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Dynamic Sliding Mode Control of Ship Rudder-fin Joint Nonlinear System

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Abstract

In order to realize good track keeping and roll reduction of the ship while navigating. A multiple input multiple output rudder-fin control system based on dynamic sliding mode algorithm is proposed. Rudder-fin joint system nonlinear mathematical model is established. Because the designed dynamic sliding mode controller makes sliding variables and the derivatives be zero, so it is essentially continuous and can eliminate traditional sliding mode's chattering problem. Simulation results show that while keeping the command track angle the designed control system can make the average roll angle within ±2°and realize good stabilization effect.

Keywords: *rudder-fin joint control, dynamic sliding mode, multiple input multiple output nonlinear, ship engineering.*

1. Introduction

Rudder roll stabilization theory was proposed long times ago. The rudder roll stabilization method requires high steering speed and it's contradictory to rudder structure, so rudder and fin joint roll stabilization was invented. Rudder-fin joint control refers to make autopilot and fin stabilizer work together with fully considering the coupling effect of roll and yaw. The main stabilization effect comes from fin stabilizer and autopilot has auxiliary stabilization effect. Rudder-fin joint control method improves seaworthiness, safety and comfortableness [1, 2].

Recently, many scholars pay attention to rudder-fin joint control problem. Paper [3] summarized studies on rudder-fin joint control before 2008 which mainly concentrate on PID control, robust control, predictive control etc. In [4], aiming at higher requirement of steering gear velocity, the given command of autopilot and fin stabilizer are modified on line. The purpose of rudder-fin joint roll stabilization is realized within promised course error. LIU Yanwen [5] propose a H_∞ method based on positive realness and adjust controller parameters synthetically according to design results. Sliding mode algorithm is widely applied in robot control, industrial control and other fields because of its robustness to parameter perturbation, un-modeled dynamics and external disturbance. The sliding mode is also used in rolling stabilization by some scholars. Aiming at ship nonlinear model with 3 freedom, Zhang Bing [6] designed a fuzzy sliding mode for rudder-fin control, but the detailed design process is missed and the simulation analysis is rough. Two different sliding mode controllers is was designed in [7]. A good effect for course keeping and roll reduction was realized, but genetic algorithm was adopted and made the whole controllers were rather complicated. Literature [8] proposed six different triple controllers including PID and sliding mode controller. The simulation results show control effects are better to autopilot, fin stabilizer and rudder stabilizer when all the three control methods are sliding mode control, but the mathematic model is linear. What is more serious is that all the above sliding mode controllers for rudder or fin stabilization exist great chattering which accelerate damage of rudder or fin mechanical parts.

Generally, the choice of sliding function in standard sliding mode control method is relied on system states and has no relation with system inputs, such that the discontinuous items in reaching law will be transferred to control variable. System states will switch back and forth under different control logics which cause the chattering. Dynamic sliding mode does not only rely on system states, but also is concerned with system input or its higher derivatives [9,

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10]. So the effect of discontinuous term in reaching law is partly transfer to first or higher derivatives of control input and the chattering is greatly alleviated.

Dynamic sliding mode is applied gradually in many fields [11]. To the authors' knowledge, dynamic sliding mode control for rudder-fin joint nonlinear system has not been studied in any literature, such that the promising theme is studied in this paper. A dynamic sliding mode controller for MIMO system is designed and rudder-fin joint nonlinear model with 4 freedoms is established. The effectiveness of the proposed controller is verified by matlab simulation. The chattering of steering and fin stabilizer is greatly weakened.

The rest of this paper is organized as follows. In section 2, The dynamic sliding mode control algorithm is described. Section 3 states the ship rudder-fin joint nonlinear mathematic model. Simulations for rudder-fin joint system is given in section 4 and at last is the conclusion.

2. Dynamic Sliding Mode Control Algorithm

Considering MIMO (Multiple Input Multiple Output) nonlinear affine system.

$$x = f(x) + g(x)u \tag{1}$$

Where $x \in R^n$ $f(x)$ $: R^n \to R^n$, $g(x) = [g_1(x),..., g_m(x)] : R^n \to R^{n\times m}$, $u \in R^m$,each vector in $f(x)$, $g(x)$ is sufficiently smooth function. Suppose $\sigma = \psi(x)$ $: R^n \to R^m$ be sliding variables of system (1). For simplifying formula, $f(x)$ is written as f and other symbols are handled like this.

