Adaptive Neural Network Approach for a Class of Uncertain Non-affine Nonlinear Systems

Hui HU^{*1}, Wang Yingjun², Xilong Qu³

¹Dept of Electrical and Information Engineering, Hunan Institute of Engineering, Hunan Xiangtan, China, ²School of Information & Engineering, Henan Institute of Science and Technology, Xinxian, China, ³Dept of Computer Science, Hunan Institute of Engineering, Hunan Xiangtan, China *Corresponding author, e-mail: onlymyhui@126.com¹, wangyingjun@sina.com², quxilong@126.com³

Abstract

The paper proposes a new output feedback adaptive tracking control scheme using neural network for a class of uncertain non-affine nonlinear systems that only the system output variables can be measured. The scheme adopts low-pass filter to transform non-affine nonlinear systems into affine in the pseudo-input dynamics. No state observer is employed and few adapting parameters to be tuned and Lipschiz assumption, SPR condition is not required. Only the output error is used in control laws and weights update laws which make the system construct simple. Boundedness for the output tracking error and all states in the closed-loop system are guaranteed, and simulation results have verified the effectiveness of the proposed approach.

Keywords: neural network, non-affine nonlinear systems, uncertain, output feedback

Copyright © 2014 Institute of Advanced Engineering and Science. All rights reserved.

1. Introduction

In the past decade, the adaptive control has seen rapid and significant development leading to global stability and asymptotic tracking results for large classes of uncertain nonlinear systems. In recent years, fuzzy logic control [1-8] and adaptive neural network [9-12] that model the functional mechanism of the human brain that can cooperate with human expert knowledge have been successfully applied to many control problems because they need no accurate mathematical models of the system under control. Likewise, for a class of nonlinear continuoustime systems, adaptive direct and indirect control using fuzzy logic have been proposed in [3, 4] by using "dominate inputs" concept. Controllers in [3, 4] using a state feedback approach is valid if all of the system states are available for measurement. In practice, however, the state feedback control does not always hold because system states are not always available. Based on [3, 4], references [5, 6] present adaptive output control algorithms based on state observer and error observer. Most of them deal with the control problem of the affine nonlinear systems. However, in practice, the control methods of affine nonlinear systems do not always hold and the control methods of the non-affine nonlinear systems are necessary. And few results are available for non-affine nonlinear systems in which the control input appears in a nonlinear fashion. In [7] authors addressed the indirect adaptive fuzzy control problem of SISO non-affine nonlinear systems. The approach is based on the approximation of the nonlinear plant dynamics by a fuzzy system and then the control action is computed based on local inversion of the fuzzy model. In [8], an indirect adaptive fuzzy controller is proposed, within this approach, the SISO non-affine nonlinear system is firstly transformed into an affine form by considering a Taylor series expansion around an operating trajectory. However, the indirect adaptive approach has the drawback of the controller singularity problem, i.e., division by zero may occur in the control law. In [9], an observer-based direct adaptive fuzzy-neural control scheme is presented for nonaffine nonlinear systems. By using implicit function theorem and Taylor series expansion and SPR Lyapunov theory, the stability of the close-loop system is verified. Recently, in [10] an output feedback-based adaptive neural controller has been presented for a class of uncertain non-affine nonlinear systems with unmodelled dynamics which reduce the complexity of control design. But in the scheme, a low-pass filter is designed to make the estimation error dynamics satisfy the strictly positive-real (SPR) condition so that they can use Meyer-Kalmon-Yakubovitz (MKY) lemma, which makes the stability analysis of the closed-loop system and real

5285

implementation very complicated. And the parameters of filter are hard to be chosen. In [11], output feedback tracking control scheme is investigated for a class of uncertain nonlinear systems. The distinguished aspect of the algorithm is that no Lipschitz assumption and SPR condition are employed which makes the system construct simple. But the observer must be employed. In order to simplify the design of controller, in [12], an output feedback-based adaptive neural controller has been proposed for a class of uncertain nonlinear systems. No state observer was employed in the algorithm and only the output error was used in control laws and weights update laws.

