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Anti-interference Tracking Methods of Maneuvering Target for Structure Random Jump Systems

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Abstract

In this article, a study of anti-interference tracking method of maneuvering target for nonlinear structure random jump systems (SRJSs) with random interference was investigated. Inview of the random interference problem of the tracking system, the nonlinear Gaussian approximation filtering (NGAF) was applied to achieve anti-interference tracking of maneuvering target in observation noises environment with the pip interference signal. Inview of the defects of the NGAF algorithm, bootstrap filtering (BSF) algorithm of SRJSs was applied to avoid the loss of information caused by neglecting the higher-order terms. A meaningful example of radar/IR dual-mode compound seeker is presented to illustrate the effectiveness of the authors'methods, the performances of extended Kalman filtering (EKF), NGAF and BSF in terms of stability, accuracy and computational complexity were compared. The purpose of this paper was to demonstrate the effectiveness of applying the NGAF and BSF on anti-interference target tracking problems of SRJSs, which in the past the factors of randomness and structure uncertainties characteristics had been rarely considered for the studies of maneuvering target tracking, mostly the structures and the parameters of the tracking system were invariant and certainty, and had typically been solved by Kalman or extended Kalman filters.

***Keywords**: structure random jump systems (SRJSs), target tacking, anti-interference, nonlinear Gaussian approximate filtering (NGAF), bootstrap filtering (BSF)*

1. Introduction

Structure random jump systems (SRJSs) extensively exist in practice, which involve both randomness and structure uncertainties characteristics [1]. SRJSs have been used to model the system with variable parameters and structures caused by sudden environment changes, working modes switch, interference exists, faults occurred in components and sudden target motorizes etc. in many fields such as target tracking [2, 3], process monitoring and fault detection [4]. However, in the past, the studies of maneuvering target tracking rarely consider factors above mentions, mostly the structures and the parameters of the tracking system are invariant and certainty. Moreover, information processing problem in intherference environment also is one of the hot spots in the field of randomness system [5].

For the maneuvering target tracking problem, the Kalman filter (KF) method is one of the most popular tools used to estimate states from systems [6, 7]. It may be applied on linear dynamic systems in the presence of Gaussian white noise, and it provides an elegant and statistically optimal solution by minimizing the mean-squared estimation error. However, in practice, all systems in nature are in fact nonlinear, especially in manerving target tracking system, such that linear estimation techniques may not be used to provide optimal solutions. For this reason, suboptimal techniques may be applied to handle the nonlinearities, its main idea is linear filtering for nonlinear system. Such techniques include the extended Kalman filter (EKF), it is a popular extension of the KF and is commonly used in target tracking [8, 9]. It uses partial derivatives of the nonlinearities in the state dynamic and measurement models, such that linearized approximations are obtained and then used in the estimation process [10]. But due to the information loss in the linearization process, the performances of EKF algorithm are difficult to satisfy the actual requirements in some applications, and more important is that it difficult to cope with random interference.

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In actual process of maneuvering target tracking, the target discharges the random interference (such as photoelectric interference, active or passive interference etc), the information of target tracking breaks off or resumes, the detecting sensors switch each other and so on, all above mention situations can lead to sharp jump of the system structure and paremeters at random times, which makes great challenge in solving the target tracking problems. In this case, the performances of classical Kalman filter will decline rapidly, and even has the phenomenon of divergence. However, nonlinear Gaussian approximation filtering (NGAF) algorithm of SRJSs has satisfactory performances in accuracy and stability, and the caculation is much less than the optimal filtering algorithm of the discrete-time SRJSs [11]. Therefore, NGAF algorithm is a kind algorithm of more suitable for practical engineering.

Although the random interference problems are solved by the anti-interference tracking algorithm based on NGAF algorithm, but it has the defects in solving the system analysis problems [11, 12]. Since the unconditional posterior probability density functions (PDFs) of the system states are the weighted sum of the structure jump vectors, so even the subsystem is linear under each structure state, but its initial states and the noise distributions are the Gaussian distribution, the unconditional and conditional PDFs of the system states are no longer the Gaussian distribution. If the Gaussian distribution is applied to approximate, it will inevitably lead to the decline of the performances of the filter. Inview of the defects existing in the NGAF method, Bootstrap filtering (BSF) is a new nonlinear filtering method based on Smith sampling theorem, which is not strict limited by the system initial state and noise distribution for SRJSs.

Inspired by the above motivations, in this paper, three filters (the commonly used EKF, NGAF, and the relatively new BSF) are applied to deal with anti-interference target tracking problem of radar/IR dual-mode compound seeker [13], and the performances in terms of stability, accuracy and computation are compared.

