list of θ and the quantum revolving door's angle $\nabla \theta$ all determine ant's optimization speed, there are many documents about this issue [10, 11]. $S(\alpha, \beta)$ is the revolving angle's direction that ensures algorithm convergence. For example, Figure 2 is the schematic diagram of quantum revolving door, assume that there are x individual ants, the optimal number is x_{best} . When the Fitness value $f(x) > f(x_{best})$, it is supposed to increase probability of current solution 0, namely to enlarge $|\alpha|^2$, if $(\alpha\beta)$ exists in the first and the third quadrant, θ should clockwise revolve, however, if $(\alpha\beta)$ exists in the second and the forth quadrant, θ should anticlockwise revolve, so different quantum conversions are designed by different situation.

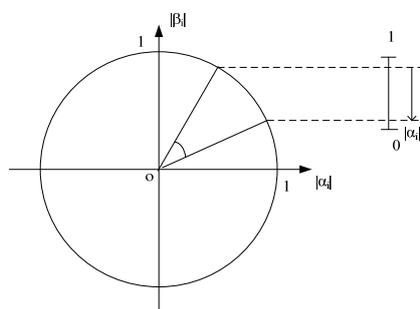


Figure 2. Schematic Diagram of Quantum Gate

2.3. Mutagenic Factors

To prevent pheromone from early-falling into the local extreme value on the path found at the initial stage, the mutagenic factors achieved by quantum NOT gate are introduced into QACA. First, assume that the mutation probability is a constant, then give a random number

ranged from 0 to 1 to each ant, if this number is less than the constant assumed before, then choose $n/2$ quantum bits in the ant individuals, so exchange the quantum NOT gate with two probability amplitudes, its principles are shown as follows:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos \theta_i \\ \sin \theta_i \end{pmatrix} = \begin{pmatrix} \sin \theta_i \\ \cos \theta_i \end{pmatrix} \quad (7)$$

Actually, the mutating operation is to exchange the two quantum bits' probability amplitudes, make the original liable-collapsed stated "1" collapse at state "0", and vice versa. Another mutation is the rotation of the quantum bit angle, the No. i Quantum bit rotating angle is θ_i , so the angle will become $(\frac{\pi}{2}) - \theta_i$ after mutation, this rotation does not compare with the current best individual after the forward-rotating, it has nothing to do with its optimal location memory and the location, so the population diversity has been increased, and avoided falling into local extreme values.

2.4. Analyzing Mechanism of Pheromone Updating

The pheromone includes ants' current optimal location information level, while gradient information is contained in heuristic information in QACA. When every ant completes a search, ant's current position will be mapped from the unit space to the optimal solution space. When quantum optimization algorithm will handle the relationship between quantum bit and solution vector, a random has been given firstly, and then compare with a probability amplitude of quantum bits to determine the results of the quantum bit transforming, finally a binary solution has been got, then convert it into a decimal solution vector. The operation is with a certain degree of randomness, and prone to self-degradation. Given this, the quantum bit probability amplitude can be directly applied as a solution vector encoding to avoid the randomness of the transforming. Integrating the fitness function values into pheromone and gradient into the heuristic information makes the optimum position of the pheromone become higher; arouses more information, and speeds up the search process and convergence.

It is necessary to update the pheromone on each node after ants finish one cycle at t , which is shown as the following:

$$\tau_{ij}(t+1) = \rho \tau_{ij}(t) + \sum_{k=1}^n \Delta \tau_{ij}^k \quad (8)$$

In above formula (8), ρ is pheromone attenuation coefficient, $\Delta \tau_{ij}^k$ is the pheromone left on nodes of the No. k ant finishes one cycle.

3. Chaotic Quantum Ant Colony Algorithm

3.1. The Overview of Chaos

Chaos is a common nonlinear phenomenon, it is symbolized with randomness, ergodicity and regularity, it is very sensitive to the initial conditions, it repeatedly traverses all states within a certain range according to its own law. These properties of chaos can be made to optimized-search so chaos can be integrated with the other algorithms.

