TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol.12, No.7, July 2014, pp. 5379 ~ 5386
DOI: 10.11591/telkomnika.v12i7.5529

Multi-user Detection Based on Gaussian Sum Particle Filter in Impulsive Noise

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Abstract

In order to improve the performance of multi-user detector, this paper analyses a new algorithm, Gaussian sum particle filter (GSPF). This algorithm approximates the filtering and predictive distributions by weighted Gaussian mixtures and is basically banks of Gaussian particle filters (GPF). Then, GSPF is used in dynamic state space (DSS) models with non-Gaussian noise. The non-Gaussian noise is approximated by Laplace noise and Alpha noise. As a result, GSPF can effectively reduce the bit error rate of the system. The simulation results show that the GSPF has the versatility and super performance in MUD. It proves that the improved algorithm has important value for the research of MUD system.

Keywords: *particle filter, Gaussian sum particle filter, multi-user detection, Bayesian estimation*

1. Introduction

CDMA suffers from multiple-access interference (MAI) with all cellular system, and the fact of strongly powered users masking weaker users is known as the near-far effect. Since the multi-user detection which is served as one of the key technology of 3G and B3G mobile communications is put forward by K.S.Schneider in 1979, it quickly received a great deal of attention due to its potential for reducing the effects of MAI, combating with the near-far problem and thereby increasing the capacity of CDMA system. In 1986, optimum multi-user detection algorithm is put forward by Verdu [1]. The performance of this new algorithm can approach the single-user receiver. Because the optimum MUD is exponential in complexity, this method can not be achieved basically in engineering. Then many approximate detectors have been presented to reduce the complexity, such as decorrelation detector and MMSE detector [2]. However, these detectors use interim hard decisions, there performance is suboptimal. In these algorithms, the background noise is assumed to be Gaussian white noise. In actual life, a lot of noises are non-Gaussian noise [3]. So the performances of these algorithms will reduce seriously in non-Gaussian noise environment. Multi-user detection technology still needs continuous improvement.

In 1993, N.J.Gordon proposed particle filter (PF) method, which is used for tricking signals in the reference [4]. Most recently, particle filter is used for simultaneous localization and mapping and human tracking[5][6]. This technique can be applied to non-linear non-Gaussian system. In other words, the algorithm has a strong robustness. PF algorithm, a Bayesian-based solution, is applied to CDMA MUD in [7]. But particle degradation is a significant drawback of the particle filter. Gaussian sum particle filter (GSPF) algorithm can solve this problem. In this paper, we analyze the standard particle filter and Gaussian sum particle filter. The simulation results show that GSPF can effectively reduce the bit error rate of the system.

This paper is organized as follow: Section 2 includes the model of the CDMA system and the presentation of two kinds of non-Gaussian noises; the application of the standard particle filter algorithm for multi-user detection is presented in Section 3; Multi-user detection based on GSPF is presented in Section 4; Simulation results are described in Section 5 and conclusion is presented in Section 6.

Received November 21, 2013; Revised February 18, 2014; Accepted March 6, 2014

2. Introduction of System Model
2.1. CDMA System Model

Considering a synchronous CDMA system with K users and symbol interval is T. The received signal can be given by:

$$r(t) = \sum_{k=1}^{K} A_k(t) g_k(t) b_k(t) + n(t) \tag{1}$$

In the above formula, A_k is amplitude of the k-th user's signal; $g_k \in \{-1,+1\}$ is the normalized deterministic signature waveform assigned to k-th user; $b_k \in \{-1,+1\}$ is the k-th user symbol; $n(t)$ is background noise.

The matched filter output y_k of the k-th user is expressed as [8]:

$$\begin{aligned} y_k &= \int_0^T r(t) g_k(t) dt \\ &= A_k b_k + \sum_{\substack{i=1 \\ i \neq k}}^{K} \rho_{i,k} A_i b_i + \frac{1}{T_b} \int_0^{T_b} n(t) g_k(t) dt \\ &= A_k b_k + MAI_k + z_k \end{aligned} \tag{2}$$

Where $A_k b_k$ is the signal of the k-th user; MAI_k is multiple access interference (MAI); z_k is noise. $\rho_{i,k}$ is the cross-correlation between the i-th user and the k-th user. It is defined to be:

$$\rho_{i,k} = \frac{1}{T_b} \int_0^{T_b} g_k(t) g_i(t) dt \tag{3}$$

When $i \neq k$, $0 \leq \rho_{i,k} < 1$; when $i = k$, $\rho_{i=k} = 1$. The larger the $\rho_{i=k}$ is, the stronger the cross-correlation between different users is.

