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Stochastic Synchronization of Neutral-type Chaotic Markovian Neural Networks with Impulsive Effects

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Abstract

This paper studies the globally stochastic synchronization problem for a class of neutral-type chaotic neural networks with Markovian jumping parameters under impulsive perturbations. By virtue of drive-response concept and time-delay feedback control techniques, by using the Lyapunov functional method, Jensen integral inequality, a novel reciprocal convex lemma and the free-weight matrix method, a novel sufficient condition is derived to ensure the asymptotic synchronization of two identical Markovian jumping chaotic delayed neural networks with impulsive perturbation. The proposed results, which do not require the differentiability and monotonicity of the activation functions, can be easily checked via Matlab software. Finally, a numerical example with their simulations is provided to illustrate the effectiveness of the presented synchronization scheme.

Keywords: Stochastically asymptotic synchronization, chaotic neural networks, Markovian jump, impulse, reciprocal convex

1. Introduction

The problem of synchronization arises in numerous practical problems in physics, ecology, and physiology. In 1990, the pioneering work of Pecora and Carroll [6] brought attention to the importance of control and synchronization of chaotic systems. In their seminal paper, Pecora and Carrol proposed the drive-response concept for constructing synchronization of coupled chaotic systems. The idea is to use the output of the driving system to control the response system so that they oscillate in a synchronization manner. Since then, chaos synchronization has been widely investigated with a view to its applications in secure communication systems [8].

Markovian jump system, introduced by Krasovskii and Lidskii in 1961, is a special class of hybrid systems. In a Markovian jump system, the random jump of parameters is governed by a Markov process which takes values in a finite set. Thus, Markov jump systems can describe some physical systems with abrupt variations very well, e.g., solar thermal central receivers, economic systems [5], and so on. Recently, a lot of research results on the stability analysis for delayed neural networks with Markovian jumping parameters have been reported, see, for instance, [8].

Impulsive effect is likely to exist in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time in the fields such as medicine and biology, economics, electronics and telecommunications. Neural networks are often subject to impulsive perturbation that in turn affect dynamical behaviors of systems. Therefore, it is necessary to consider both the impulsive effect and delay effect when investigating the stability of neural networks. So far, several interesting results have been reported that have focused on the impulsive effect of delayed neural networks [1, 7].

Motivated by aforementioned discussion, this paper investigates the globally stochastic synchronization of a class of neutral-type chaotic neural networks with Markovian jumping parameters under impulsive perturbations. The mixed delays consists of discrete and distributed time-varying delays. By virtue of drive-response concept and time-delay feedback control techniques, by using the Lyapunov functional method, Jensen integral inequality, a novel reciprocal

convex lemma and the free-weight matrix method, a novel sufficient condition is derived to assure the stochastic synchronization of two identical Markovian jumping chaotic delayed neural networks with impulsive perturbation. The proposed results, which do not require the differentiability and monotonicity of the activation functions, can be easily checked via Matlab software. Finally, a numerical example with their simulations is provided to illustrate the effectiveness of the presented synchronization scheme.

Notations: Throughout this paper, W^T, W^{-1} denote the transpose and the inverse of a square matrix W, respectively. $W > 0 (< 0)$ denotes a positive (negative) definite symmetric matrix, I denotes the identity matrix with compatible dimension, the symbol "*" denotes a block that is readily inferred by symmetry. The shorthand $\mathrm{col}\{M_1, M_2, ..., M_k\}$ denotes a column matrix with the matrices $M_1, M_2, ..., M_k$. $\mathrm{sym}(A)$ is defined as $A + A^T$, $\mathrm{diag}\{\cdot\}$ stands for a diagonal or block-diagonal matrix. For $\tau > 0, \mathcal{C}([-\tau, 0]; \mathbb{R}^n)$ denotes the family of continuous functions ϕ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the norm $||\phi|| = \sup_{-\tau \leq s \leq 0} |\phi(s)|$. Moreover, let $(\Omega, \mathbb{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathbb{F}_t\}_{t \geq 0}$ satisfying the usual conditions and $\mathbb{E}\{\cdot\}$ representing the mathematical expectation. Denote by $\mathcal{C}^p_{\mathbb{F}_0}([-\tau, 0]; \mathbb{R}^n)$ the family of all bounded, $\mathbb{F}_0$-measurable, $\mathcal{C}([-\tau, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(s) : -\tau \leq s \leq 0\}$ such that $\sup_{-\tau \leq s \leq 0} \mathbb{E}|\xi(s)|^p < \infty$. $|| \cdot ||$ stands for the Euclidean norm; Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem description and preliminaries