According to the following assumptions to choose $\psi(x)$.

Assumption 1: When system moves on $\sigma = 0$ sliding mode manifold, x is guaranteed to converge to equilibrium point.

Assumption 2: For every x ,

$$\sum_{j=1}^{m} L_{g_i} L_f^{k_i} \psi_i(x) = 0, \sum_{j=1}^{m} L_{g_i} L_f^{r_i - 1} \psi_i(x) \neq 0, \quad i = 1,..., m; 0 \le k_i < r_i - 1 \tag{2}$$

Assumption 3: Matrix $\begin{bmatrix} L_{g_1} L_f^{r_1 - 1} \psi_1(x) & ...... & L_{g_m} L_f^{(r_1 - 1)} \psi_1(x) \\ ...... & ...... & ...... \\ L_{g_1} L_f^{r_m - 1} \psi_m(x) & ...... & L_{g_m} L_f^{(r_m - 1)} \psi_m(x) \end{bmatrix}$ is invertible.

According to above assumptions:

$$\sigma_i^{(k_i)} = L_f^{k_i} \psi_i(x) \tag{3}$$

Where $k_i = 0,1,..., r_i - 1; i = 1,..., m.$

$$\sigma_i^{(r_i)} = L_f^{r_i} \psi_i(x) + \sum_{j=1}^{m} L_{g_f} L_f^{r_i - 1} \psi_i(x) u_j \tag{4}$$

Derivative of formula (4) is calculated and then formula (5) which explicitly includes first order differentiation of control variable is:

$$\sigma_i^{(r_i+1)} = L_f^{r_i+1} \psi_i(x) + \sum_{j=1}^{m} L_{g_f} L_f^{r_i} \psi_i(x) u_j + \sum_{j=1}^{m} \sum_{j=1}^{m} L_{g_f} L_{g_f} L_f^{r_i - 1} \psi_i(x) u_j u_j + \sum_{j=1}^{m} L_{g_j} L_f^{r_i - 1} \psi_i(x) \dot{u}_j \tag{5}$$

Write formula (5) as vector and matrix form.

$$\sigma^{(r+1)} = \mathrm{M}_1(x,u) + \mathrm{M}_2(x)\dot{u} \tag{6}$$

Where $\sigma^{(r+1)} = (\sigma_1^{(r_1+1)} \ \ ...... \ \ \sigma_m^{(r_m+1)})^T$, $\sigma^{(r+1)} \in R^m$, $\mathrm{M}_1(x,u) = \mathrm{M}_{11}(x) + \mathrm{M}_{12}(x)u + \mathrm{M}_{13}(x)u + \mathrm{M}_{14}(x,u)u$, $\mathrm{M}_1(x,u) \in R^m$, $\mathrm{M}_{11}(x) \in R^m$, $\mathrm{M}_{12}(x) \in R^{m\times m}$, $\mathrm{M}_{13}(x) \in R^{m\times m}$, $\mathrm{M}_{14}(x,u) \in R^{m\times m}$, $\mathrm{M}_2(x) \in R^{m\times m}$.

Every concrete expression is as follow:

$$\mathrm{M}_{11}(x) = (L_f^{r_1+1}\Psi_1(x) \ \ ... \ \ L_f^{r_m+1}\Psi_m(x))^T, \mathrm{M}_{12}(x) = \begin{bmatrix} L_{g_1}L_f^{r_1}\psi_1(x) & ... & L_{g_m}L_f^{r_1}\psi_1(x) \\ ... & ... & ... \\ L_{g_1}L_f^{r_m}\psi_m(x) & ... & L_{g_m}L_f^{r_m}\psi_m(x) \end{bmatrix},$$

$$\mathrm{M}_{13}(x) = \begin{bmatrix} L_fL_{g_1}L_f^{r_1-1}\psi_1(x) & ... & L_fL_{g_m}L_f^{r_1-1}\psi_1(x) \\ ... & ... & ... \\ L_fL_{g_1}L_f^{r_m-1}\psi_m(x) & ... & L_fL_{g_m}L_f^{r_m-1}\psi_m(x) \end{bmatrix}, \mathrm{M}_2(x) = \begin{bmatrix} L_{g_1}L_f^{(r_1-1)}\psi_1(x) & ... & L_{g_m}L_f^{(r_1-1)}\psi_1(x) \\ ... & ... & ... \\ L_{g_1}L_f^{(r_m-1)}\psi_m(x) & ... & L_{g_m}L_f^{(r_m-1)}\psi_m(x) \end{bmatrix}$$