Based on the above observation, a novel systematic design procedure is developed for non-affine nonlinear systems without state observer to simplify the design of control system. First, a low-pass filter is employed to transform the normal form non-affine nonlinear system into affine in the pseudo-input dynamics. No state observer is employed and the neural weights update laws is tuned according to only the output tracking error. The stability analysis depends heavily on the universal function approximation property, only one RBFN is employed to approximate the lumped uncertain nonlinear function. There are no restrictive conditions on the design constants. The proposed scheme has few adapting parameters to be tuned and Lipschiz assumption, SPR condition are not required.

The paper is organized as follows. First, the problem is formulated in Section II. Adaptive neural network controller design is given in III. In Section IV, stability analysis is included. Simulation results are presented to confirm the effectiveness and applicability of the proposed method in Section V. Finally, conclusions are included.

2. Problem Formulation

The following notations and definitions will be used extensively throughout this paper. Let *R* be the real number, and *Rⁿ* represent the real *n*-vectors. |k| denotes the usual Euclidean norm of a vector *k*. In case where *k* is a scalar, |k| denotes its absolute value.

We consider the following non-affine nonlinear system:

$$\dot{x}_i = x_{i+1} \quad i = 1, \cdots, n-1$$

$$\dot{x}_n = f(\underline{x}, u)$$

$$y = x_1$$

$$(1)$$

Where $y \in R$, $u \in R$ are the outputs and input of the system and $\underline{x} = [x_1, \dots, x_n]^T \in R^n$ is the system state vector. The smooth function $f(\Box)$ is unknown. The states are not measurable, only y is available for control design.

For the controllability issue, the following assumption must be made.

Assuption 1: The value of $\frac{\partial f}{\partial u}$ is nonzero. Without loss of generality, we assume that

for all $\underline{x} \in \mathbb{R}^n$, $\frac{\partial f}{\partial u} \ge f_u > 0$.

The control objective is to design an adaptive neural network controller for a class of non-affine nonlinear systems (1) such that the system output y follows a desired trajectory y_d , while all signals in the closed-loop system are bounded.

In the followings, we will adopt low-pass filter to transform (1) into affine in the pseudoinput dynamics ^[13]. The overall scheme is illustrated in Figure 1.

The transfer function of the low-pass filter is:

$$L(s) = \frac{\kappa}{s + \kappa} \tag{2}$$

Where κ is a positive design constant. Then, although the pseudo-control u_p shows a chattering phenomenon due to a switching function, the actual control input u applied to the real plant is smooth because u is made by low pass filtering of u_p ,

Adaptive Neural Network Approach for a Class of Uncertain Non-affine Nonlinear... (Hui HU)

$$\dot{u} = -\kappa u + \kappa u_{p}$$

$$\underbrace{u_{p}}_{\text{Pseudo-control}} \underbrace{K}_{S+K} \underbrace{u}_{\text{Control}} \underbrace{\text{Real}}_{\text{plant}} \underbrace{y}_{\text{output}}$$

Figure 1. Basic Idea for Smoothing Control

We define the augmented state variable as

$$\underline{\varsigma} = [x_1, x_2, \dots, x_n, x_{n+1}]^T = [y, \dot{y}, \dots, y^{(n-1)}, y^{(n)}]^T \in \mathbb{R}^m \text{ and } \underline{\eta} = [\underline{x}, u]^T \in \mathbb{R}^m \text{ with } m = n+1. \text{ Then:}$$

$$\begin{aligned} \dot{x}_{i} &= x_{i+1} \quad i = 1, 2, \cdots, n-1 \\ \dot{x}_{n} &= f(\underline{x}, u) =: x_{n+1} \\ \dot{x}_{n+1} &= \frac{\partial}{\partial \underline{\eta}} f(\underline{x}, u) \underline{\dot{\eta}} \\ &= \sum_{i=1}^{n} \frac{\partial f}{\partial x_{i}} \dot{x}_{i} + \frac{\partial f}{\partial u} \dot{u} \\ &= \left(\sum_{i=1}^{n} \frac{\partial f}{\partial x_{i}} x_{i+1} - \kappa \frac{\partial f}{\partial u} u\right) + \kappa \frac{\partial f}{\partial u} u_{p} \end{aligned}$$

$$(4)$$

If we define the functions $a(\eta)$ and $b(\eta)$ as:

$$a(\underline{\eta}) = \left(\sum_{i=1}^{n} \frac{\partial f}{\partial x_{i}} x_{i+1} - \kappa \frac{\partial f}{\partial u} u\right)$$

$$b(\underline{\eta}) = \kappa \frac{\partial f}{\partial u}$$
(5)