In this paper, we consider the following neutral-type chaotic neural networks with Markovian jumping parameters under impulsive perturbations

$$\begin{cases} \dot{x}(t) = & -C(\eta(t))x(t) + A(\eta(t))g(x(t)) + B(\eta(t))g(x(t - \tau(t, \eta(t)))) \\ & + D(\eta(t)) \int_{t-\sigma(t)}^{t} g(x(s))\mathrm{d}s + E(\eta(t))\dot{x}(t - \rho(t)) + J, \\ x(t) = & \varphi_1(t), \quad s \in [-\hat{\tau}, 0], \end{cases} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T \in \mathbb{R}^n$ is the state vector associated with n neurons, real constant matrices $C(\eta(t)), A(\eta(t)), B(\eta(t)), D(\eta(t)), E(\eta(t))$ are the interconnection matrices representing the weight coefficients of the neurons. $g(x(t)) = (g_1(x_1(t)), g_2(x_2(t)), ..., g_n(x_n(t)))^T \in \mathbb{R}^n$ denotes the neural activation function. The bounded functions $\tau(t), \rho(t)$ represent unknown time-varying delays with $0 \leq \tau(t, \eta(t)) \leq \bar{\tau}(\eta(t)) \leq \bar{\tau}, \dot{\tau}(t, \eta(t)) \leq \tau'(\eta(t)) \leq \tau' < 1, 0 \leq \sigma(t) \leq \bar{\sigma}, \dot{\sigma}(t) \leq \sigma' < 1, 0 \leq \rho(t) \leq \bar{\rho}, \dot{\rho}(t) \leq \rho' < 1$, where $\bar{\tau}, \bar{\sigma}, \bar{\rho}$ are positive scalars, $\hat{\tau} = \max\{\bar{\tau}, \bar{\sigma}, \bar{\rho}\}$. J is an external input, $\varphi_1(t)$ is a real-valued initial vector function that is continuous on the interval $[-\hat{\tau}, 0]$. $\{\eta(t), t \geq 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in finite set $\mathcal{N} = \{1, 2, ..., N\}$ based on given probability space $(\Omega, \mathbb{F}, \mathbb{P})$ with and the initial model η_0. Let $\Pi = [\pi_{ij}]_{N \times N}$ denote the transition rate matrix with transition probability:

$$\mathbb{P}(\eta(t + \delta) = j | \eta(t) = i) = \begin{cases} \pi_{ij}\delta + o(\delta), & i \neq j, \\ 1 + \pi_{ii}\delta + o(\delta), & i = j, \end{cases}$$

where $\delta > 0, \lim_{\delta \to 0^+} \frac{o(\delta)}{\delta} = 0$ and π_{ij} is the transition rate from mode i to mode j satisfying $\pi_{ij} \geq 0$ for $i \neq j$ with

$$\pi_{ii} = -\sum_{j=1, j \neq i}^{N} \pi_{ij}, \quad i, j \in \mathcal{N}.$$

For convenience, each possible value of $\eta(t)$ is denoted by $\iota(\iota \in \mathcal{N})$ in the sequel. Then we have

$$A_\iota = A(\eta(t)), \quad B_\iota = B(\eta(t)), \quad C_\iota = C(\eta(t)), \quad D_\iota = D(\eta(t)), \quad E_\iota = E(\eta(t)).$$

Throughout this paper, we make the following assumptions:

Assumption 1. Each neural activation function $g_j(\cdot)(j = 1, 2, ..., n)$ is bounded, differentiable and satisfies the following condition

$$\delta_j^- \leq \frac{g_j(\xi) - g_j(\zeta)}{\xi - \zeta} \leq \delta_j^+, \quad \forall \, \xi, \zeta \in \mathbb{R}, \, \xi \neq \zeta,$$

where δ_j^-, δ_j^+ are known real constants.

For simplicity, we denote $\Delta_1 = \mathrm{diag}\left\{\delta_1^-, \delta_2^-, \cdots, \delta_n^-\right\}$, $\Delta_2 = \mathrm{diag}\left\{\delta_1^+, \delta_2^+, \cdots, \delta_n^+\right\}$, $\Delta_3 = \mathrm{diag}\left\{\delta_1^- \delta_1^+, \delta_2^- \delta_2^+, \cdots, \delta_n^- \delta_n^+\right\}$, $\Delta_4 = \frac{1}{2}\mathrm{diag}\left\{\delta_1^- + \delta_1^+, \delta_2^- + \delta_2^+, \cdots, \delta_n^- + \delta_n^+\right\}$.