$$\mathrm{M}_{14}(x,u) = \begin{bmatrix} L_{g_1}\sum_{j=1}^{m}L_{g_j}L_f^{(r_1-1)}\psi_1(x)u_j & ... & L_{g_m}\sum_{j=1}^{m}L_{g_j}L_f^{(r_1-1)}\psi_1(x)u_j \\ ... & ... & ... \\ L_{g_1}\sum_{j=1}^{m}L_{g_j}L_f^{(r_m-1)}\psi_m(x)u_j & ... & L_{g_m}\sum_{j=1}^{m}L_{g_j}L_f^{(r_m-1)}\psi_m(x)u_j \end{bmatrix}.$$

Choose a new sliding mode function [12]:

$$\Theta_i = \sigma_i^{(r_i)} + \lambda_{i1}\sigma_i^{(r_i-1)} + ... + \lambda_{ir_i-1}\dot{\sigma}_i + \lambda_{ir_i}\sigma_i + \lambda_{ir_i+1} \quad (7)$$

Choose $\dot{\Theta} = (\dot{\Theta}_1 \ \ ...... \ \ \dot{\Theta}_m)^T$ and take it to formula (6).

$$\dot{\Theta} = \sigma^{(r+1)} + \mathrm{M}_3(x,u) \quad (8)$$

Where $\mathrm{M}_3(x,u) = \begin{bmatrix} \lambda_{11}\sigma_1^{(r_1)} + ... + \lambda_{1r_1-1}\dot{\sigma}_1 + \lambda_{1r_1}\sigma_1 \\ ...... \\ \lambda_{m1}\sigma_1^{(r_m)} + ... + \lambda_{mr_m-1}\dot{\sigma}_m + \lambda_{mr_m}\sigma_m \end{bmatrix}$.

Choose parameters $\lambda_{ij}, i = 1,...,m; j = 1,...,r_i+1,$ by using pole assignment method and make each polynomial be *Hurwitz* polynomial.

For satisfying sliding mode reaching condition, Exponential approach law is chosen as:

$$\dot{\Theta} = -\tilde{\lambda}_1\Theta - \tilde{\lambda}_2\,\mathrm{sgn}(\Theta) \quad (9)$$

Where $\tilde{\lambda}_{\tilde{i}} = diag(\tilde{\lambda}_{\tilde{i}1},...,\tilde{\lambda}_{\tilde{i}m}), \tilde{\lambda}_{\tilde{i}\tilde{j}} > 0, \tilde{i} = 1,2, \tilde{j} = 1,...,m$.

Take (6) and (8) to (9):

$$\mathrm{M}_1(x,u) + \mathrm{M}_2(x)\dot{u} + \mathrm{M}_3(x,u) = -\tilde{\lambda}_1\Theta - \tilde{\lambda}_2\,\mathrm{sgn}(\Theta) \quad (10)$$

Choose lyapunov function V to design controller according to Lyapunov stability theorem and then formula (9):

$$\dot{u} = \Gamma(t,x,u,\Theta,\mathrm{sgn}(\Theta)) \quad (11)$$

$\sigma_i = \dot{\sigma}_i = ... = \sigma_i^{(r_i)} = 0, \ \ i = 1,...,m$.

According to assumption 1, x will converge to equilibrium point.

In formula (11), take integration of $\dot{u}$:

$$u(t) = u(0) + \int_0^{\tau} \dot{u}(\tau)d\tau = u(0) + \int_0^{\tau} \Gamma(\tau, x, u, \Theta, \mathrm{sgn}(\Theta))d\tau \tag{12}$$

The discontinuous terms $\mathrm{sgn}(\Theta)$ are included in first order derivate of controller and the controller becomes continuous function in time domain. Theoretically, it needs infinite time for x converging to equilibrium point because traditional switching functions are linear. In order to accelerate convergence rate, combining terminal sliding mode. Designing switching function $\dot{\sigma} = \dot{s} + as^{q/p}$, where, a is diagonal matrix, p, q are design parameters of terminal sliding mode. The designed controller can not only eliminate the chattering but also make x converge to equilibrium point in finite time.

3. Mathematical Model of Rudder-fin Joint System

Actual motion of ship is rather complicated and the complex motion can be divided into six motions including rolling, pitching, yawing, surging, swaying and heaving. For researching rudder-fin joint control, ship course keeping and rolling reducing are mainly considered and so pitching and heaving are ignored. Nonlinear mathematical model of rudder-fin joint system which is established based on dynamics theorem [13].