Where $b(\underline{\eta})$ is nonzero and positive according to Assumption 1. Thus there exist positive constant b_0 such that $b(\underline{\eta}) \ge b_0$ for all $\underline{\eta} \in R^m$. And we can see that the original *n*th-order non-affine nonlinear system becomes the *m*th-order affine in the pseudo-input nonlinear system:

$$\dot{x}_i = x_{i+1} \quad i = 1, 2, \cdots, n$$

 $\dot{x}_m = a(\eta) + b(\eta)u_p$
(6)

3. Adaptive Neural Network Controller Design

Define the reference vector $\underline{y}_d = [y_d \quad \dot{y}_d \quad \cdots \quad y_d^{(n)}]^T \in \mathbb{R}^m$. The reference signal y_d and its time derivative are assumed to be smooth and bounded. We also define the tracking error as $e = y_d - y$ and corresponding error vector as $\underline{e} = \underline{y}_d - \underline{\zeta} = [e, \dot{e}, \quad \cdots \quad e^{(n)}]^T \in \mathbb{R}^m$. A filtered tracking error is defined as:

$$s = \left(\frac{d}{dt} + \lambda\right)^{m-1} e = \left[\tau^T \ 1\right] \underline{e} = \Lambda \underline{e}$$
⁽⁷⁾

Where $\lambda > 0$ is a design constant. $\tau = \left[\lambda^{m-1}, (m-1)\lambda^{m-2}, \cdots, (m-1)\lambda\right]^T$, $\Lambda = \left[\tau^T 1\right]$. The time derivative of *s* is derived as:

(3)

$$\dot{s} = \Lambda_1^T \underline{e} + y_d^{(m)} - y^{(m)}$$

$$= \Lambda_1^T \underline{y}_d - \Lambda_1^T \underline{\varsigma} + y_d^{(m)} - a(\underline{\eta}) - b(\underline{\eta})u_p$$

$$= -a(\underline{\eta}) - \Lambda_1^T \underline{\varsigma} - b(\underline{\eta})u_p + v_1$$
(8)

Where $\Lambda_1 = \begin{bmatrix} 0 \ \tau^T \end{bmatrix}^T$, $v_1 = y_d^{(m)} + \Lambda_1^T \underline{y}_d$. For the system (1) satisfies Assumption 1, if the ideal control is designed as:

$$u_p^* = k(t)\lambda^{m-1}e + \frac{\underline{a}(\underline{\eta}) + v}{b(\eta)}$$
(9)

Where $k(t) > \frac{1}{2}$ is a design parameter, $\underline{a}(\underline{\eta}) = -a(\underline{\eta}) - \Lambda_1^T \underline{\varsigma} - b(\underline{\eta})k\tau_2^T \underline{\varsigma}$, $v(\underline{\eta}, v_1, v_2) = v_1 + b(\underline{\eta})v_2$, $v_2 = k\tau_2^T \underline{y}_d$, $\tau_2 = \begin{bmatrix} 0, (m-1)\lambda^{m-2}, \cdots, (m-1)\lambda, 1 \end{bmatrix}^T$, Then, s converges to zero.

Proof: Consider the Lyapunov function $V_s = \frac{1}{2}s^2$. Taking the time derivative of V_s along (8) yields:

$$\dot{V}_{s} = s\dot{s} = s\left(-a(\underline{\eta}) - \Lambda_{1}^{T}\underline{\varsigma} - b(\underline{\eta})u_{p} + v_{1}\right)$$

$$= s\left[-a(\underline{\eta}) - \Lambda_{1}^{T}\underline{\varsigma} - b(\underline{\eta})\left(k(t)\lambda^{n-1}e + \frac{a(\underline{\eta}) + v}{b(\underline{\eta})}\right) + v_{1}\right]$$

$$= s\left[-b(\underline{\eta})k(t)\lambda^{m-1}e + b(\underline{\eta})k\tau_{2}^{T}\underline{\varsigma} - b(\underline{\eta})k\tau_{2}^{T}\underline{y}_{d}\right]$$

$$= -b(\eta)k(t)s^{2}$$
(10)

According to the Lyapunov theorem, the results implies that $\lim s = 0$.