The organization of this paper is as follows. In Section 2, we describe NGAF algorithm to deal with random interference problem. In Section 3, we propose BSF algorithm to deal with the loss of information caused by neglecting the higher-order terms in NGAF algorithm. The objective of Section 4 is to demonstrate the effectiveness of the methods with a simulation example. Finally, the conclusions are drawn in Section 5.

2. Nonlinear Gaussian Approximate Filtering

2.1. Gaussian Approximate Filtering

For the case of linear system with Gaussian white noise, the state equation and observation equation of the target motion may be described separately as follows:

$$\begin{array}{c} \boldsymbol{X}(k+1)=\boldsymbol{F}(s,k)\boldsymbol{X}(k)+\boldsymbol{w}(s,k) \\ \boldsymbol{Z}(k+1)=\boldsymbol{H}(s,k+1)\boldsymbol{X}(k+1)+\boldsymbol{v}(s,k+1) \end{array} (s\in S=\overline{1,M}) \tag{1}$$

Where $\boldsymbol{X}(k)$ is the state vector and $\boldsymbol{Z}(k)$ is the correponding observation vector at time k, $\boldsymbol{w}(s,k)$ and $\boldsymbol{v}(s,k)$ denote process noise and observation noise, s is the structure label of the system and described by conditional Markov chain with M finite states, $\boldsymbol{F}(s,k)$ and $\boldsymbol{H}(s,k)$ are known function matrix.

Assume that all noises are zero-mean Gaussian random sequences and independent of each other. In addition, the initial values of the target state also obey Gaussian distribution, and are independent with all noises, namely:

$$\begin{cases} \boldsymbol{w}(s,k) \sim \boldsymbol{N}[\boldsymbol{w}(s,k)\,|\,\boldsymbol{\mu}(s,k),\Xi_w(s,k)] \\ \boldsymbol{v}(s,k) \sim \boldsymbol{N}[\boldsymbol{v}(s,k)\,|\,\boldsymbol{\varepsilon}(s,k),\Xi_v(s,k)] \\ \boldsymbol{X}_0^{(s)} \sim \boldsymbol{N}[\boldsymbol{X}_0^{(s)}\,|\,\boldsymbol{m}_0^{(s)},\boldsymbol{\Theta}_0^{(s)}] \end{cases} \tag{2}$$

Where $X_0^{(s)}$ is the target initial state of s structure, $m(t) = E\{X(t)\}$, $\Theta(t) = E\left\{[X(t) - m(t)][X(t) - m(t)]^T\right\}$, the function $N\{\alpha|\bar{\alpha}, \Xi_\alpha\}$ denotes that the random vectors obey $\bar{\alpha}$ -mean Gaussian distribution with a variance of Ξ_α.

In order to simplicity for writing, the time k is omitted, thus assume that $s = s(k+1)$, $r = s(k)$, $z_{k+1} = Z(k+1)$, $Z^k = [Z(0), Z(1), \cdots, Z(k-1), Z(k)]$.

According to Equation (1)~(2), the conditional transition PDF $f[X(k+1)|X, r]$ of the target motion state $X(k+1)$ is defined as follows:

$$\begin{aligned} f[X(k+1)|X, r] &= f[X(k+1)|X(k), s(k)] \\ &= N[X(k+1)|F(r,k)X(k|r,k) + \mu(r,k), \Xi_w(r,k)] \end{aligned} \quad (3)$$

The conditional transition PDF $f[z_{k+1}|X(k+1), s]$ of the observation vector $Z(k+1)$ is defined as follows:

$$f[z_{k+1}|X(k+1), s] = N[z_{k+1}|H(s,k+1)m(k+1|s,k) + \varepsilon(s,k+1), \Xi_v(s,k+1)] \quad (4)$$

Assume that the process of structure jump has nothing to do with the system state, it only depends on the former structure state of the system, and thus the transition probability of structure state has the following relation established.

$$q[s, k+1|X(k), r, k] = q[s, k+1|r, k] = q^{(sr)}(k+1, k) \quad (5)$$

Since the random jump and switching of the system structure, the distribution of the target motion state no longer obeies Gaussian type, so the conditional PDF $f[x(k)|r, Z^k]$ may be obtained by Gaussian approximate method as follows:

$$f[X(k)|r, Z^k] = N[X(k)|m(k|r,k), \Theta(k|r,k)] \quad (6)$$

Therefore, Gaussian approximate filtering equations can be derived by using optimal filtering equations of discrete time SRJSs as follows [14].