In order to simplify analysis, the received vector can be expressed as matrix.

$$y = RAb + z \tag{4}$$

$A = diag\{A_1, A_2, \cdots, A_K\}$ is a diagonal matrix of the power of the corresponding received signal. $b = [b_1, b_2, \cdots, b_K]^T$ is users' data, $\mathbf{R}$ is a symmetric correlation matrix. $z = [z_1, z_2, \cdots, z_K]^T$, it is a complex-valued vector with independent real and imaginary components and covariance matrix equal to $\sigma^2 R$.

$\mathbf{R}$ is a symmetric matrix, Colicky factorization can be employed. There is a unique lower triangular matrix F such that $R = F^T F$. We apply F^{-T} to Equation (4), we can obtain [9]:

$$\overline{y} = F^{-T} y = FAb + \overline{z} \tag{5}$$

It can be proved that the covariance matrix of $\overline{z}$ is $\sigma^2 I$, where I is the identity matrix. Because the noise becomes independent and identically distributed, white noise, $\overline{y}$ is called the whitened matched filter output. Scalar expression of the received signal can be expressed as:

$$\overline{y_k} = \sum_{l=1}^{K} F_{k,l} a_l b_l + \overline{z_k} \tag{6}$$

The purpose of the detection is to detect signals of the users $b_{1:K} = \{b_1, b_2, \cdots, b_K\}$ from the matched filter output signals $\bar{y}_{1:K} = \{\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_K\}$.

2.2. Non-Gaussian Noise Simulation

In order to simplify mathematical analysis, background noise is often assumed as Gaussian noise. But this assumption is not very accurate. In practice, many kinds of noises do not have Gaussian nature, such as thunder and lightning, ice avalanches, all kinds of machine motors, neon signs, etc. These kinds of noise show significant peak amplitude in the time domain. In order to prevent the detection performance decline under the “noise spikes”. It is very necessary to establish a more accurate model. Two models of the non-Gaussian noise will be discussed briefly in the followings.

Firstly, introduce the Laplace noise which is one kind of non-Gaussian noise. Laplace probability density function (PDF) can be expressed as [3]:

$$p(x) = \frac{1}{\sqrt{2\sigma^2}} \exp(-\sqrt{\frac{2}{\sigma^2}}|x|) \tag{7}$$

In the above formula, σ^2 is variance or power of the noise. Laplace PDF has an obvious smearing. This is the main difference between the Laplace noise and the Gaussian noise.

Secondly, introduce another kind of non-Gaussian noise, Alpha stable noise. If the random variable X was subject to the Alpha stable distribution, its characteristic function is expressed as [10, 11]:

$$\phi(u) = \exp\{jau - \gamma|u|^\alpha [1 + j\beta \operatorname{sgn}(u)\omega(u,\alpha)]\} \tag{8}$$

$$\omega(u,\alpha) = \begin{cases} \tan(\pi\alpha/2) \cdots\cdots \alpha \neq 1 \\ (2/\pi)\log|u| \cdots \alpha = 1 \end{cases} \tag{9}$$

$$\operatorname{sgn}(u) = \begin{cases} 1, \cdots\cdots u > 0 \\ 0, \cdots\cdots u = 0 \\ -1, \cdots u < 0 \end{cases} \tag{10}$$

In these formulas, $\alpha \in (0,2]$ is called characteristic index, it determines the degree of the distribution pulse characteristics. The smaller of the α, the more obvious of the pulse characteristics. When $0 < \alpha < 2$, this distribution is named fractional lower order Alpha stable distribution. When $\alpha = 2$, it is Gaussian distribution which its mean is a and variance is $2\sigma^2$. In other words, the Gaussian distribution is a special case of the α-stable distribution. $-1 < \beta < 1$, it is symmetry parameter which can control the gradient of the distribution. When $\beta = 0$, this distribution is a symmetric α-stable distribution and denoted as $S\alpha S$. When $\alpha = 1$ and $\beta = 0$, this distribution is called Cauchy distribution. γ is scattering coefficient. It is similar to the variance of the Gaussian distribution. Alpha stable noise’s power can be expressed approximately as 2γ, but 2γ is not equal completely to the true power. Signal to Noise Ratio (SNR) can be expresses as $SNR = S/2\gamma$ (S is the signal’s power). a is a real number. When $a = 0$, $\gamma = 1$, this distribution is named standard α-stable distribution.