However, $\underline{a}(\underline{\eta}) \ b(\underline{\eta})$ are unknown in ideal controller (9), and the state vector $\underline{\varsigma}$ can not be measured. u_p^* is not available. The ideal controller (9) can be rewrittened as:

$$u^* = k(t)\lambda^{m-1}e + u^*_{ad}$$
(11)

Where $u_{ad}^* = \frac{\underline{a}(\underline{\eta}) + v}{b(\eta)}$ is an unknown function.

In this paper, a radial basis function (RBF) neural network (NN) is used to capture the unknown nonlinearity u_{ad}^* in (11). In general, the output of the multiple-input-single-output RBFNN is described by:

$$h(\xi) = W^T \phi(\xi) \tag{12}$$

Where $h(\xi) \in R$ is the RBFN output, $W \in R^{L}$ is the adjustable parameter vector, $\phi(\square) : R^{n+1} \to R^{L}$ is a nonlinear vector function of the inputs with *L* being the number of RBFs. The *i*th element of *W*, $\omega_{i}, i = 1, \dots, L$, is the synaptic weight between the *i*th neuron in the hidden layer and output neuron and $\phi_{i}(\xi)$ is a Gaussian function in the form of:

$$\phi_i(\xi) = \exp\left(-\frac{|\xi - v_i|}{2\delta_i^2}\right) \tag{13}$$

Where v_i is a m-dimensional vector representing the center of the *i*th basis function and δ_i is the variance representing the spread of the basis function.

The key advantage of RBFN is that it has the capability to approximate nonlinear mappings to any degree of accuracy. So:

$$u_{ad}^{*} = \frac{\underline{a}(\underline{\eta}) + v}{b(\underline{\eta})} = W^{*^{T}} \phi(\xi) + \varepsilon$$
(14)

Where approximation error ε satisfy $|\varepsilon| \le \varepsilon_0$, $\xi = [y(t), y(t-d_1), \dots, y(t-(m-1)d_1), v_1(t), v_2(t)]^T$ is the input vector to the RBFNN and $d_1 > 0$ is a positive time delay W^* is an ideal parameter vector which minimizes the function $|\varepsilon|$ and be defined as:

$$W^* = \underset{W \in \Omega_{\omega}}{\operatorname{arg\,min}} \left\{ \sup \left| W^T \phi(\xi) - u^*_{ad} \right| \right\}$$
(15)

Where $\Omega_{\omega} = \{W \mid ||W|| \le \varepsilon_{\omega}\}, \varepsilon_{\omega} > 0$ is the design constant. So the neural network output feedback controller can be described as:

$$u_p = k\lambda^{m-1}e + \hat{u}_{ad}(\xi) \tag{16}$$

Where $\hat{u}_{ad}(\xi) = \hat{W}^T \phi(\xi)$ is the output of RBFNN, \hat{W} is the estimated value of the optimal weight W^* .

The adaptive law for the estimated parameters of the NN is determined as the following:

$$\hat{W} = \gamma (e\phi - \sigma |e|\hat{W}) \tag{17}$$

Where adaptive gain $\gamma, \sigma > 0$ and the *e*-modification term is introduced to improve the robustness of adaptive law in the presence of the approximation error, and there exists compact set.

$$\Theta_{\omega} = \left\{ \hat{W} \mid \left\| \hat{W} \right\| \le \frac{\phi_m}{\sigma} \right\}$$
(18)

Where $\|\phi(\xi)\| \le \phi_m$, ϕ_m is a constant. If $\hat{W}(0) \in \Theta_\omega$, then $\hat{W}(t) \in \Theta_\omega, \forall t \ge 0$.