(a) the state prediction:

$$m(k+1|r,k) = F(r,k)m(k|r,k) + \mu(r,k) \quad (7)$$

(b) the covariance prediction:

$$\Theta(k+1|r,k) = F(r,k)\Theta(k|r,k)F^T(r,k) + \Xi_w(r,k) \quad (8)$$

(c) the mixed state prediction:

$$m(s,k+1|r,k) = m(k+1|r,k) + K_{k+1}^{(sr)} e_{k+1}^{(sr)} \quad (9)$$

(d) the mixed covariance prediction:

$$\Theta(s,k+1|r,k) = [I - K_{k+1}^{(sr)} H(s,k+1)]\Theta(k+1|r,k) \quad (10)$$

(e) the state estimation:

$$\boldsymbol{m}(k+1|s,k+1)=\frac{\sum_{r=1}^{M} q^{(sr)}(k+1,k)\boldsymbol{N}[\boldsymbol{e}_{k+1}^{(sr)}|0,\boldsymbol{\Xi}_{ek+1}^{(sr)}]f(r|\boldsymbol{Z}^{k})\boldsymbol{m}(s,k+1|r,k)}{\sum_{r=1}^{M} q^{(sr)}(k+1,k)\boldsymbol{N}[\boldsymbol{e}_{k+1}^{(sr)}|0,\boldsymbol{\Xi}_{ek+1}^{(sr)}]f(r|\boldsymbol{Z}^{k})} \tag{11}$$

(f) the covariance estimation:

$$\begin{aligned}\boldsymbol{\Theta}(k+1|s,k+1)=&\frac{\sum_{r=1}^{M} q^{(sr)}(k+1,k)\boldsymbol{N}[\boldsymbol{e}_{k+1}^{(sr)}|0,\boldsymbol{\Xi}_{ek+1}^{(sr)}]f(r|\boldsymbol{Z}^{k+1})}{\sum_{r=1}^{M} q^{(sr)}(k+1,k)\boldsymbol{N}[\boldsymbol{e}_{k+1}^{(sr)}|0,\boldsymbol{\Xi}_{ek+1}^{(sr)}]f(r|\boldsymbol{Z}^{k+1})}\{\boldsymbol{\Theta}(s,k+1|r,k)\\&+[\boldsymbol{m}(s,k+1|r,k)-\boldsymbol{m}(k+1|s,k)][\boldsymbol{m}(s,k+1|r,k)-\boldsymbol{m}(k+1|s,k)]^{T}\}\end{aligned} \tag{12}$$

(g) synthesize the state estimation:

$$\boldsymbol{m}(k+1)=\sum_{s=1}^{M} f(s|\boldsymbol{Z}^{k+1})\boldsymbol{m}(k+1|s,k+1) \tag{13}$$

(h) synthesize the covariance estimation:

$$\begin{aligned}\boldsymbol{\Theta}(k+1)=\sum_{s=1}^{M} f(s|\boldsymbol{Z}^{k+1})\{&\boldsymbol{\Theta}(k+1|s,k+1)+[\boldsymbol{m}(k+1|s,k+1)\\&-\boldsymbol{m}(k+1)][\boldsymbol{m}(k+1|s,k+1)-\boldsymbol{m}(k+1)]^{T}\}\end{aligned} \tag{14}$$

(i) the conditional PDF of the system structure state:

$$f(s|\boldsymbol{Z}^{k+1})=\frac{\sum_{r=1}^{M} q^{(sr)}(k+1,k)\boldsymbol{N}[\boldsymbol{e}_{k+1}^{(sr)}|0,\boldsymbol{\Xi}_{ek+1}^{(sr)}]f(r|\boldsymbol{Z}^{k})}{\sum_{s=1}^{M}\sum_{r=1}^{M} q^{(sr)}(k+1,k)\boldsymbol{N}[\boldsymbol{e}_{k+1}^{(sr)}|0,\boldsymbol{\Xi}_{ek+1}^{(sr)}]f(r|\boldsymbol{Z}^{k})} \tag{15}$$

Where,

$$\begin{cases}\boldsymbol{e}_{k+1}^{(sr)}=\boldsymbol{z}_{k+1}-\boldsymbol{H}(s,k+1)\boldsymbol{m}(k+1|r,k)-\boldsymbol{\varepsilon}(s,k+1)\\ \boldsymbol{\Xi}_{ek+1}^{(sr)}=\boldsymbol{H}(s,k+1)\boldsymbol{\Theta}(k+1|r,k)\boldsymbol{H}^{T}(s,k+1)+\Xi_{v}(s,k+1)\\ \boldsymbol{K}_{k+1}^{(sr)}=\boldsymbol{\Theta}(k+1|r,k)\boldsymbol{H}^{T}(s,k+1)[\boldsymbol{\Xi}_{ek+1}^{(sr)}]^{-1}\end{cases} \tag{16}$$