Chaotic variable is usually generated by the Logistic. The chaos sequence formula is following:

$$x(t+1) = \mu x(t)(1-x(t)) \quad (9)$$

$x(t+1)$ is chaotic variable, $t = 0, 1, 2, \dots, \mu$ is a chaotic attractor. When $\mu = 4$, the system falls into the chaotic state, and generates chaotic variables ranging from 0 to 1.

3.2. The Chaotic Quantum Ants Searching

When the quantum ant colony algorithm begins to initialize, the pheromone on each path is equal. Therefore, individual direction is only determined by the heuristic information, it is difficult to quickly find a good solution in a large number of candidates, so the convergence speed is slowed down at the initial stage. Using the initial pheromone distribution formed in the chaos optimization speeds up the convergence. In this paper, the improved Tent mapping mentioned in Document [14] is adopted to do chaos optimization.

$$x_{t+1} = \begin{cases} 2(x_t + 0.1rand(0,1)), 0 \leq x_t \leq 0.5 \\ 2(1 - (x_t + 0.1rand(0,1))), 0.5 < x_t \leq 1 \end{cases} \quad (10)$$

Here, $0 \leq t \leq k-1$, the first path of quantum diversity population is initialized by chaotic variables, and the quantum bit is following:

$$\alpha_i = \cos(2\pi x_i), \beta_i = \sin(2\pi x_i), \quad 1 \leq i \leq n \quad (11)$$

$n-1$ paths will be generated according the method above, find the optimal paths from them, and initialize the pheromone on these optimal paths.

Considering the gradient is the important feature of the image edges, this paper adjusts the step of θ by using the gradient function, it is most likely on the image edge when the pixel gradient value occurs, we must make small angle to prevent the extreme points. If the value of the gradient of the pixel is smaller, a larger angular will cross the area within the image, $\alpha_r, \beta_r, \alpha_d, \beta_d$ probability amplitude and the target quantum bit for the current bit respectively. When $A = \alpha_r \beta_d - \alpha_d \beta_r, |A| \neq 0$, the direction of θ is $-\text{sgn}(|A|)$, while, $|A| = 0$, the direction of θ can be plus or minus. So dynamic adjustment of design is shown below:

$$\Delta\theta = -\text{sgn}(\mathbf{A})\theta_0 \frac{\|\nabla f(x) - \nabla f_{\min}\|}{\|\nabla f_{\max} - \nabla f_{\min}\|} \quad (12)$$

$f(x)$ is an individual fitness function, $\nabla f(x)$ is a gradient value at the point of x . The location of the initial optimal solution will be found by $\nabla \theta$, searches the optimal solution in the current near location, If a better solution is found, it will replace the original optimal solution. The positive feedback of ant colony algorithm pheromone is used to avoid chaos searching blindness, and improve search efficiency. Disturbance $10 \Delta\theta$ chaotic sequence is used in this paper, thus, different individuals are subjected to different disturbance by fatnesses. a single and fixed-size quantum rotation gate has been avoided, ants population is prevented from falling into local extreme values. The sequence perturbation amplitude is $\lambda_t = \lambda_0 e^{-t}$, λ_0 is the controlling-parameter, t is the iterations.

$$\Delta\theta_t = \lambda_t x(t+1) \quad (13)$$

3.3. Algorithm Flow Chart

Step 1: Initializing chaos, chaotic variables are generated chaotic variables according to the Formula (11), and initializing paths are generated according to the Formula (10). The initial population size is m , iteration maximum number of is $IterMax$, attenuation coefficient of pheromone is α , the pheromone update index is ρ , heuristic information of updating the index is β , mutation probability is p_m , A maximum pheromone is τ_{\max} .

Step 2: calculating the fitness of each individual quantum value according to (3), (4), and calculating the heuristic information according to (5).