Proof: Consider the Lyapunov function $V_{\omega} = \frac{1}{2\gamma} \hat{W}^T \hat{W}$. The time derivative of the function V_{ω} along (19) is derived as:

$$\begin{split} \dot{V}_{\omega} &= \frac{1}{\gamma} \hat{W}^{T} \dot{\hat{W}} = \hat{W}^{T} \left(e\phi - \sigma \left| e \right| \hat{W} \right) \\ &= \hat{W}^{T} e\phi - \sigma \left| e \right| \left\| \hat{W} \right\|^{2} \\ &\leq \left\| \hat{W} \right\| \left\| \phi \right\| \left| e \right| - \sigma \left| e \right| \left\| \hat{W} \right\|^{2} \\ &\leq -\left| e \right| \left\| \hat{W} \right\| \left(\sigma \left\| \hat{W} \right\| - \phi_{m} \right) \end{split}$$
(19)

Thus, it follows that if $\|\hat{W}\| > \frac{\phi_m}{\sigma}$ then $\dot{V}_{\omega} \le 0$. So $\hat{W}(t) \in \Theta_{\omega}, \forall t \ge 0$.

From (8), the time derivative of the filtered tracking error can be derived as:

$$\dot{s}_{i} = -a(\underline{\eta}) - \Lambda_{1}^{T} \underline{\varsigma} - b(\underline{\eta})u_{p} + v_{1}$$

$$= -a(\underline{\eta}) - \Lambda_{1}^{T} \underline{\varsigma} - b(\underline{\eta})u_{p} - b(\underline{\eta})u_{ad}^{*} + b(\underline{\eta})u_{ad}^{*} + v_{1}$$

$$= -a(\underline{\eta}) - \Lambda_{1}^{T} \underline{\varsigma} - b(\underline{\eta})u_{p} - b(\underline{\eta})\frac{\underline{a}(\underline{\eta}) + v}{b(\underline{\eta})} + b(\underline{\eta})u_{ad}^{*} + v_{1}$$

$$= b(\underline{\eta})(-k\lambda^{m-1}e - \hat{W}^{T}\phi + W^{*^{T}}\phi + \varepsilon - k\tau_{2}^{T}\underline{e})$$

$$= b(\underline{\eta})(-ks - \tilde{W}^{T}\phi + \varepsilon)$$
(20)

Where $\tilde{W} = \hat{W} - W^*$.

4. Stability Analysis

We are now ready to present our main theorem which is summarized in Theorem 1.

Theorem 1: Consider the pure-feedback nonlinear system (1) with the controller input (16) and adaptive law (17). Then, all the signals in the closed-loop system are bounded and the state vector x remains in:

$$\Omega_{x} = \left\{ \underline{x}(t) \mid \left| e_{i}(t) \right| \leq 2^{i} \lambda^{i-m} \frac{b_{\omega} \phi_{m} + \varepsilon_{0}}{\sqrt{k - 0.5}}, i = 1, 2, \cdots, m \right\}, \forall t \geq T$$

Where $b_{\omega} = \frac{\phi_m}{\sigma} + \left\| W^* \right\|$.

Proof: Let the Lyapunov function $V_s = \frac{1}{2}s^2$. Taking the time derivative of V_s according to (20), we get:

$$\begin{aligned} \dot{V}_{s} &= s\dot{s} = b(\underline{\eta})(-ks - \tilde{W}^{T}\phi + \varepsilon)s \\ &\leq -b(\underline{\eta})ks^{2} + b(\underline{\eta}) \left\| \tilde{W} \right\| \|\phi\| |s| + b(\underline{\eta}) |\varepsilon| |s| \\ &\leq -b(\underline{\eta})ks^{2} + b(\underline{\eta}) \left\| \tilde{W} \right\| \|\phi\| |s| + b(\underline{\eta})\varepsilon_{0} |s| \\ &\leq -b(\underline{\eta})ks^{2} + b(\underline{\eta}) |s| \left(\left\| \tilde{W} \right\| \|\phi\| + \varepsilon_{0} \right) \end{aligned}$$

$$(21)$$

Since \hat{W} is bounded as shown in (18), it follows that $\left\|\tilde{W}\right\| \le b_{\omega}$, $b_{\omega} = \frac{\phi_m}{\sigma} + \left\|W^*\right\|$, then:

$$\dot{V}_{s} \leq -b(\underline{\eta})ks^{2} + b(\underline{\eta})\left|s\right|\left(b_{\omega}\phi_{m} + \varepsilon_{0}\right)$$
(22)

