

Controlled Synchronization by Limited Capacity Communication Channel

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Abstract

The work focuses on synchronization of time-delay chaos systems under information constraints. A theoretical analysis for time-delayed system by limited communication channel is provided. By using the Lyapunov-Krasovskii approach, a synchronization condition is first obtained. Encoder of binary coding is designed by this condition, and synchronization error of this systems tends to zero. Finally, analytical conditions are used for synchronization of time-delay chaos systems under information constraints, and simulations verify the validity of obtained result.

Keywords: chaos synchronization, limited capacity channel, coder

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1. Introduction

Chaos is an unusual sophisticated nonlinear behavior. Though chaos systems are deterministic systems, their behavior is extremely sensitive to initial factors and is unpredictable. Chaos synchronization is to control a chaos system (named responsive system) so that it follows another chaos system (named drive system) [1]. Recently, the applications are widely applied in a variety of areas, see e.g. [2-15].

Under ideal conditions, the standard assumption is that the communication channel is faultless, that is, the obtained output is the same as the input of the observer [16]. Nevertheless it is just not the situation in some actual conditions. For example, in networked control systems, physical plant and controller are not situated in same place, and measured control signals are transmitted via information networks [17]. Recently, limitations of controlling synchronization under constraints imposed by a finite capacity information channel have been well analyzed. Considering the limited information channel, conditions are used to analyze chaos systems synchronization. In this job, synchronization in time-delayed chaos systems by a limited capacity information channel is investigated. We present a theoretical analysis for the coupled systems with time delay. Assuming that input signals are coded and sent under information constraints, the decoder gets the finite coded signals by nonideal information capacity channel. Since only the finite-valued signals are transmitted, ideal analysis cannot be applied to implement stabilization. The Lyapunov-Krasovskii approach and encoding processes are discussed to deal with synchronization for the time-delayed chaotic system by the nonideal capacity information channel.

2. Description of Problem

Considering the coupled system with time delay is of the form:

$$\dot{x} = -ax + \sum_{i=1}^N m_i f(x_{\tau_i}) \quad (1)$$

$$\dot{y} = -ay + \sum_{i=1}^N n_i f(y_{\tau_i}) + K(\bar{x} - y) - u_R \quad (2)$$

Where m_i and n_i are parameters, K represents coupling strength. u_R means error compensation. τ_i is a time function of the form:

$$\tau_i(t) = \tau_0 + \alpha_i \sin(\omega_i t), \quad i = 1, 2, \dots, N \quad (3)$$

And τ_0 means zero-frequency component, α_i represents amplitudes. A flow chart to illustrate the system is given in Figure 1.

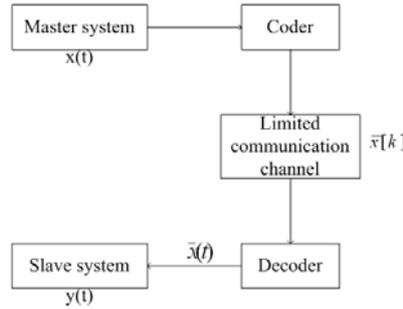


Figure 1. Flow Chart for Master-slave Controlled Synchronization under Information Constraints

Defining synchronization error as following:

$$e(t) = x(t) - y(t) \quad (4)$$

Now question arises when transmission error is considered. At the transmitter side the signal $x(t)$ should be coded and codewords are sent via the limited capacity communication channel at discrete sampling time instants $t_k = kT_s$, $k = 0, 1, 2, \dots$, as T_s represents sampling time. To simplify this analysis, we assume that channel noise and transmission delay may be ignored. Since coded signal be sent via the nonideal capacity channel, namely, $\bar{x}[k] = \bar{x}(t_k)$, transmission error occurs. Assume the coded signals can be on the receiving end by similar sampling time t_k , zero-order extrapolation can be applied to convert the digital sequence $\bar{x}[k]$ to the controller side $\bar{x}(t)$. So, we can obtain $\bar{x}(t) = \bar{x}[k], kT_s \leq t < (k+1)T_s$. And transmission error can be describe in the form of:

$$\delta_x(t) = x(t) - \bar{x}(t) \quad (5)$$

From (4) and (5), synchronization can be designed as:

$$e(t) = \bar{x}(t) - y(t) + \delta_x(t) \quad (6)$$

A positive definite Lyapunov-Krasovskii functional is defined as:

$$V(t) = \frac{1}{2} e^2(t) + p(t) \sum_{i=1}^N \int_{-\tau_i(t)}^0 e^2(t+\theta) d\theta \quad (7)$$