The initial conditions are calculated as follows:

$$\begin{aligned}&\boldsymbol{e}_{0}^{(s)}=\boldsymbol{Z}(0)-\boldsymbol{H}(s,0)\boldsymbol{m}_{0}^{(s)}-\boldsymbol{\varepsilon}_{0}^{(s)}\\&\boldsymbol{\Xi}_{e0}^{(s)}=\boldsymbol{H}(s,0)\boldsymbol{\Theta}_{0}^{(s)}\boldsymbol{H}^{T}(s,0)+\Xi_{v}(s,0)\\&\boldsymbol{m}(0|s,0)=\boldsymbol{m}_{0}^{(s)}+\boldsymbol{K}_{0}^{(s)}\boldsymbol{e}_{0}^{(s)}\\&\boldsymbol{\Theta}(0|s,0)=[\boldsymbol{I}-\boldsymbol{K}_{0}^{(s)}\boldsymbol{H}(s,0)]\boldsymbol{\Theta}_{0}^{(s)}\\&\boldsymbol{K}_{0}^{(s)}=\boldsymbol{\Theta}_{0}^{(s)}\boldsymbol{H}^{T}(s,0)[\boldsymbol{\Xi}_{e0}^{(s)}]^{-1}\\&f[s(0)=i]=\frac{q[s(0)=i]\boldsymbol{N}[\boldsymbol{e}_{0}^{(s)}|0,\boldsymbol{\Xi}_{e0}^{(s)}]}{\sum_{j=1}^{N} q[s(0)=j]\boldsymbol{N}[\boldsymbol{e}_{0}^{(s)}|0,\boldsymbol{\Xi}_{e0}^{(s)}]}\qquad(i=1,2,\cdots,M)\end{aligned} \tag{17}$$

2.2. Improved Gaussian Approximate Filtering for Nonlinear systems

The observation equation is linear above discussed algorithm. However, the observation equations of radar/IR dual-mode compound seeker are nonlinear [13], here Taylor series expansion method is used by centered on $\hat{\boldsymbol{X}}(k+1|r,k)$, and the higher-order terms (more than two-order) are neglected [15].

$$\begin{aligned}\boldsymbol{Z}(k+1) &= \boldsymbol{H}(s,\boldsymbol{X}(k+1)) + \boldsymbol{v}(s,k+1)\\ &= \boldsymbol{H}(s,\hat{\boldsymbol{X}}(k+1|r,k)) + \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{X}}\Big|_{\boldsymbol{X}=\hat{\boldsymbol{X}}(k+1|r,k)}(\boldsymbol{X}(k+1) - \hat{\boldsymbol{X}}(k+1|r,k)) + \boldsymbol{v}(s,k+1)\end{aligned} \tag{18}$$

So the difference between the measurement value and the prediction value in polar coordinate is as follows:

$$\begin{aligned}\tilde{\boldsymbol{Z}}(k+1) &= \boldsymbol{Z}(k+1) - \hat{\boldsymbol{Z}}(k+1|k)\\ &= \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{X}}\Big|_{\boldsymbol{X}=\hat{\boldsymbol{X}}(k+1|r,k)}(\boldsymbol{X}(k+1) - \hat{\boldsymbol{X}}(k+1|r,k)) + \boldsymbol{v}(s,k+1)\end{aligned} \tag{19}$$

From above disscussed, we can see that NGAF algorithm is a manevuing target tracking method based on “soft swithing” between the different models, it belongs to nonlinear parameters self-adptive filtering algorithm, the outputs of these models are synthetically caculated by the probability whights, this will improve the estimation accuracy and convergence of the filtering algorithm.

3. Bootstrap Filtering

The main idea of Bootstrap filtering method is that PDFs are represented to a set of random samples, the Bootstrap filter uses random sampling to transmit and update these samples, and to ensure that these samples are concentrated in the high probability density range [11, 12].

3.1. Smith Sampling Theorem

Smith sampling theorem is described as follows. Assume the random samples $\{\boldsymbol{X}_i^*(k), i=\overline{1,N}\}$ can be obtained from the continous PDFs $\boldsymbol{\chi}(\boldsymbol{x})$, and these samples are proportion to $\boldsymbol{L}(\boldsymbol{x})\boldsymbol{\chi}(\boldsymbol{x})$ in accordance with the requirements of PDFs, where $\boldsymbol{L}(\boldsymbol{x})$ is a known function. A sample is obtained by the discrete distribution of $\{\boldsymbol{X}_i^*(k), i=\overline{1,N}\}$, the corresponding probability factors of $\boldsymbol{X}_i^*(k)$ may be caculated as follows:

$$q_i(k) = \frac{\boldsymbol{L}[\boldsymbol{X}_i^*(k)]}{\sum_{j=1}^{N}\boldsymbol{L}[\boldsymbol{X}_j^*(k)]} \tag{20}$$

From Smith sampling theorem, it can be seen that as N tends to infinity, the distribution tends to be needed probability density. The random samples $\{\boldsymbol{X}_i^*(k), i=\overline{1,N}\}$ are obtained by random sampling PDFs $f[\boldsymbol{X}(k)|\boldsymbol{Z}^k]$, and the forecasting samples are obtained according to known state equation, then according to the approximation distribution of PDFs $f[\boldsymbol{X}(k+1)|\boldsymbol{Z}^{k+1}]$, the random samples $\{\boldsymbol{X}_i^*(k+1), i=\overline{1,N}\}$ are obtained after weighted and update.