In (13), u, v, w, r, p, q respectively are: surge velocity, sway velocity, heave velocity, yaw rate, roll rate and pitch rate in body- fitted coordinate system. ψ, φ, θ respectively are: heading angle, rolling angle and pitching angle in inertial coordinate system which determine the geometric position relation between body- fitted and inertial coordinate system. X, Y, Z, K, M, N are: longitudinal force, transverse force, vertical force, rolling moment, pitching moment and yawing moment. m is ship weight. I_{xx}, I_{yy}, I_{zz} respectively are: inertia moment about the X,Y,Z axis. Other parameters are defined in [13].

$$\begin{cases}
\dot{u} = \{(m' + m'_x)v_c r + (m' + m'_y)v_r r + X'_{uu} u_r^2 / L + X'_{vv} v_r^2 / L + L \cdot X'_{rr} r^2 + \\
\quad 2[(1 - t_p)\rho n^2 D_p^4 k_T(J_p) - (1 - t_R)F_N \sin\delta + X_{WIND} + X_{WAVE}] / (\rho L^2 d)\} / (m' + m'_x) \\
\dot{v} = \{-(m' + m_y{}')u_c r - (m' + m_x{}')u_r r + V \cdot Y_r{}' v_r / L + V \cdot Y_r{}' r + Y'_{vv} v_r |v_r| / L + \\
\quad Y_{vr}{}' v_r |r| + L \cdot Y_{rr}{}' r|r| - 2[(1 + a_H)F_N \cos\delta - \rho V^2 A_F C_{la} \alpha_f \cos\beta_f - \\
\quad Y_{WIND} - Y_{WAVE}] / (\rho L^2 d)\} / (m' + m_y{}') \\
\dot{p} = \{2 - K_p p - K_{pp} p|p| - W \cdot GM \cdot \varphi + W \cdot GM \cdot \varphi^3 / \varphi_v^2) / (\rho L^4 d) - \\
\quad z'_H \cdot (V \cdot Y'_v v_r / L^2 + V \cdot Y'_r r / L + Y'_{vv} v_r |v_r| / L^2 + Y'_{vr} v_r |r| / L + Y'_{rr} r|r|) + \\
\quad 2[(1 + a_H)z_H F_N \cos\delta - \rho V^2 A_F C_{La} \alpha_f l_f + K_{WIND} + K_{WAVE}] / (\rho L^4 d)\} / (I'_{xx} + J'_{xx}) \\
\dot{r} = \{V \cdot N'_v v_r / L^2 + V \cdot N'_r r / L + N'_{rr} r|r| + N'_{vvr} v_r^2 r / (VL) + N'_{vrr} v_r r^2 / V + \\
\quad V^2 \cdot N'_{\varphi} \varphi / L^2 + V \cdot N'_{v\varphi} v_r |\varphi| / L^2 + V \cdot N'_{r\varphi} r|\varphi| / L - x'_c (V \cdot Y'_v v_r / L^2 + \\
\quad V \cdot Y'_r r / L + Y'_{vv} v_r |v_r| / L^2 + Y'_{vr} v_r |r| / L + Y'_{rr} r|r|) - 2[(1 + a_H)x_R F_N \cos\delta + \\
\quad \rho V^2 A_F C_{La} \alpha_f l_{fl} \cos\beta_f - N_{WIND} - N_{WAVE}] / (\rho L^4 d)\} / (I'_{zz} + J'_{zz}) \\
\dot{x}_0 = u\cos\psi - v\cos\varphi\sin\psi \\
\dot{y}_0 = u\sin\psi + v\cos\varphi\cos\psi \\
\dot{\varphi} = p \\
\dot{\psi} = r\cos\varphi
\end{cases} \tag{13}$$

Rudder-fin joint control study in this paper needn't consider the change in position, so x0 and y0 can be neglected. The influence of current disturbance on course and rolling is rather small, so current disturbance can be neglected and make $u_c = v_c = 0$, $u_r = u, v_r = v$. In addition, suppose the ship is sailing straightly under constant speed which means it is not need consider the change of u.