Then,

$$\begin{aligned} \dot{V}(t) &= e(t)\dot{e}(t) + \dot{p}(t) \sum_{i=1}^N \int_{-\tau_i(t)}^0 e^2(t+\theta) d\theta \\ &+ p(t) \times \sum_{i=1}^N [e^2(t) - e^2(t-\tau_i) + e^2(t-\tau_i)\tau'(t)] \end{aligned} \quad (8)$$

Assuming that $\dot{p}(t) \leq 0$ for all t .

$$\begin{aligned} \dot{V}(t) &\leq -(a+K)e^2(t) + \sum_{i=1}^N n_i f'(y_{\tau_i}) e(t)e(t-\tau_i) + p(t)e^2(t) \\ &- p(t) \sum_{i=1}^N (1-\tau'_i) e^2(t-\tau_i) + Ke(t)\delta_x(t) - u_R e(t) \\ &= -(a+K-p(t))e^2(t) + \frac{n_i^2 f'^2(y_{\tau_i})}{4p(t)(1-\tau'_i)} e^2(t) - p(t) \sum_{i=1}^N (1-\tau'_i) \times (e(t-\tau_i) \\ &- \frac{n_i f'(y_{\tau_i})}{2p(t)(1-\tau'_i)} e(t))^2 + K\delta_x(t)e(t) - u_R e(t) \\ &< -(a+K-p(t) - \frac{1}{4p(t)} \sum_{i=1}^N \frac{n_i^2 f'^2(y_{\tau_i})}{1-\tau'_i}) e^2(t) + K\delta_x(t)e(t) - u_R e(t) \end{aligned} \quad (9)$$

We choose error compensation u_R as:

$$u_R = K\delta_x(t) \quad (10)$$

Then,

$$\begin{aligned} V(t) &< -(a+K-p(t) - \frac{1}{4p(t)} \sum_{i=1}^N \frac{n_i^2 f'^2(y_{\tau_i})}{1-\tau'_i}) e^2(t) \\ &= -F(p(t), Q) e^2(t) \end{aligned} \quad (11)$$

Where $Q = \sum_{i=1}^N \frac{n_i^2 f'^2(y_{\tau_i})}{1-\tau'_i}$ and $F(p(t), Q) = a+K-p(t) - \frac{Q}{4p(t)}$. To show $\dot{V}(t) < 0$, that is enough to prove $F_{\min} > 0$. It only happens when $p(t) = \sqrt{Q}/2$ is satisfied with $F_{\min} = a+K - \sqrt{Q}$. At last, a necessary condition for synchronization is described of the form:

$$a+K > \left(\sum_{i=1}^N \frac{|n_i| \|\sup f'^2(y_{\tau_i})\|}{1-\tau'_i} \right)^{1/2} \quad (12)$$

In this way, the system can transmit finite information under the nonideal capacity information channel.

Based on the stable condition, encoding processes for the necessary transmission rate can be designed. Introduce memoryless binary coder to be a discretized map as follows:

$$q_M(x) = M \operatorname{sgn}(x) \quad (13)$$

The range interval is $[-M, +M]$ and $\operatorname{sgn}(\cdot)$ of the form:

$$\text{sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases} \quad (14)$$

Evidently, $|x - q_M(x)| \leq M$ for all x such that $x: |x| \leq 2M$. What is more, every codeword signal includes $R = 1$ bit. Then output of this coder is described as $\bar{x} = q_M(x)$. Consider the sequence of the central numbers $c[k]$, $k = 0, 1, 2, \dots$, where primary condition is $c[0] = 0$. At step k the coder compares the current obtained output $x[k]$ with the number $c[k]$, forming the deviation symbol $\partial x[k] = x[k] - c[k]$. It can be discretized with a given $M = M[k]$. Then output signal:

$$\bar{\partial}x[k] = q_{M[k]}(\partial x[k]) \quad (15)$$

Represents an R -bit communication symbol. Then we can describe $c[k+1]$, $M[k]$ as follow:

$$c[k+1] = c[k] + \bar{\partial}x[k], \quad c[0] = 0, \quad k = 0, 1, \dots, \quad (16)$$

$$M[k] = (M_0 - M_\infty)\rho^k + M_\infty, \quad k = 0, 1, \dots, \quad (17)$$

$0 < \rho \leq 1$ represents decay parameter, M_0 is initial value and M_∞ stands for extremum of $M[k]$. To meet all the range of initial values of x_0 , M_0 must be large enough. Equations (15), (16), and (17) discuss the encoder procedure. The procedures for this decoder are similar to the coder.