The system (1) is considered as a drive system, the corresponding response system of (1) is given in the following form:

$$\begin{cases} \dot{y}(t) = & -C_\iota y(t) + A_\iota g(y(t)) + B_\iota g(y(t - \tau_\iota(t))) \\ & + D_\iota \int_{t-\sigma(t)}^{t} g(y(s))\mathrm{d}s + E_\iota \dot{y}(t - \rho(t)) + J + u_\iota(t), \quad t > 0, \, t \neq t_k, \\ \Delta y(t_k) = & y(t_k) - y(t_k^-) = -\Gamma_k \left\{y(t_k^-) - x(t_k^-)\right\}, \qquad\qquad k \in \mathbb{Z}_+, \\ y(t) = & \varphi_2(t), \quad s \in [-\hat{\tau}, 0], \end{cases} \quad (2)$$

where $y(t) = (y_1(t), y_2(t), ..., y_n(t))^T \in \mathbb{R}^n$ is the state vector associated with n neurons, $u_\iota(t) = (u_{\iota 1}(t), ..., u_{\iota n}(t))^T \in \mathbb{R}^n$ is the state feedback controller given to achieve the exponential synchronization between the drive and response systems, Γ_k is a known matrix, $\varphi_2(t)$ is a real-valued continuous vector function on the interval $[-\hat{\tau}, 0]$.

In order to investigate the synchronization for the chaotic delayed neural networks with impulsive perturbation, $e_j(t) = y_j(t) - x_j(t)$ is defined as the synchronization error, where $x_j(t)$ and $y_j(t)$ are the i-th state variables of drive system (1) and response system (2), respectively. Therefore, the error dynamical system between (1) and (2) is given as follows:

$$\begin{cases} \dot{e}(t) = & -C_\iota e(t) + A_\iota f(e(t)) + B_\iota f(e(t - \tau_\iota(t))) \\ & + D_\iota \int_{t-\sigma(t)}^{t} f(e(s))\mathrm{d}s + E_\iota \dot{e}(t - \rho(t)) + u_\iota(t), \quad t > 0, \, t \neq t_k, \\ \Delta e(t_k) = & e(t_k) - e(t_k^-) = -\Gamma_k e(t_k^-), \qquad\qquad\qquad k \in \mathbb{Z}_+, \\ e(t) = & \varphi(t) \doteq \varphi_2(t) - \varphi_1(t), \quad t \in [-\hat{\tau}, 0], \end{cases} \quad (3)$$

where $e(t) = (e_1(t), e_2(t), ..., e_n(t))^T$, $f_j(e_j(t)) = g_j(y_j(t)) - g_j(x_j(t))$.

In this paper, the control input vector with state feedback is designed as follows:

$$u_\iota(t) = Y_{1\iota} e(t) + Y_{2\iota} e(t - \tau_\iota(t)). \qquad (4)$$

Therefore, it follows from [2] that system (3) admits a trivial solution $e(t) = 0$.

The development of the work in this paper requires the following lemmas.

Lemma 1 (see [4]). *Let $z(t) \in \mathbb{R}^n$ has continuous derived function $\dot{z}(t)$ on interval $[a, a + \omega]$, then for any $n \times n-$matrix $\Theta > 0$, the following inequality holds:*

$$-\int_a^{a+\omega} \dot{z}^T(s)\Theta\dot{z}(s)\mathrm{d}s \leq -\frac{2}{\omega}\left(\frac{1}{\omega}\int_a^{a+\omega} z(s)\mathrm{d}s - z(a)\right)^T \Theta \left(\frac{1}{\omega}\int_a^{a+\omega} z(s)\mathrm{d}s - z(a)\right).$$

Lemma 2 (see [9]). *Assume that $\nu, \mu, \underline{\vartheta}, \bar{\vartheta}$ are real scalars such that $\nu \leq 1, \nu + \mu \leq 4$, and $\underline{\vartheta} < \bar{\vartheta}$. Let $\vartheta : \mathbb{R} \to (\underline{\vartheta}, \bar{\vartheta})$ be a real function. Then for any non-negative scalars a, b, the following inequality holds*

$$-\frac{a}{\vartheta(t) - \underline{\vartheta}} - \frac{b}{\bar{\vartheta} - \vartheta(t)} \leq \frac{1}{\bar{\vartheta} - \underline{\vartheta}}\max\{-\nu a - \mu b, -\mu a - \nu b\}.$$

3. Main result

Now, we begin to state our result for error system (3) with input (4).