The state equation and observation equation of SRJSs are separately as follows:

$$\begin{aligned}\boldsymbol{X}(k+1) &= \boldsymbol{f}[\boldsymbol{X}(k), s(k), \boldsymbol{w}(k)]\\ \boldsymbol{Z}(k+1) &= \boldsymbol{h}[\boldsymbol{X}(k+1), s(k+1), \boldsymbol{v}(k+1)]\end{aligned} \tag{21}$$

Where $\boldsymbol{X}(k)$ denote the state vector, $\boldsymbol{w}(k)$ and $\boldsymbol{v}(k)$ denote system noise and observation noise, $\boldsymbol{F}(\cdot)$ and $\boldsymbol{H}(\cdot)$ are given nonlinear function, $s(k) \in S = \overline{1, M}$ is Markov chain with M finite states, its transition probability is as follows:

$$q[s, k+1 | \boldsymbol{X}(k), r, k] = q^{sr}(\boldsymbol{X}, k+1, k) \quad (22)$$

3.2. The steps of Bootstrap Filtering

Bootstrap filtering of SRJSs mainly consists of seven steps as follows [11, 14].

Step 1: the random samples $\{\boldsymbol{X}_i(0), i = \overline{1, N}\}$ are obtained by random sampling according to known initial PDF $q[\boldsymbol{X}(0)]$ of the system state vector, its initial condition is $q[\boldsymbol{X}(0) | s(0)]$, $q[s(0)]$, $q[\boldsymbol{X}(0)]$ is defined as follows:

$$q[\boldsymbol{X}(0)] = \sum_{i=1}^{M} q[\boldsymbol{X}(0) | s(0) = i] q[s(0) = i] \quad (23)$$

Step 2: the random samples $\{\boldsymbol{w}_i^{(s)}(k), i = \overline{1, N}, s = \overline{1, M}\}$ are obtained by random sampling according to known PDF $q[\boldsymbol{w}(k) | s(k)]$ of the system noise vector.

Step 3: one step precasting samples $\bar{\boldsymbol{X}}_i^{(s)}(k+1)$ are obtained by the system state equation, $\bar{\boldsymbol{X}}_i^{(s)}(k+1)$ are defined as follows:

$$\bar{\boldsymbol{X}}_i^{(s)}(k+1) = \boldsymbol{f}[\boldsymbol{X}_i(k), s(k), \boldsymbol{w}_i^{(s)}(k)] \quad (24)$$

Step 4: the samples $\boldsymbol{X}_i^*(k+1)$ are obtained as follows:

$$\boldsymbol{X}_i^*(k+1) = \sum_{j=1}^{M} \bar{\boldsymbol{X}}_i^{s}(k+1) f[s(k) = j | \boldsymbol{Z}^k] \quad (25)$$

Step 5: the normalized weighted factors $\{\boldsymbol{X}_i(0), i = \overline{1, N}\}$ are caculated by equation (24), namely PDFs of probability factors $q_i(k+1)$ and the system structure $f(s(k+1) | \boldsymbol{Z}^{k+1})$ are defined as follows:

$$q_i(k+1) = f[\boldsymbol{X}_i^*(k+1) | \boldsymbol{Z}^{k+1}] = \frac{\sum_{j=1}^{M} \boldsymbol{A}^*[\boldsymbol{X}_i^*(k+1), s(k+1) = j, \boldsymbol{z}_{k+1}]}{\sum_{j=1}^{M} \sum_{i=1}^{N} \boldsymbol{A}^*[\boldsymbol{X}_i^*(k+1), s(k+1) = j, \boldsymbol{z}_{k+1}]} \quad (26)$$

$$f(s(k+1) | \boldsymbol{Z}^{k+1}) = \frac{\sum_{i=1}^{N} \boldsymbol{A}^*[\boldsymbol{X}_i^*(k+1), s(k+1), \boldsymbol{z}_{k+1}]}{\sum_{j=1}^{M} \sum_{i=1}^{N} \boldsymbol{A}^*[\boldsymbol{X}_i^*(k+1), s(k+1) = j, \boldsymbol{z}_{k+1}]} \quad (27)$$