Step 3: Replacing with current position, when ant the location is superior to the optimal location of its memory, replacing with the current global optimal location, when the current global optimal location is better than that of the global optimal position before the search.

Step 4: Updating ant location with quantum revolving door.

Step 5: Ascending the individuals according their fitness value, mutation-operating the after $m/2$ individuals' mutation probability mutation by the quantum gate.

Step 6: Chaos quantum searching near the current optimal solution according to (13), if optimal solution exists, replacing step 3 to get the optimal solution.

Step 7: Updating the pheromone on the nodes according to (8).

Step 8: Returning to Step2 cycle-calculating, until the convergence condition or the maximum limit of number of iteration is got.

Step 9: Tagging the node (i, j) as the goal, if its pheromones meet $\tau_{ij} \geq \tau_{max}$ after reaching or a specified number of iterations of cycling times. Otherwise tagging it as the background, and completing the image segmentation.

4. Analyzing the Algorithm's Convergence

Theorem 1: The population sequence $\{X_t, t \geq 0\}$ of chaotic quantum ant colony algorithm (CQACA) is a limited and homogeneous Markov chain.

In CQACA, quantum bits have been adopted, although the space population space is theoretically infinite, yet in this paper the population size is a predetermined value, the spatial dimension is 3, so the population is limited. However, quantum mutation, chaotic quantum searching and pheromone updating all have nothing to do with the number of iterations, so

X_{t+1} is only related to $A^* = \{\max(f(A) = f^*, A \in S)\}$, not about the number of iterations.

So as mentioned, the population sequence $\{X_t, t \geq 0\}$ of chaotic quantum ant colony algorithm (CQACA) is a limited and homogeneous Markov chain.

Theorem 2: CQACA converges to the globally optimal solution with probability 1.

Suppose that S is the states space, f^* is the global optimal solution, and order

$A^* = \{\max(f(A) = f^*, A \in S)\}$, if and only if $\lim_{t \rightarrow \infty} p\{A(t) \in A^* | A(0) = S_0\} = 1$, so the algorithm is convergent.

Assume the individual state transition probability is:

$$p_{ij}(t) = p\left(\frac{X_{t+1}^j}{X_t^i}\right)$$

Because the algorithm uses the optimal retention strategy, there are two types of the transition probabilities.

(1) When $i \in A^*, j \notin A^*$, for any $t \geq 0$, there is $f(X_{t+1}) \geq f(X_t)$, so $p_{ij}(t) = 0$;

(2) When $i \in A^*, j \in A^*$,

$$p_t = \sum p_i(t) = p\{A(t) = i | A(0) = S_0\}$$

$$p_{t+1} = \sum_{i \in A^*} \sum_{j \in A^*} P_i(t) p_{ij}(t) + \sum_{i \in A^*} \sum_{j \notin A^*} P_i(t) p_{ij}(t)$$

Because,

$$\sum_{j \in A^*} p_{ij}(t) + \sum_{j \notin A^*} p_{ij}(t) = 1$$

We get,

$$\begin{aligned}
 p(t) &= \sum_{i \in A^*} p_i(t) \left(\sum_{j \in A^*} p_{ij}(t) + \sum_{j \notin A^*} p_{ij}(t) \right) \\
 &= \sum_{i \in A^*} \sum_{j \in A^*} p_i(t) p_{ij}(t) + \sum_{i \in A^*} \sum_{j \notin A^*} p_i(t) p_{ij}(t)
 \end{aligned}$$

Form the first case,,

$$\sum_{i \in A^*} \sum_{j \notin A^*} p_i(t) p_{ij}(t) = 0$$

And,

$$\begin{aligned}
 p(t) &= \sum_{i \in A^*} \sum_{j \in A^*} p_i(t) p_{ij}(t) \\
 p_{t+1} &= p(t) + \sum_{i \in A^*} \sum_{j \notin A^*} p_i(t) p_{ij}(t)
 \end{aligned}$$

So,

$$0 \leq p_{t+1} < p_t,$$

Because,

$$\lim_{t \rightarrow \infty} \{f_t = f^*\} = 1 - \lim_{t \rightarrow \infty} \sum_{i \in A} p_i(t) = 1 - \lim_{t \rightarrow \infty} p_t$$

Then,

$$\lim_{t \rightarrow \infty} \{f_t = f^*\} = 1$$

End.