Theorem 1. *Assume that Assumption* 1 *hold, the drive system* (1) *and the response system* (2) *with* (4) *can be stochastically asymptotically synchronized in mean square if there exist positive definite matrices* $P_\iota, Q_\iota, R_\iota, S_\iota, Z_\iota, U_i (i = 1, ..., 7)$, *positive diagonal matrices* $\Lambda, \Gamma, T_\iota, W_\iota$, *real matrices* $X_{1\iota}, X_{2\iota}$ *of appropriate dimensions such that*

$$\sum_{j=1}^{N} \pi_{\iota j} Q_j < U_1, \qquad \sum_{j=1}^{N} \pi_{\iota j} R_j < U_2, \tag{5}$$

$$\bar{\tau}_\iota \sum_{j=1}^{N} \pi_{\iota j} S_j < U_3, \qquad \bar{\tau}_\iota \sum_{j=1}^{N} \pi_{\iota j} \bar{\tau}_j S_\iota < U_4, \tag{6}$$

$$\begin{bmatrix} P_\iota & (I - \Gamma_k) P_\iota \\ * & P_\iota \end{bmatrix} \geq 0, \quad k \in \mathbb{Z}_+, \tag{7}$$

$$(I - \Gamma_k)^T P_\iota (I - \Gamma_k) \leq P_l, \quad l \neq \iota, \quad l, \iota \in \mathcal{N}, \tag{8}$$

$$\Phi_\iota - 4(\varpi_8 - \varpi_2)^T S_\iota (\varpi_8 - \varpi_2) < 0, \tag{9}$$

$$\Phi_\iota - 4(\varpi_9 - \varpi_3)^T S_\iota (\varpi_9 - \varpi_3) < 0, \tag{10}$$

where

$$\Phi_\iota = [\, \Phi_{ij} \,]_{9 \times 9}, \quad \varpi_i = [\, 0_{(i-1)n \times n} \quad I \quad 0_{(10-i)n \times n} \,], \ i = 1, 2, ..., 10,$$

with

$$\Phi_{1,1} = Q_\iota + \bar{\tau} U_1 + U_6 - W_\iota \Delta_3 + \sum_{j=1}^{N} \pi_{\iota j} P_j, \quad \Phi_{1,4} = W_\iota \Delta_4,$$

$$\Phi_{1,7} = P_\iota - \Delta_1 \Lambda + \Delta_2 \Gamma - C_\iota Z_\iota + X_{1\iota}, \ \Phi_{2,2} = -(1 - \tau'_\iota) Q_\iota - T_\iota \Delta_3 - 2S_\iota + \sum_{j=1}^{N} \bar{\pi}_{\iota j} \bar{\tau}_j Q_\iota,$$

$$\Phi_{2,5} = T_\iota \Delta_4, \quad \Phi_{2,7} = X_{2\iota}, \quad \Phi_{2,8} = 2S_\iota, \quad \Phi_{3,3} = -U_6 - 2S_\iota, \quad \Phi_{3,9} = 2S_\iota,$$

$$\Phi_{4,4} = R_\iota + \bar{\tau} U_2 + \bar{\sigma}^2 U_5 - W_\iota, \ \Phi_{4,7} = \Lambda - \Gamma + A_\iota^T Z_\iota, \ \Phi_{5,5} = -(1 - \tau'_\iota) R_\iota - T_\iota + \sum_{j=1}^{N} \bar{\pi}_{\iota j} \bar{\tau}_j R_\iota,$$

$$\Phi_{5,7} = B_\iota^T Z_\iota, \quad \Phi_{6,6} = -U_5, \quad \Phi_{6,7} = D_\iota^T Z_\iota, \quad \Phi_{7,7} = \bar{\tau}_\iota^2 S_\iota + \frac{\bar{\tau}^2}{2} U_3 + \bar{\tau} U_4 + U_7 - 2Z_\iota,$$

$$\Phi_{7,10} = Z_\iota E_\iota, \quad \Phi_{8,8} = -2S_\iota, \quad \Phi_{9,9} = -2S_\iota, \quad \Phi_{10,10} = -(1 - \rho') U_7, \qquad \bar{\pi}_{\iota j} = \max\{\pi_{\iota j}, 0\},$$

and the control gain matrices $Y_{1\iota}$ *and* $Y_{2\iota}$ *in* (4) *are given as* $Y_{1\iota}^T = X_{1\iota} Z_\iota^{-1}$, $Y_{2\iota}^T = X_{2\iota} Z_\iota^{-1}$.

Proof. Construct a Lyapunov-Krasovskii functional in the following form

$$V_\iota(t, e(t)) = e(t)^T P_\iota e(t) + \sum_{i=1}^{3} V_{i\iota}(t, e(t)),$$

where

$$V_{1\iota}(t, e(t)) = 2 \sum_{j=1}^{n} \int_0^{e_i(t)} \left\{ \lambda_i [f_i(s) - \delta_i^- s] + \gamma_i [\delta_i^+ s - f_i(s)] \right\} \mathrm{d}s$$

$$+ \int_{t-\tau_\iota(t)}^t [e(s)^T Q_\iota e(s) + f(e(s))^T R_\iota f(e(s))] \, \mathrm{d}s + \bar{\tau}_\iota \int_{t-\bar{\tau}_\iota}^t \int_\theta^t \dot{e}(s)^T S_\iota \dot{e}(s) \mathrm{d}s \mathrm{d}\theta,$$