From the inequality $|\alpha||\beta| \le (\alpha^2 + \beta^2)/2$, it follows that:

$$\begin{split} \dot{V}_{s} &\leq -b(\underline{\eta})ks^{2} + 0.5b(\underline{\eta})\left((b_{\omega}\phi_{m} + \varepsilon_{0})^{2} + s^{2}\right) \\ &= -b(\underline{\eta})\left[(k - 0.5)s^{2} - \frac{(b_{\omega}\phi_{m} + \varepsilon_{0})^{2}}{2}\right] \\ &= -2b(\underline{\eta})(k - 0.5)\left[V_{s} - \frac{(b_{\omega}\phi_{m} + \varepsilon_{0})^{2}}{4(k - 0.5)}\right] \end{split}$$
(23)

Let
$$\overline{V_s} = V_s - \frac{(b_\omega \phi_m + \varepsilon_0)^2}{4(k - 0.5)}$$
 and note $\frac{(b_\omega \phi_m + \varepsilon_0)^2}{4(k - 0.5)}$ is constant. Using the comparison

principle, it follows that:

$$\overline{V}_{s} \leq \overline{V}_{s}(0) e^{-2(k-0.5) \int_{0}^{t} b(\underline{\eta}(\tau))d\tau}$$
(24)

Therefore,

$$V_{s} - \frac{(b_{\omega}\phi_{m} + \varepsilon_{0})^{2}}{4(k - 0.5)} \leq \left[V_{s}(0) - \frac{(b_{\omega}\phi_{m} + \varepsilon_{0})^{2}}{4(k - 0.5)}\right] e^{-2(k - 0.5)\int_{0}^{t} b(\underline{\eta}(\tau))d\tau}$$
(25)

Since $b(\underline{\eta}) \ge b_0 > 0$ and $-\frac{(b_{\omega}\phi_m + \varepsilon_0)^2}{4(k-0.5)}e^{-2(k-0.5)b_0t} \le 0$, it follows that:

$$V_{s} \leq \overline{V}_{s}(0) \boldsymbol{\varrho}^{-2(k-0.5)b_{0}t} + \frac{(b_{\omega}\phi_{m} + \varepsilon_{0})^{2}}{4(k-0.5)}$$
(26)

Therefore,

$$s^{2} \leq s^{2}(0)\boldsymbol{\varrho}^{-2(k-0.5)b_{0}t} + \frac{(b_{\omega}\phi_{m} + \varepsilon_{0})^{2}}{2(k-0.5)}$$
(27)

From the above equation, *s* is bounded and it implies that \underline{x} is bounded. Following [14], the state vector \underline{x} will remain in Ω_x for all $t \ge T$. This completes the proof.

5. Simulation Study

In this part, the following non-affine nonlinear system is simulated to illustrate the effectiveness of the proposed adaptive neural network output feedback tracking controller. The non-affine nonlinear system is described as follows:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_1^2 + 0.15u^3 + 0.1(1 + x_2^2)u + \sin(0.1u)$$

$$y = x_1$$
(28)

The tracking objective is to make the system output y follow the desired trajectory $y_d = 0.5 \sin(\pi(t - 0.5))$.

In (28), $\frac{\partial f}{\partial u} > 0$ which satisfy the assumption. The simulation parameters are selected as follows: $\kappa = 20.0$, $\lambda = 2.0$, k = 22.0. The adaptive gain $\gamma = 95.0$, $\sigma = 0.02$. According to the design process, we can get controller and weights update law as follows:

$$\dot{u} = -20 \times u + 20 \times u_p$$
$$u_p = 22 \times 2^2 \times e + \hat{W} \times \phi(\xi)$$
$$\dot{\hat{W}} = 95 \times (e\phi - 0.02 \times |e|\hat{W})$$

The system initial conditions are $x_1(0) = 0$, $x_2(0) = 0$. The simulation result using MATLAB is shown in Figure 1-4.

15

10

5

0

1

2

IIWII

5

6



Figure 1. Plots of Output Tracking of System



З

t(s)

4



Figure 1 and Figure 4 shows the results of output tracking. It can be seen that the actual trajectory converges rapidly to the desired one. The weights norm is shown in Figure 2 and the bounded control input is indicated in Figure 3. These simulation results demonstrate the tracking capability of the proposed controlled and its effectiveness for control tracking of uncertain non-affine nonlinear systems.