3. Multi-user Detection Based on Standard Particle Filter Algorithm

Particle filter (PF) algorithm is a Monte Carlo method which is based on Bayesian theory. In this algorithm, posterior distribution of the states is represented by importance sampling and resampling. Its core idea is to express the posterior distribution by a set of samples with associated weights, and using them to compute estimates of the signals. All the receiving data and the prior information are combined in the posterior distribution $p(b_{1:k} \mid \overline{y}_{1:k})$. If the samples were from the accurate posterior probability distribution $p(b_{1:k} \mid \overline{y}_{1:k})$, each sample has the same weight. However, in practice, $p(b_{1:k} \mid \overline{y}_{1:k})$ has not the typical solution. So the sampling process is very difficult to achieve. In order to solve this problem, the particles are often obtained from an importance density function $q(b_{1:k} \mid \overline{y}_{1:k})$. The weights of particles are defined as [12, 13]:

$$\omega_k^i \propto \frac{p(x_{1:k}^i \mid \overline{y}_{1:k})}{q(x_{1:k}^i \mid \overline{y}_{1:k})} \quad i = 1,2,\cdots,N_s \tag{11}$$

Where i is the trajectory index. The importance density function can be decomposed into:

$$q(x_{1:k} \mid \overline{y}_{1:k}) = q(x_k \mid x_{1:k-1}, \overline{y}_{1:k}) q(x_{1:k-1} \mid \overline{y}_{1:k-1}) \tag{12}$$

Assume that the particles $x_{1:k-1}^i$ from $p(x_{1:k-1}^i \mid \overline{y}_{1:k-1})$ have been generated with weights $\omega_{1:k-1}^i$. If particles x_k^i are sampled from the proposal distribution and appended to $x_{1:k-1}^i$, $x_{1:k}^i$ can be obtained.

The posteriori probability density function can be expressed as:

$$p(x_{1:k} \mid \overline{y}_{1:k}) \propto p(\overline{y}_k \mid x_k) p(x_k \mid x_{k-1}) p(x_{1:k-1} \mid \overline{y}_{1:k-1}) \tag{13}$$

By substituting (14) with (12) and (11), we can get the updated formula of importance weights.

$$\omega_k^i \propto \frac{p(\overline{y}_k \mid x_k^i) p(x_k^i \mid x_{k-1}^i) p(x_{1:k-1}^i \mid \overline{y}_{1:k-1})}{q(x_k^i \mid x_{1:k-1}^i, \overline{y}_{1:k}) q(x_{1:k-1}^i \mid \overline{y}_{1:k-1})} = \omega_{k-1}^i \frac{p(\overline{y}_k \mid x_k^i) p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid x_{1:k-1}^i, \overline{y}_{1:k})} \tag{14}$$

In the standard particle filter algorithm, we choose the priori probability density function as the importance density function.

$$q(x_k^i \mid x_{1:k-1}^i, \overline{y}_{1:k}) = p(x_k^i \mid x_{k-1}^i) \tag{15}$$

By substituting Equation (15) with Equation (16), the formula is simplified as:

$$\omega_k^i \propto \omega_{k-1}^i p(\overline{y}_k \mid x_k^i) \tag{16}$$

Normalize the weights:

$$\omega_k^i = \omega_k^i / \sum_{i=1}^{N_s} \omega_k^i \tag{17}$$

When the course of algorithm is finished with the last user K, we can obtain the generated particles and there weights that can approximate the posterior probability density function. For example, the marginalized posterior probability density function $p(x_k \mid \overline{y}_{1:k})$ can be expressed as:

$$p(x_k \mid \overline{y}_{1:K}) \approx \sum_{i=1}^{N_s} \omega_K^i \delta(x_k - x_k^i) \tag{18}$$

In the above formula, $\delta(\cdot)$ is Dirac delta function. We define two vectors, $x_k = [x_k^1, x_k^2, \cdots, x_k^{N_s}]^T$, $\omega_k = [\omega_k^1, \omega_k^2, \cdots, \omega_k^{N_s}]^T$. According to the Maximum A Posterior (MAP) rule, we have:

$$\hat{b}_k = sign({x_k}^T \omega_k) \tag{19}$$

A major problem with the PF algorithm is that most of the particles' weights except for a very few are negligible. This problem is called particles degradation. The degeneracy problem implies that a large computation is wasted on updating the particles whose contribution to the approximation to $p(x_k \mid \overline{y}_{1:k})$ is almost zero. Resampling can solve this problem. The basic idea of resampling is that sample N_s times from the posterior probability density function $p(x_k \mid \overline{y}_{1:K}) \approx \sum_{i=1}^{N_s} \omega_K^i \delta(x_k - x_k^i)$, then new sample set $\{x_k^{i^*}\}_{i^*=1}^{N_s}$ can be obtained. The weight of each particle is $\omega_k^j = 1/N_s$.