Where,

$$\boldsymbol{A}^*[\boldsymbol{X}_i^*(k+1), s(k+1), \boldsymbol{z}_{k+1}] = \sum_{l=1}^{M} \sum_{j=1}^{N} \boldsymbol{A}[\boldsymbol{X}_i^*(k+1), \boldsymbol{x}_j(k), s(k+1), s(k) = l, \boldsymbol{z}_{k+1} | \boldsymbol{Z}^k] \quad (28)$$

$$\begin{aligned} & \boldsymbol{A}[\boldsymbol{X}_i^*(k+1), \boldsymbol{X}_j(k), s(k+1), s(k), \boldsymbol{z}_{k+1} | \boldsymbol{Z}^k] \\ & = f[\boldsymbol{z}_{k+1} | \boldsymbol{X}_i^*(k+1), s(k+1)] f[\boldsymbol{X}_i^*(k+1) | \boldsymbol{X}_j(k), s(k)] \\ & \cdot q[s(k+1) | s(k), \boldsymbol{X}_j(k)] f[\boldsymbol{X}_j(k) | \boldsymbol{Z}^k] f[s(k) | \boldsymbol{Z}^k] \end{aligned} \quad (29)$$

$$\begin{aligned} & f[\boldsymbol{z}_{k+1} | \boldsymbol{X}_i^*(k+1), s(k+1)] \\ & = \int_{-\infty}^{+\infty} \delta\{\boldsymbol{z}_{k+1} - \boldsymbol{h}[\boldsymbol{X}_i^*(k+1), s(k+1), \boldsymbol{v}(k+1)]\} q[\boldsymbol{v}(k+1) | s(k+1)] d\boldsymbol{v}(k+1) \end{aligned} \quad (30)$$

Where $\delta(\cdot)$ is Dirac function. Since the probability of each random sample $\{\boldsymbol{X}_i^*(k+1), i = \overline{1, N}\}$ is equal, so $f[\boldsymbol{X}_i^*(k+1) | \boldsymbol{X}_j(k), s(k)] = 1$, $f[\boldsymbol{X}_j(k) | \boldsymbol{Z}^k] = \frac{1}{N}$.

Step 6: the random samples $\{u(i), i = \overline{1, N}\}$ are extracted by the uniform distribution $(0,1)$, and new random samples $\boldsymbol{X}_i(k+1) = \boldsymbol{X}_l^*(k+1)$ are obtained by resampling according to the probability factors $q_i(k+1)$, so as to realize the update and transmit process of the random samples. The random samples $\{u(i), i = \overline{1, N}\}$ satisfy the relational expression as follows:

$$\sum_{j=0}^{l-1} q_j(k+1) < u(i) \le \sum_{j=0}^{l} q_j(k+1), \quad q_0(k+1) = 0 \quad (l = \overline{1, N}) \quad (31)$$

Step 7: the estimation value and variance of the system states and structure states are calculated separately as follows:

$$\boldsymbol{m}(k+1) = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{X}_i(k+1) \quad (32)$$

$$\hat{s}(k+1) = \arg\max_{s \in S} f[s(k+1) | \boldsymbol{Z}^{k+1}] \quad (33)$$

$$\boldsymbol{\Theta}(k+1) = \frac{1}{N} \sum_{i=1}^{N} [\boldsymbol{X}_i(k+1) - \boldsymbol{m}(k+1)][\boldsymbol{X}_i(k+1) - \boldsymbol{m}(k+1)]^T \quad (34)$$

So much for that, BSF algorithm of SRJSs can be realized by loop running the process of step 2 to step 7.

4. Results and Analysis

Assume that the target motion with constant acceleration in two-dimensional plane, the scanning period of radar and IR seeker are *T*. The constant acceleration (CA) model used for the state equation of the target motion is given by Equation (35) [16, 17].

$$\boldsymbol{X}(k+1) = \begin{bmatrix} 1 & T & 0 & 0 & T^2/2 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & T & 0 & T^2/2 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \boldsymbol{X}(k) + \begin{bmatrix} T^2/4 & 0 \\ T/2 & 0 \\ 0 & T^2/4 \\ 0 & T/2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{w}(k) \quad (35)$$

Where $\boldsymbol{w}=\begin{bmatrix} w_1 & w_2 \end{bmatrix}^T$, $\boldsymbol{w}(k)$ denote Gauss random sequence with zero mean and variance $\boldsymbol{Q}$, w_1 and w_2 are independent and have the same variance σ_α^2, so $\boldsymbol{Q}=\sigma_\alpha^2\boldsymbol{I}$, and satisfy $E[\boldsymbol{w}(k)]=\boldsymbol{0}$, $E[\boldsymbol{w}(k)\boldsymbol{w}^{\mathrm{T}}(j)]=\boldsymbol{Q}\delta_{kj}$.

The state vector of the target may be defined as follows:

$$\boldsymbol{X}(k)=\begin{bmatrix} x(k) & \dot{x}(k) & y(k) & \dot{y}(k) & \ddot{x}(k) & \ddot{y}(k) \end{bmatrix}^T \tag{36}$$

The first two states refer to the position and the velocity along the *x*-axis, respectively, and the next two states refer to the position and the velocity along the *y*-axis, respectively, and the last two states refer to the acceleration along the *x*-axis and *y*-axis, respectively.