$$V_{2\iota}(t, e(t)) = \int_{t-\bar{\tau}}^t \int_\theta^t [e(s)^T U_1 e(s) + f(e(s))^T U_2 f(e(s))] \, \mathrm{d}s \mathrm{d}\theta$$

$$+ \int_{t-\bar{\tau}}^t \int_\theta^t \int_\eta^t \dot{e}(s)^T U_3 \dot{e}(s) \mathrm{d}s \mathrm{d}\eta \mathrm{d}\theta + \int_{t-\bar{\tau}}^t \int_\theta^t \dot{e}(s)^T U_4 \dot{e}(s) \mathrm{d}s \mathrm{d}\theta,$$

$$V_{3\iota}(t, e(t)) = \bar{\sigma} \int_{t-\sigma(t)}^t \int_\theta^t f(e(s))^T U_5 f(e(s)) \mathrm{d}s \mathrm{d}\theta + \int_{t-\bar{\tau}}^t e(s)^T U_6 e(s) \mathrm{d}s + \int_{t-\rho(t)}^t \dot{e}(s)^T U_6 \dot{e}(s) \mathrm{d}s.$$

Denoting $e_\iota = e(t - \tau_\iota(t))$, calculating the weak infinitesimal operator along the system (3) gives

$$\mathfrak{L}V_\iota(t, e(t)) = 2e(t)^T P_\iota \dot{e}(t) + \sum_{j=1}^{N} \pi_{\iota j} e(t)^T P_j e(t) + \sum_{i=1}^{3} \mathfrak{L}V_{i\iota}(t, e(t)), \qquad (11)$$

where

$$\begin{aligned}
\mathfrak{L}V_{1\iota}(t, e(t)) = & 2\dot{e}(t)^T \left\{ \Lambda \left[f(e(t)) - \Delta_1 e(t) \right] + \Gamma \left[\Delta_2 e(t) - f(e(t)) \right] \right\} + e(t)^T Q_\iota e(t) \\
& + f(e(t))^T R_\iota f(e(t)) - (1 - \dot{\tau}_\iota(t)) \left[e_\iota^T Q_\iota e_\iota + f(e_\iota)^T R_\iota f(e_\iota) \right] \\
& + \sum_{j=1}^{N} \pi_{\iota j} \int_{t-\tau_\iota(t)}^{t} \left[e(s)^T Q_j e(s) + f(e(s))^T R_j f(e(s)) \right] \mathrm{d}s \\
& + \sum_{j=1}^{N} \pi_{\iota j} \tau_j(t) \left[e_\iota^T Q_\iota e_\iota + f(e_\iota)^T R_\iota f(e_\iota) \right] + \bar{\tau}_\iota^2 \dot{e}(t)^T S_\iota \dot{e}(t) - \bar{\tau}_\iota \int_{t-\bar{\tau}_\iota}^{t} \dot{e}(s)^T S_\iota \dot{e}(s) \mathrm{d}s \\
& + \bar{\tau}_\iota \sum_{j=1}^{N} \pi_{\iota j} \int_{t-\bar{\tau}_\iota}^{t} \int_{\theta}^{t} \dot{e}(s)^T S_\iota \dot{e}(s) \mathrm{d}s \mathrm{d}\theta + \bar{\tau}_\iota \sum_{j=1}^{N} \pi_{\iota j} \bar{\tau}_j \int_{t-\bar{\tau}_\iota}^{t} \dot{e}(s)^T S_\iota \dot{e}(s) \mathrm{d}s, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
\mathfrak{L}V_{2\iota}(t, e(t)) = & \bar{\tau} \left[e(t)^T U_1 e(t) + f(e(t))^T U_2 f(e(t)) \right] - \int_{t-\bar{\tau}}^{t} \left[e(s)^T U_1 e(s) + f(e(s))^T U_2 f(e(s)) \right] \mathrm{d}s \\
& + \frac{\bar{\tau}^2}{2} \dot{e}(t)^T U_3 \dot{e}(t) - \int_{t-\bar{\tau}}^{t} \int_{\theta}^{t} \dot{e}(s)^T U_3 \dot{e}(s) \mathrm{d}s \mathrm{d}\theta + \bar{\tau} \dot{e}(t)^T U_4 \dot{e}(t) - \int_{t-\bar{\tau}}^{t} \dot{e}(s)^T U_4 \dot{e}(s) \mathrm{d}s,
\end{aligned}$$
$$(13)$$

$$\begin{aligned}
\mathfrak{L}V_{3\iota}(t, e(t)) = & \bar{\sigma} \sigma(t) f(e(t))^T U_5 f(e(t)) - \bar{\sigma} \int_{t-\sigma(t)}^{t} f(e(s))^T U_5 f(e(s)) \mathrm{d}s + e(t)^T U_6 e(t) \\
& - e(t-\bar{\tau})^T U_6 e(t-\bar{\tau}) + \dot{e}(t)^T U_7 \dot{e}(t) - (1 - \dot{\rho}(t)) \dot{e}(t-\rho(t))^T U_4 \dot{e}(t-\rho(t)). \qquad (14)
\end{aligned}$$