A suitable measure of degeneracy of the algorithm is defined as:

$$N_{eff} = 1/\sum_{i=1}^{N_s} (\omega_k^i)^2 \tag{20}$$

We set a threshold $N_{threshold}$. If $N_{eff} < N_{threshold}$, resample. As a result, we have no need to resampling at every moment. The complexity of the algorithm can be reduced to a certain extent.

4. Multi-user Detection Based on Gaussian Sum Particle Filter

In order to improve system performance, this paper discusses a new filter, Gaussian sum particle filter (GSPF). GSPF combines Gaussian sum filter and Particle filter. This algorithm can increase the diversity of the particles then improve the system performance.

Assume that we have the predictive distribution:

$$p(x_k \mid \overline{y}_{1:k-1}) = \sum_{j=1}^{G} \omega_{kj} N(x_k; u_{kj}, \varepsilon_{kj}) \tag{21}$$

Where u is mean of the particles, ε is covariance of the particles. They are based on a priori information. G is the number of parallel GPF. After receiving the observation $\overline{y}_k$, the filtering distribution can be approximated as [14]:

$$p(x_k \mid \overline{y}_{1:k}) = C_k \sum_{j=1}^{G} \omega_{kj} p(\overline{y}_k \mid x_k) N(x_k; u_{kj}, \varepsilon_{kj}) \tag{22}$$

Usually, the mean and covariance of $p(x_k \mid \overline{y}_{1:k})$ cannot be obtained accurately. GSPF samples from the importance function $q(x_k \mid \overline{y}_{1:k})$, and then calculates the weights ω_{kj}^i of the particles x_{kj}^i, where i is the trajectory index of the particle. Then the Monte Carlo estimates of the mean and covariance of the state can be expressed as:

$$u_{kj} = \sum_{i=1}^{N_s} \omega_{kj}^i x_{kj}^i, \varepsilon_{kj} = \sum_{i=1}^{N_s} \omega_{kj}^i (u_{kj} - x_{kj}^i)(u_{kj} - x_{kj}^i)^T \quad (23)$$

In this formula, N_s is the total number of the particles. The updated filtering distribution can be expressed as:

$$p(x_k \mid \overline{y}_{1:k}) \approx \sum_{j=1}^{G} \omega_{kj} N(x_k; u_{kj}, \varepsilon_{kj}) \quad (24)$$

Prediction probability distribution can be obtained through the updated filtering distribution.

$$p(x_{k+1} \mid \overline{y}_{1:k}) = \sum_{j=1}^{G} \omega_{kj} \int p(x_{k+1} \mid x_k) N(x_{k;} u_{kj}, \varepsilon_{kj}) dx_k$$

$$\approx \sum \omega_{(k+1)j} N(x_{k+1}; u_{(k+1)j}, \varepsilon_{(k+1)j}) \quad (25)$$

In this formula, $u_{(k+1)j}$ and $\varepsilon_{(k+1)j}$ are obtained from GSPF. In this algorithm, the importance function is expressed as:

$$q(\cdot) = p(x_k \mid \overline{y}_{1:k}) = N(x_k; u_k, \varepsilon_k) \quad (26)$$

In summary, the steps of multi-user detection based on Gaussian sum particle filter are as follows [15]:

(1) For $j = 1, 2, \cdots G$, Sample particles $\{x_{kj}^i\}_{i=1}^{N_s}$ from the importance function $p(x_k \mid \overline{y}_{1:k})$.

(2) For $i = 1, 2, \cdots N_s$, $j = 1, 2, \cdots G$, calculate the weight of the every particle.

$$\omega_{kj}^i = \frac{p(\overline{y}_k \mid x_{kj}^i) N(x_k = x_{kj}^i; u_{kj}, \varepsilon_{kj})}{q(x_{kj}^i \mid \overline{y}_{1:k})} \quad (27)$$

(3) For $j = 1, 2, \cdots G$, calculate the mean and variance.

$$u_{kj} = \frac{\sum_{i=1}^{N_s} \omega_{kj}^i x_{kj}^i}{\sum_{i=1}^{N_s} \omega_{kj}^i}, \varepsilon_{kj} = \frac{\sum_{i=1}^{N_s} \omega_{kj}^i (x_{kj}^i - u_{kj})(x_{kj}^i - u_{kj})^T}{\sum_{i=1}^{N_s} \omega_{kj}^i} \quad (28)$$

(4) Update the weights as:

$$\omega_{kj} = \omega_{(k-1)j} \frac{\sum_{i=1}^{N_s} \omega_{kj}^i}{\sum_{j=1}^{G} \sum_{i=1}^{N_s} \omega_{kj}^i}, \; j = 1,2,\cdots G \tag{29}$$

(5) Normalize the weights:

$$\overline{\omega}_{kj} = \frac{\omega_{kj}}{\sum_{j=1}^{G} \omega_{kj}} \tag{30}$$

(6) For $j = 1,2,\cdots G$, Sample particles from the filter probability distribution $N(x_k; u_{kj}, \varepsilon_{kj})$, denote them as $\{x_{kj}^i\}_{i=1}^{N_s}$.