Due to the influence of external environment (such as radio interference), the radar measurement errors exist the dramatic jump at random times, namely the pip interference signal exist in the observation system, so the observation equation is as follows:

$$\boldsymbol{Z}_1(k+1)=\boldsymbol{h}_1(\boldsymbol{X}(k+1))+\boldsymbol{v}_1(s,k+1) \qquad (s\in S=1,2) \tag{37}$$

Where,

$$\boldsymbol{Z}_1(k+1)=\begin{bmatrix} r^*(k+1) \\ \theta(k+1) \end{bmatrix} \tag{38}$$

$$\boldsymbol{h}_1(\boldsymbol{X}(k+1))=\begin{bmatrix} \sqrt{x^2(k+1)+y^2(k+1)} \\ \arctan\dfrac{y(k+1)}{x(k+1)} \end{bmatrix} \tag{39}$$

Here in order to distinguish the structure label *r* of the system, the distance measurement is marked r^*, $\boldsymbol{v}_1(2,k)$ denote the pip interference signal. Since the infrared measurement is not affected by the pip interference signal, thus the measurement noise is zero-mean Gaussian distribution with a variance of $R_2(k)$.

The initial conditions of the system noise, observation noise and system state are described as follows:

$$\begin{aligned} &\boldsymbol{w}(k) \sim \boldsymbol{N}[\boldsymbol{w}(k)|0,\boldsymbol{Q}] \\ &\boldsymbol{X}_0^{(s)} \sim \boldsymbol{N}[\boldsymbol{X}_0^{(s)}|\boldsymbol{m}_0^{(s)},\boldsymbol{\Theta}_0^{(s)}] \\ &\boldsymbol{v}_1(1,k) \sim \boldsymbol{N}[\boldsymbol{v}_1(1,k)|\boldsymbol{0},\boldsymbol{R}_1(1,k)] \\ &\boldsymbol{v}_1(2,k) \sim \boldsymbol{N}[\boldsymbol{v}_1(2,k)|\boldsymbol{0},\boldsymbol{R}_1(2,k)] \\ &\boldsymbol{R}_1(1,k) \ll \boldsymbol{R}_1(2,k) \end{aligned} \tag{40}$$

The transition probability of the system state is described as follows:

$$q^{(sr)}(k+1,k)=q(s,k+1|r,k)=q(s) \qquad (s=1,2) \tag{41}$$

Assume that the probability of the pip jamming signal of the observation noise is $q(s=2)=\lambda \ll 1$. The initial conditions of the simulation can be written respectively as follows: *T*=20ms, λ=0.05, $R_1(1,k)$=diag[0.36×10^4(m)^2, 3×10^{-6}(rad)^2], $R_1(2,k)$=diag[9.0×10^4(m)^2, 7.5×10^{-6}(rad)^2], $R_2(k)$=1×10^{-6}(rad)^2, m_0^s =[10000m, 300m/s, 4000m, 150m/s, 5m/s^2, 4m/s^2]^T, σ_α^2 =100.

All error curves applying NGAF algorithm are obtained by Monte Carlo simulation with 50 runs, and the simulation results are shown in Figure 1-5. We can see that whether the target position estimation, velocity estimation and acceleration estimation are very close to the actual trajectory, the root mean square error (RMSE) of the position estimation of *x*-axis is basically maintained at about 20m. From Figure 5, we can clearly see that the error of NGAF algorithm is much smaller than EKF algorithm that without considering the pip interference signal, it effectively overcomes the target tracking difficulties result in the random interference. Obviously, under the condition of random jump and variable in noise characteristics, anti-interference tracking algorithm based on NGAF can accurately track the maneuvering target, its performances are significantly higher than EKF.