For $0 < \tau_\iota(t) \leq \bar{\tau}_\iota$, define $\zeta_1(t) = \frac{1}{\tau_\iota(t)} \int_{t-\tau_\iota(t)}^{t} e(s) \mathrm{d}s$. It is easy to see that $\zeta_1(t) \to e(t)$ while $\tau_\iota(t) \to 0$. Therefore we can define $\zeta_1(t) = e(t)$ when $\tau_\iota(t) = 0$. Similarly, for $0 \leq \tau_\iota(t) < \bar{\tau}_\iota$, define $\zeta_2(t) = \frac{1}{\bar{\tau}_\iota - \tau_\iota(t)} \int_{t-\bar{\tau}_\iota}^{t-\tau_\iota(t)} e(s) \mathrm{d}s$; when $\tau_\iota(t) = \bar{\tau}_\iota$, define $\zeta_2(t) = e(t-\bar{\tau}_\iota)$.

For $0 < \tau_\iota(t) < \bar{\tau}_\iota$, utilizing Lemma 1 gives

$$\begin{aligned}
& - \int_{t-\bar{\tau}_\iota}^{t} \dot{e}(s)^T S_\iota \dot{e}(s) \mathrm{d}s \\
& = - \int_{t-\tau_\iota(t)}^{t} \dot{e}(s)^T S_\iota \dot{e}(s) \mathrm{d}s - \int_{t-\bar{\tau}_\iota}^{t-\tau_\iota(t)} \dot{e}(s)^T S_\iota \dot{e}(s) \mathrm{d}s \leq -\frac{2}{\tau_\iota(t)} \Xi_1 - \frac{2}{\bar{\tau}_\iota - \tau_\iota(t)} \Xi_2, \qquad (15)
\end{aligned}$$

where $\Xi_1 = [\zeta_1(t) - e_\iota]^T S_\iota [\zeta_1(t) - e_\iota]$, $\Xi_2 = [\zeta_2(t) - e(t-\bar{\tau}_\iota)]^T S_\iota [\zeta_2(t) - e(t-\bar{\tau}_\iota)]$.

It is easy to see that inequality (15) holds for any $t > 0$ with $\tau_\iota(t) = 0$ or $\tau_\iota(t) = \bar{\tau}_\iota$.

From inequality (15) and Lemma 2, we get the following inequality

$$-\bar{\tau}_\iota \int_{t-\bar{\tau}_\iota}^{t} \dot{e}(s)^T S_\iota \dot{e}(s) \mathrm{d}s \leq 2 \max \left\{ -\Xi_1 - 3\Xi_2, \ -3\Xi_1 - \Xi_2 \right\}. \qquad (16)$$

Furthermore, from the Jensen inequality we have

$$-\bar{\sigma} \int_{t-\sigma(t)}^{t} f(e(s))^T U_5 f(e(s)) \mathrm{d}s \leq - \left(\int_{t-\sigma(t)}^{t} f(e(s)) \mathrm{d}s \right)^T U_5 \left(\int_{t-\sigma(t)}^{t} f(e(s)) \mathrm{d}s \right). \qquad (17)$$

Moreover, based on (H2), the following matrix inequalities hold for any positive diagonal matrices L_ι, T_ι

$$0 \leq -e(t)^T W_\iota \Delta_3 e(t) + 2e(t)^T W_\iota \Delta_4 f(e(t)) - f(e(t))^T W_\iota f(e(t)), \tag{18}$$

$$0 \leq -e_\iota^T T_\iota \Delta_3 e_\iota + 2e_\iota^T T_\iota \Delta_4 f(e_\iota) - f(e_\iota)^T T_\iota f(e_\iota). \tag{19}$$

Furthermore, the following equality is true for any real matrix Z_ι

$$0 = 2\dot{e}(t)^T Z_\iota^T \bigg[-\dot{e}(t) - C_\iota e(t) + A_\iota f(e(t)) + B_\iota f(e(t - \tau_\iota(t)))$$

$$+ D_\iota \int_{t-\sigma(t)}^t f(e(s)) \mathrm{d}s + E_\iota \dot{e}(t - \rho(t)) + Y_{1\iota} e(t) + Y_{2\iota} e(t - \tau_\iota(t)) \bigg]. \tag{20}$$