6. Conclusion

This paper proposes a new output feedback adaptive neural network adaptive controller for a class of uncertain non-affine nonlinear systems. The distinguished aspect of the proposed control algorithm is that no state observer is employed. Only the output error is used to generate control input and update laws. The stability analysis depends heavily on the universal function approximation property, only one RBFN is employed to approximate the lumped uncertain nonlinear function (16). There are no restrictive conditions on the design constants. So the system construct is very simple. Outputs tracking error and all states in the closed-loop system are guaranteed to be bounded by Lyapunov approach. Simulation results have verified the effectiveness of the proposed approach.

Acknowledgement

It is a project supported by Provincial Natural Science Foundation of Hunan, China (Grant No. 13JJ9022), the Research Foundation of Education Bureau of Hunan Province, China (Grant No.09B022), the Planned Science and Technology Project of Hunan Province, China (Grant No.2011FJ3126). Supported by the Construct Program of the Key Discipline in Hunan Province: Control Science and Engineering Science and Technology Innovation Team of Hunan Province: Complex Network Control.

References

- [1] Fitrian Imaduddin, Khisbullah Hudha, Janatul Islah Mohammad, Hishamuddin Jamaluddin. Simulation and experimental investigation on adaptive multi-order proportional-integral control for pneumatically actuated active suspension system using knowledge-based fuzzy. *Int. J. of Modelling, Identification and Control.* 2011; 14(1/2): 73 – 92.
- [2] Mrinal Kanti Sarkar, Subrata Banerjee, Sakti Prasad Ghoshal, Tapas Kumar Saha. Adaptive fuzzy parameter scheduling scheme for GSA based optimal proportional integral derivative and lag-lead control of a DC attraction type levitation system. *Int. J. of Automation and Control.* 2012; 6(2): 174 – 192.
- [3] LIU Guo-rong, WAN Bai-wu. Direct adaptive fuzy robust control for a class of nonlinear MIMO systems. Control Theory&Applications. 2002; 19: 693-698.
- [4] LIU Guo-rong, WAN Bai-wu. Indirect adaptive fuzy robust control for a class of nonlinear MIMO systems. *Control and Decision.* 2002; 17: 676-680.
- [5] TONG Shao-cheng, QU Lian-jiang. Fuzzy adaptive output feedback control for a class of MIMO nonlinear systems. *Control and Decision*. 2005; 20: 781-793.
- [6] TONG Shao-cheng, CHAI Tian-you. Adaptive fuzzy output feedback control for a class of MIMO nonlinear systems. Acta Electronica Sinica. 2005; 23: 987-990.
- [7] R Boukezzoula, S Galichet, L Foullor. *Fuzzy adaptive control for non-affine systems*. Proc. IEEE Internat. Conf. Fuzzy Systems. 2003; 23: 543-548.
- [8] J Wang, SS Ge, TH Lee. Adaptive fuzzy sliding mode control of a class of nonlinear systems. Proc.Third Asian Control Conf. 2000; 170: 599-604.
- [9] Yih-Guang Leu, Wei-Yen Wang, Tsu-Tian Lee. Observer-based direct adaptive fuzzy-neural control for non-affine nonlinear systems. *IEEE Transactions on Neural Networks*. 2005; 16: 853-861.
- [10] H Du, SS Ge, JK Liu. Adaptive neural network output feedback control for a class of non-affine nonlinear systems with unmodelled dynamics. *IET Control Theory & Application*. 2010; 5: 465-477.
- [11] Hui Hu, Peng Guo. Output Feedback Neural Network Adaptive Tracking Control for Pure- Feedback Nonlinear Systems. *IJACT*. 2012; 4(18): 655-663.
- [12] Hui Hu, Guo-rong Liu, Peng Guo, Feng Huang. Neural network adaptive tracking control for strict-feedback nonlinear systems without backstepping. *Journal of Central South University (Science and Technology)*. 2012; 42(10): 3900-3905.
- [13] SWKKB Park, JJ Lee. Smooth variable structure controller for robot manipulator using virtual plant. Electronics Letters. 1995; 31(23): 2134-2136.
- [14] PA Ioannou, J Sun. Robust adaptive control. Englewook Cliffs, NJ: Prentice-Hall. 1996.