Conclusion: the CQACA converges to the global optimal solution with probability 1.

5. Simulation and Results

In order to verify the feasibility of CQACA algorithm for image segmentation, this papers has segmented LENA image whose size is 256*256 respectively via Sobel operator, Canny operator, Ant Colony Algorithm and Chaotic Quantum Ant Colony Algorithm of the in the Matlab7.0 simulation environment and on the platform that is equipped with the Intel (R) Core (TM) Duo CPU, dominant frequency 1.83GHz, 987MHz, 0.99GB memory. The results are shown in Figure 4. Initializing parameters are: Ant colony size^m is 40, the evaporation coefficient ρ is 0.7, α is 0.5, β is 0.5, the maximum number of iterations *IterMax* is 100, P_m is 0.02, the pheromones τ_{\max} is 0.9, chaotic disturbance factor λ_0 is 1.6. Quantum ants are randomly distributed at the initializing stage, the pheromone at the optimal solution begins to increase during searching process, until achieve the maximum at last, so the ants will be concentrated at these nodes just as shows in Figure 3, when the image edge is being checked, the image edge point is the optimal solution, so the pheromone density is much higher, and the pheromone density matrix will be obtained, then the image edge point can be extracted, the quantum ants position transferring diagram is shown as Figure 4. Figure 5 is the picture from

edge segmentation, Figure (a) is the result of being edge-checked by Soble operator, Figure (b) Figure (a) is the result of being edge-checked by Canny operator, Figure (c) is the result of ant algorithm, and Figure (d) is the result of chaotic quantum ant algorithm. It is proved that Soble operator has lost some low level of gray in some details, the Canny operator is better, but it has some too much details. Figure (c) is the result of ACA and Figure (d) is CQACA's result. It is showed that gray parts has been checked and have a relative accuracy. Because of the diversity of the CQACA's searching space, a lot of details have been remained. What's more, with comparing the iteration times of two algorithms, the iteration times of ACA is 46, its running time is 98.6s, while CQACA's iteration times is 18, its running time is 31.6. Obviously, CQACA is much better than ACA.