(7) For $i = 1,2,\cdots N_s$, $j = 1,2,\cdots G$, draw particles from the state transition distribution $p(x_{(k+1)j} \mid x_{kj} = x_{kj}^i)$, then denote them as $\{x_{(k+1)j}^i\}_{i=1}^{N_s}$.

(8) For $j = 1,2,\cdots G$, update the weights.

$$\omega_{(k+1)j} = \overline{\omega}_{kj} \tag{31}$$

(9) Calculate the predicted mean $u_{(k+1)j}$ and covariance $\varepsilon_{(k+1)j}$.

$$u_{(k+1)j} = \frac{1}{N_s} \sum_{i=1}^{N_s} \omega_{(k+1)j} x_{(k+1)j}^i \tag{32}$$

$$\varepsilon_{(k+1)j} = \frac{1}{N_s} \sum_{i=1}^{N_s} (u_{(k+1)j} - x_{(k+1)j}^i)(u_{(k+1)j} - x_{(k+1)j}^i)^T \tag{33}$$

(10) By substituting Equation (24, 25) with Equation (32, 33), Posterior probability density function can be obtained. Turn to (1), and estimate the signals of the next user.

5. Simulation Results

In this simulation, PF and GSPF are applied to synchronous DS_CDMA system. We select 8 users, 31-bit gold spread-spectrum code. Channel noises are additive Gaussian noise, Laplace noise and Alpha stable noise. Alpha stable noise parameters: $\alpha = 1.8$, $\beta = 0$, $\gamma = 1$, $a = 0$. The range of signal to noise ratio (SNR) for all users is -4~10 dB. The number of particles is 50; the number of parallel GPF is 5.

From the simulation, it is clear that the performance of GSPF is better than PF in Gaussian noise environment. GSPF algorithm can obviously improve the performance of the system.

Figure 2 analyzes the error code performance of GSPF detection aiming to Gaussian noise, Laplace noise and Alpha stable noise. We can find that the error code performance of the Gaussian noises and the Laplace noise are almost the same. The error code performance of the Alpha stable noises slightly weakened, as the true power of the Alpha stable noise is not 2γ. But, in the simulation, we assume its power is 2γ. The result also proves that GSPF algorithm has strong robustness.

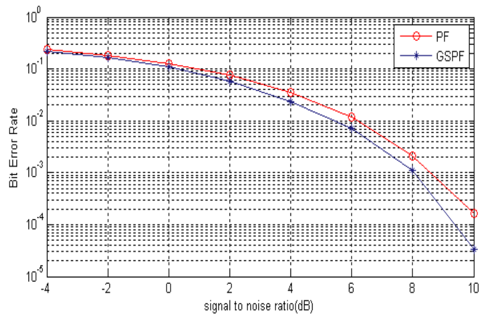

Figure 1. The BER of PF Detection and GSPF Detection

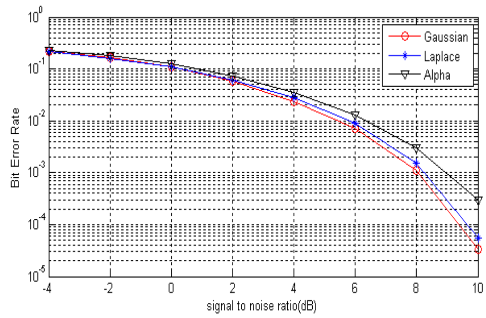

Figure 2. The BER of GSPF Detection under Three Kinds of Noise

6. Conclusion

This paper analyzes the error code performance of PF detection and GSPF detection aiming to Gaussian noise, Laplace noise, and Alpha stable noise. It can be seen that the error performance of GSPF is better than PF. It also can be seen from Figure 2 that the GSPF provide near-optimum performance in non-Gaussian noise environment, which is consistent with multi-user detection result in the real condition. Therefore, GSPF algorithm can obviously improve the performance of the system. Meanwhile, this new algorithm has a strong adaptability in non-Gaussian noise environment. All in all, the research results have important reference value for the research of MUD system. The next word to be done is to modify algorithm so that detection performance achieves better results in non-Gaussian.

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