In order to study the relationship between transmission rate and the achievable accuracy of the coder and decoder, we suppose that growth rate of $x(t)$ is uniformly bounded. The accurate bound L_x for the rate of $x(t)$ is $L_x = \sup_{x \in \Omega} |C\dot{x}|$, where \dot{x} is from (1). We assume the upper bound of the transmission error is $Z = \sup_t |\delta_x(t)|$. The total transmission error for each interval $[t_k, t_{k+1}]$ should satisfy $|\delta_x(t)| \leq M + L_x T_s$. $|\delta_x(t)| \leq Z = 2M$ is the sufficient condition for all t . Sampling interval T_s must meet $T_s < Z/L_x$. If above conditions hold, coding interval δ should satisfy $\delta < 2Z - 2L_x T_s$. We hope transmission rate in shortest sampling time is large enough to achieve high efficiency. Transmission rate R need be greater than or equal to $\log_2\left(\frac{M}{Z - L_x T_s} + 1\right)$, so it satisfies the inequality.

$$R \geq \log_2\left(\frac{M}{Z - L_x T_s} + 1\right) \quad (18)$$

If T_s is small and R is large enough, then an arbitrarily small value of Z can be assured.

3. Simulation Results

For simulation we think about the time delayed system in the following form:

$$\dot{x} = -ax + m_1 \sin x_{\tau_1(t)} \quad (19)$$

$$\dot{y} = -ay + n_1(t) \sin y_{\tau_1(t)} + K(\bar{x} - y) - u_R \quad (20)$$

Following parameter values of this coupled system are chosen: $a = 1.0$, $m_1 = 4.0$, $\tau_0 = 2.0$, $a_1 = 0.05$, $\omega_1 = 0.001$, $K = 2.01$, $t_{fin} = 1000s$, $L_x = 45$, $M_0 = 5$, $M_\infty = Z/2$, $\rho = \exp(-0.1T_s)$, and $Z = 0.2, 0.4, \dots, 3.0$. The system exhibits a synchronization behavior, see in Figure 2.

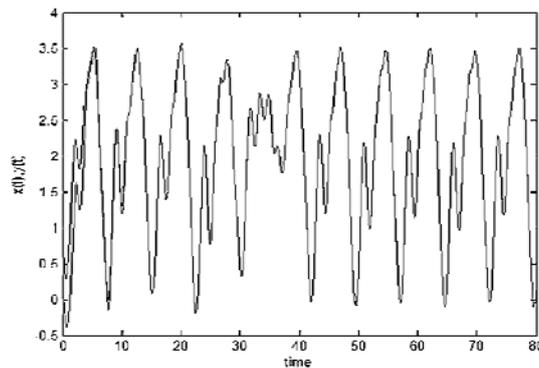


Figure 2. Time Histories of State Variables of Drive and Master Systems (19) and (20)

Figure 3 shows the state of the synchronization error system.

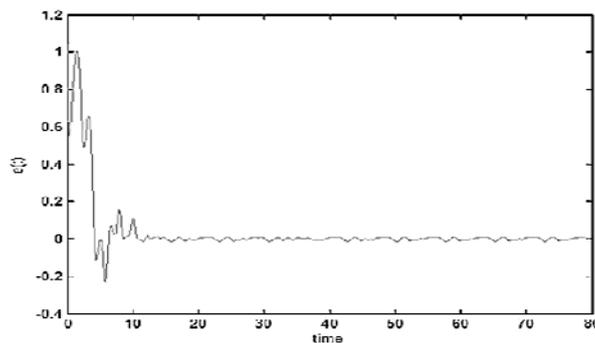


Figure 3. The Synchronization Error

4. Conclusion

Questions of synchronization of the time delayed chaotic systems under information constraints are considered. Based on the Lyapunov-krasovskii approach, a simple condition is obtained to ensure synchronization. Furthermore, encoder analysis deal with these chaotic systems clearly to make sure synchronization error tends to 0. Above proposed method is successfully used to the system with time delay. Moreover, numerical simulations illustrate the feasibility.

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