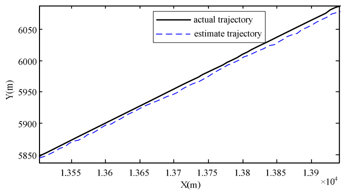

Figure 1. Target Trajectory

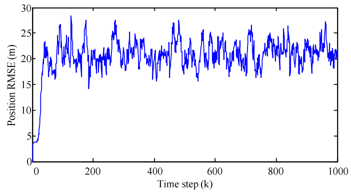

Figure 2. RMS Position Error

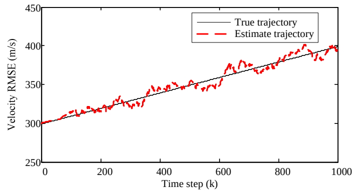

Figure 3. RMS Velocity Error

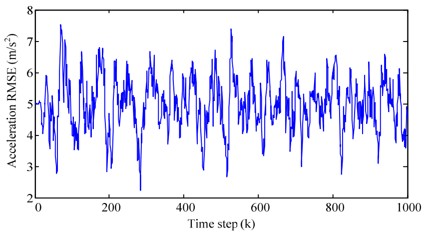

Figure 4. RMS Acceleration Error

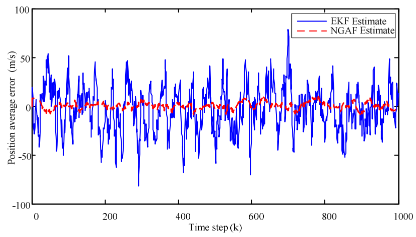

Figure 5. Position Average Error

The simulation conditions as above mentions, the simulation results comparing BSF algorithm with NGAF algorithm are shown in Figure 6-8. 500 groups of samples are used in BSF algorithm. Compared with NGAF algorithm, we can see that the filtering results of BSF algorithm are more close to the true value from the simulation results. Furthermore, RMSE of BSF algorithm is much less than NAGF algorithm, on this account we can see that the performances of BSF algorithm is significantly superior to NGAF algorithm.

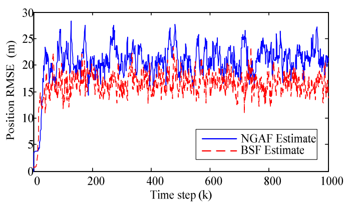

Figure 6. RMS Position Error

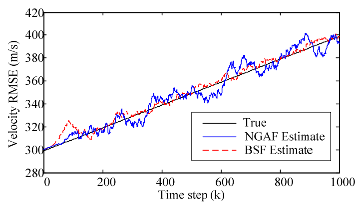

Figure 7. RMS Velocity Error

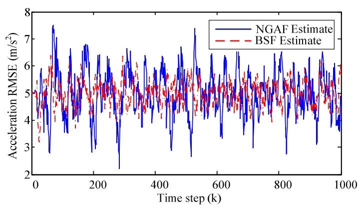

Figure 8. RMS Acceleration Error

However, BSF algorithm is required to operation for each sample in the samples set, thus the computation of BSF algorithm is much larger than NGAF algorithm, and its computation is multiplied increased with the increase of the number of samples. As the law of large number can be seen that the true-values of the samples set are close to real values when the number of samples tends to infinity, therefore the number of samples has a certain influence on the performances of BSF algorithm.

The comparison results are shown in Figure 9-11, which the number of samples is *N*=500 and *N*=1000 respectively. As can be seen from the simulation results, when the sampling number is doubled, the performances of BSF algorithm are almost nothing to improve, but the computation is significantly increased. Therefore, increasing the number of samples may not significantly improve the performances of BSF algorithm, the resolvement method should consider selecting the appropriate sampling numbers to significantly reduce the computation without much loss of the performances of BSF algorithm.

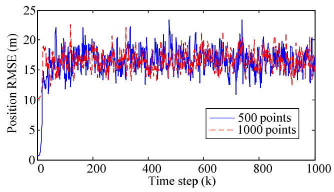

Figure 9. RMS Position Error Comparision

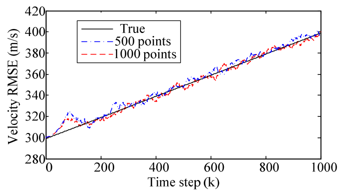

Figure 10. RMS Velocity Error Comparision

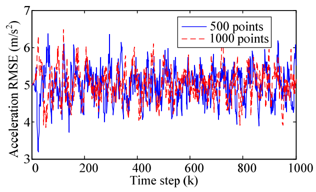

Figure 11. RMS Acceleration Error Comparision

5. Conclusion and future work

The results of applying NGAF and BSF algorithm on anti-interference target tracking problem of SRJSs demonstrate its stability and robustness. It is shown that EKF performs poorly in the presence of random interference and structure uncertainties. However, NGAF and BSF algorithm are able to overcome these difficulties, and provide a stable estimate of the states, so both them have higher performances in accuracy and stability than EKF. The comparison results of NGAF and BSF algorithm demonstrate that BSF algorithm has better adaptability than NGAF algorithm. Firstly, NGAF algorithm is linearized by Taylor series expansion method, so it has the loss of information of higher order terms. Secondly, NGAF algorithm has the defects by Gaussian approximate method. Finally, NGAF algorithm only uses one sample of the sample sets to be the state variables, by contrast, the sample sets of BSF algorithm carries a large amount of information, so it can more accurately represent real value than NGAF algorithm. Since BSF algorithm is not strict limited by the system initial state and noise distribution, so BSF algorithm may overcome the defects of NGAF algorithm, and its performances is significantly superior to NGAF algorithm. However, the number of samples has a certain influence on the accuracy and computation of BSF algorithm. It is noted that the accuracy and computation are two designer-chosen performances and further investigation on the quantified relation between those performances is expected in future.

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