Denoting $Z_\iota Y_{1\iota} = X_{1\iota}^T, Z_\iota Y_{2\iota} = X_{2\iota}^T$, substituting (12)-(20) into (11) and taking mathematical expectation gives

$$\frac{\mathrm{d}\mathbb{E}V_\iota(t, e(t))}{\mathrm{d}t} = \mathbb{E}\zeta(t)^T \overline{\Phi}_\iota \zeta(t), \qquad t \in [t_{k-1}, t_k), k \in \mathbb{Z}_+. \tag{21}$$

where

$$\zeta(t) = \mathrm{col} \left\{ e(t), e_\iota, \ e(t - \bar{\tau}), \ f(e(t)), \ f(e_\iota), \int_{t-\sigma(t)}^t f(e(s)) \mathrm{d}s, \ \dot{e}(t), \ \zeta_1(t), \zeta_2(t), \ \dot{e}(t - \rho(t)) \right\},$$

$$\overline{\Phi}_\iota = \Phi_\iota + 4 \max \left\{ -(\varpi_8 - \varpi_2)^T S_\iota (\varpi_8 - \varpi_2), \ -(\varpi_9 - \varpi_3)^T S_\iota (\varpi_9 - \varpi_3) \right\}.$$

We deduce that inequality $\overline{\Phi}_\iota < 0$ is equivalent to inequalities (9) and (10) respectively. Therefore, if inequalities (9) and (10) hold, then from (21) we derive that

$$\frac{\mathrm{d}\mathbb{E}V_\iota(t, e(t))}{\mathrm{d}t} < 0, \qquad \forall\, t \in [t_{k-1}, t_k), \ k \in \mathbb{Z}_+. \tag{22}$$

When $t = t_k, \ k \in \mathbb{Z}_+$, from the condition (H5), we have

$$V_\iota(t_k, e(t_k)) = V_\iota(t_k^-, e(t_k^-)) + e(t_k^-)^T \left[(I - \Gamma_k)^T P_\iota (I - \Gamma_k) - P_\iota \right] e(t_k^-). \tag{23}$$

On the other hand, it follows from (7) that

$$\begin{pmatrix} I & 0 \\ 0 & P_\iota^{-1} \end{pmatrix} \begin{pmatrix} P_\iota & (I - \Gamma_k)P_\iota \\ * & P_\iota \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & P_\iota^{-1} \end{pmatrix} \geq 0,$$

that is

$$\begin{pmatrix} P_\iota & I - \Gamma_k \\ * & P_\iota^{-1} \end{pmatrix} \geq 0.$$

From the Schur complement, we have

$$P_\iota - (I - \Gamma_k)^T P_\iota (I - \Gamma_k) \geq 0. \tag{24}$$

Combining (23) with (24), we can deduce that

$$V_\iota(t_k, e(t_k)) \leq V_\iota(t_k^-, e(t_k^-)), \quad k \in \mathbb{Z}_+.$$

By simple calculation, it can be verified from (8) that

$$V_\iota(t_k, e(t_k)) \leq V_l(t_k^-, e(t_k^-)). \tag{25}$$

For $t \in [t_{k-1}, t_k], \ k \in \mathbb{Z}_+$, in view of (22) and (25), we have

$$V_\iota(t_k, e(t_k)) \leq V_l(t_k^-, e(t_k^-)) \leq V_l(t_{k-1}, e(t_{k-1})). \tag{26}$$

By the similar proof and Mathematical induction, we can derive that (26) is true for any $m, l, \eta(0) = \eta_0 \in \mathcal{N}, k \in \mathbb{Z}_+$

$$V_\iota(t_k, e(t_k)) \leq V_l(t_k^-, e(t_k^-)) \leq V_l(t_{k-1}, e(t_{k-1})) \leq \cdots \leq V_{\eta_0}(t_0, e(t_0)).$$

Therefore, the system (4) is asymptotically stable in mean square. This completes the proof of Theorem 1.

4. Illustrative example

In this section, we give a example to demonstrate the effectiveness of our theoretical results.

Example 1. Consider system (1) with $n = N = 2$ and the following parameters:

$$A_1 = \begin{bmatrix} 1.9 & -0.18 \\ 4.2 & 3.0 \end{bmatrix}, \ A_2 = \begin{bmatrix} 2.0 & -0.23 \\ 4.1 & 3.1 \end{bmatrix}, \ B_1 = \begin{bmatrix} -1.8 & -0.3 \\ 0.4 & -2.7 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -1.9 & -0.4 \\ 0.3 & -2.8 \end{bmatrix}, \ C_1 = \begin{bmatrix} 2.7 & 0 \\ 0 & 2.2 \end{bmatrix}, \ C_2 = \begin{bmatrix} 2.8 & 0 \\ 0 & 2.1 \end{bmatrix},$$

$$D_1 = 2I, \ D_2 = \begin{bmatrix} 1.8 & 0 \\ 0 & 2.1 \end{bmatrix}, \ E_1 = 0.25I, \ E_2 = \begin{bmatrix} 0.5 & -0.4 \\ -1 & 0.6 \end{bmatrix}, \ J = 0.$$