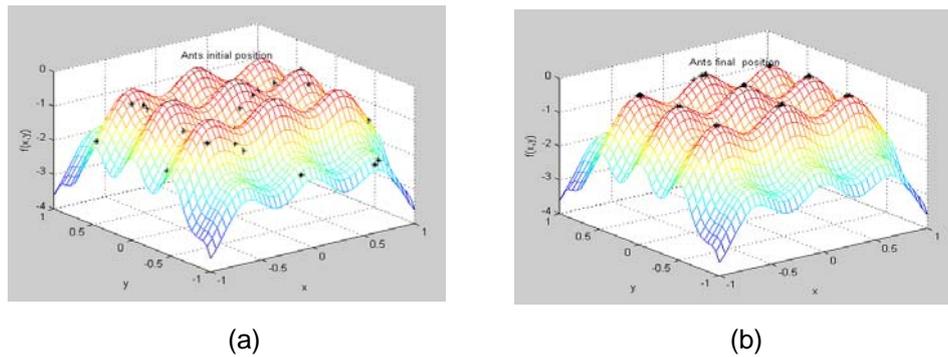


Figure 3. The Distribution Diagram of Chaotic Quantum Ants



Figure 4. The Comparison of Lena Edge Extraction

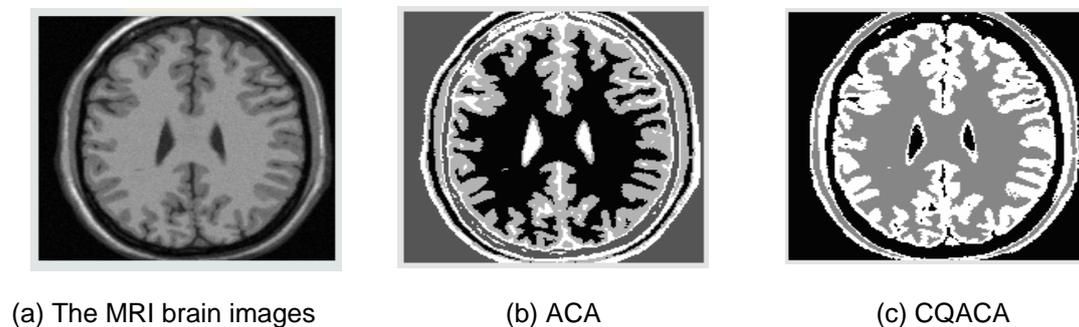


Figure 5. The Comparison of Two Kinds of Brain MRI Image Segmentation

Figure 5 is the MRI brain images, Figure (b) and Figure (c) is the result of comparing the ant colony algorithm segmentation and quantum ant colony algorithm. Parameter selection:

Colony size is 40, volatile coefficient is 0.7, pheromone update index is 0.85, heuristic information updating the index is 0.85, the maximum number of iterations of is 60, mutation probability is 0.04, pheromone maximum value is 0.7, the disturbance factor is 1.6.

Differences in the segmentation of white matter and gray matter in the brain is not big, but in the cerebrospinal fluid of segmentation, the spatial diversity of quantum search algorithm and the segmentation results are retained more detailed information. Comparison of number of iterations and the convergence time of form 2 to two kinds of algorithms.

Table 1. the Comparison of Three Algorithms' Iteration Times and Running Times

	Iterations times	Running time
ACA	38	78.3 s
CQACA	22	36.8 s

Figure 6 is the comparing results of human cells segmented by two kinds of algorithms, form segmentation effect, chaotic quantum ant colony algorithm segmentation low gray information which has more information, the target is completed, and the convergence speed is much lower than the ant colony algorithm.

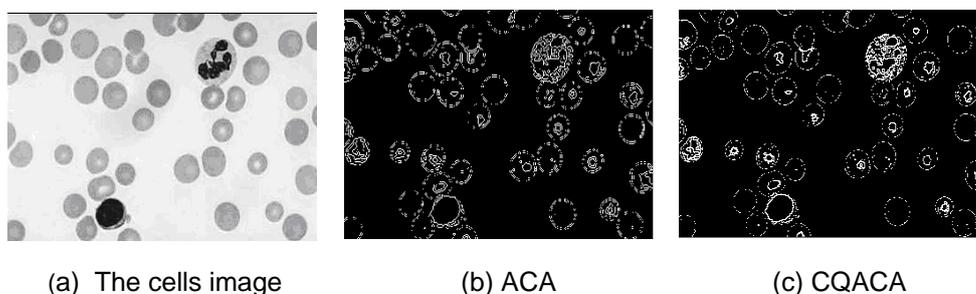


Figure 6. The Comparison of the Two Kind of Cells Image Segmentation

6. Conclusion

Image segmentation is a difficult problem in image processing. At present, there is no a common method for different image segmentation. Combining the theory of chaos optimization and referring the quantum evolutionary algorithm and ant colony algorithm, this paper uses quantum ant search space diversity and the advantage of convergence speed. After segmenting three kinds of images, it is showed that this method is effective to solve the ant colony algorithm and easy to fall into local minima and slow convergence speed problem, and on the segmentation speed and precision has been improved greatly by experiments.

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