The activation functions are $g_1(x) = g_2(x) = \tanh(x)$, and the time-varying delays are $\tau_1(t) = 0.8 + 0.3 \sin t$, $\tau_2(t) = 0.65 + 0.25 \sin t$, $\sigma(t) = 0.5 + 0.3 \cos t$, $\rho(t) = 0.6 + 0.2 \cos t$. Then Assumption 1 is satisfied with $\Delta_1 = 0, \Delta_2 = I, \Delta_3 = 0, \Delta_4 = 0.5I$ and $\bar{\tau} = \bar{\tau}_1 = 1.1$, $\bar{\tau}_2 = 0.9, \tau_1' = 0.3, \tau_2' = 0.25$, $\bar{\sigma} = 0.8, \sigma' = 0.3$, $\bar{\rho} = 0.8$, $\rho' = 0.2$.

In this paper, the transition rate matrix is given as follows

$$\Pi = \begin{bmatrix} -0.7 & 0.7 \\ 0.3 & -0.3 \end{bmatrix}.$$

Solving the LMIs (5)-(10) in Theorem 1 by resorting to the Matlab LMI Control Toolbox, we can obtain one feasible solution. The control input vector with state feedback is designed as (4) with

$$Y_{11} = -15.4038I, \ Y_{12} = -12.7158I, \ Y_{21} = 2.0655I, \ Y_{22} = 2.4185I.$$

Therefore, we conclude that system (1) and (2) with (4) can be stochastically asymptotically synchronized.
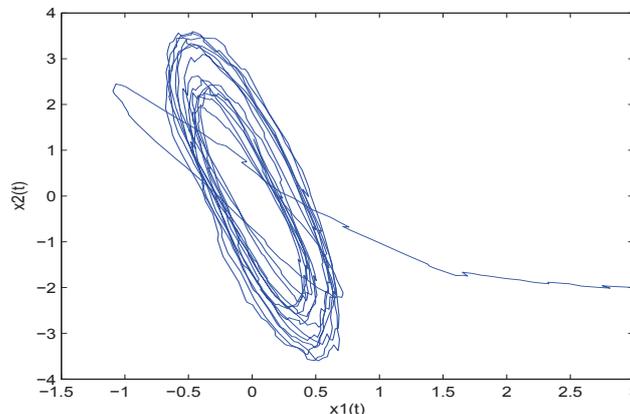
Fig. 1. Chaotic attractor of Example 1.

Fig. 1 shows the neural network model has a chaotic attractor with initial values $x_1(t) = 0.3, x_2(t) = 0.4, \ t \in [-1, 0]$. The initial values of the response system are taken as $y_1(t) = 1.1, y_2(t) = -1.6, \ t \in [-1, 0]$. Fig. 2 shows the error states. By numerical simulation, we can see that the dynamical behaviors of response system (2) synchronize with master system (1).

5. Conclusion

This paper deals with the synchronization problem for a class of neutral-type chaotic neural networks with both leakage delay and Markovian jumping parameters under impulsive
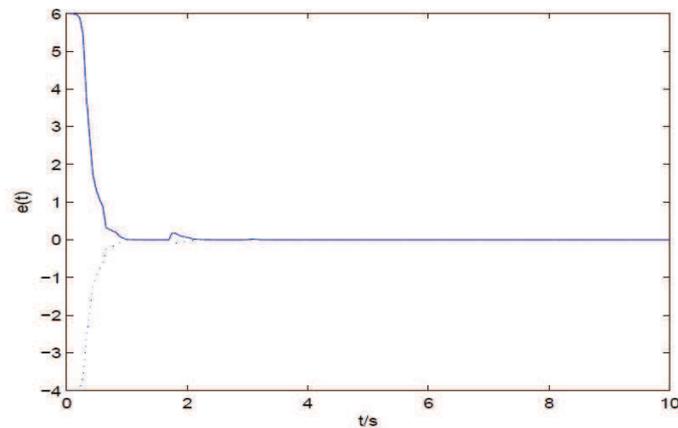
Fig. 2. The error state of $t - e_1(t) - e_2(t)$.

perturbations. By virtue of drive-response concept and time-delay feedback control techniques, by using the Lyapunov functional method, Jensen integral inequality, a novel reciprocal convex lemma and the free-weight matrix method, a novel sufficient condition is derived to assure the stochastic synchronization of two identical Markovian jumping chaotic delayed neural networks with impulsive perturbation. The proposed results, which do not require the differentiability and monotonicity of the activation functions, can be easily checked via Matlab software. Finally, a numerical example with their simulations is provided to illustrate the effectiveness of the presented synchronization scheme.

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