Choose state variables $x_1 = v$, $x_2 = p$, $x_3 = r$, $x_4 = \varphi$, $x_5 = \psi$, input variables $u_1 = \alpha_f$, $u_2 = \delta$, output variables $y_1 = \varphi, y_2 = \psi$, suppose effective angle $\alpha_R = \delta - \delta_0 - \gamma \cdot \beta_R$ flowing to rudder and rudder angle δ are rather small, then $\sin\alpha_R \approx \alpha_R = \delta - \delta_0 - \gamma \cdot \beta_R$, $\sin\delta \approx \delta$, $\cos\delta \approx 1$, meanwhile choose wind and wave disturbance, balance rudder angle and small nonlinear term as disturbance term. Then formula (13) is changed into an affine nonlinear system:

$$\begin{cases} \dot{x}_1 = a_{11}x_1 + a_{12}x_3 + a_{13}x_1|x_1| + a_{14}x_1|x_3| + b_{11}u_1 + b_{12}u_2 \\ \dot{x}_2 = a_{22}x_2 + a_{24}x_4 + a_{29}x_4^3 + b_{21}u_1 + b_{22}u_2 \\ \dot{x}_3 = a_{31}x_1 + a_{32}x_3 + a_{39}x_1x_3^2 + b_{31}u_1 + b_{32}u_2 \\ \dot{x}_4 = x_2 \\ \dot{x}_5 = x_3 \\ y_1 = x_4 \\ y_2 = x_5 \end{cases} \tag{14}$$

In formula (14):

$b_{11} = 2V^2 A_F C_{La} \cos\beta_f / [L^2 d(m' + m_y^{'})], b_{12} = -(1 + a_H)V_R^2 A_R f_a / [L^2 d(m' + m_y^{'})] a_{11} = VY_v^{'} / [L(m' + m_y^{'})] + b_{12}\gamma / V$,

$a_{12} = [Y_r^{'} - (m' + m_x^{'})]V / (m' + m_y^{'}) - b_{12}\gamma L / V, a_{13} = Y_{vv}^{'} / [L(m' + m_y^{'})], a_{14} = Y_{vr}^{'} / (m' + m_y^{'}), a_{15} = LY_{rr}^{'}(m' + m_y^{'})$,

$b_{21} = -2V^2 A_F C_{La} l_f / L^4 d(I_{xx}^{'} + J_{xx}^{'})], b_{22} = (1 + a_H) z_H^{'} V_R^2 A_R f_a / [L^3 d(I_{xx}^{'} + J_{xx}^{'})]$,

$a_{21} = -z_H^{'} VY_v^{'} / L^2 (I_{xx}^{'} + J_{xx}^{'}) + b_{22}\gamma / V, a_{22} = -2K_p / [\rho L^4 d(I_{xx}^{'} + J_{xx}^{'})], a_{23} = -z_H^{'} VY_v^{'} / [L(I_{xx}^{'} + J_{xx}^{'})] - b_{22}\gamma L / V$,

$a_{24} = -2W \cdot GM / [\rho L^4 d(I_{xx}^{'} + J_{xx}^{'})], a_{25} = -z_H^{'} Y_{vv}^{'} / [L^2 (I_{xx}^{'} + J_{xx}^{'})], a_{26} = -z_H^{'} Y_{vr}^{'} / [L(I_{xx}^{'} + J_{xx}^{'})]$

$a_{31} = (N_v^{'} - x_c^{'} Y_v^{'})V / [L^2 (I_{zz}^{'} + J_{zz}^{'})] + b_{32}\gamma / V, a_{32} = (N_r^{'} - x_c^{'} Y_v^{'})V / [L(I_{zz}^{'} + J_{zz}^{'})] - b_{32}\gamma L / V$

$a_{33} = V^2 N_{\varphi}^{'} / [L^2 (I_{zz}^{'} + J_{zz}^{'})], a_{34} = -x_c^{'} Y_{vv}^{'} / [L^2 (I_{zz}^{'} + J_{zz}^{'})], a_{35} = -x_c^{'} Y_{vr}^{'} / [L((I_{zz}^{'} + J_{zz}^{'})] a_{36} = VN_{v\varphi}^{'} / [L^2 (I_{zz}^{'} + J_{zz}^{'})]$,

$a_{37} = (N_{rr}^{'} - x_c^{'} Y_{vr}^{'}) / (I_{zz}^{'} + J_{zz}^{'}), a_{38} = VN_{r\varphi}^{'} / [L(I_{zz}^{'} + J_{zz}^{'})], a_{39} = N_{vrr}^{'} / [V(I_{zz}^{'} + J_{zz}^{'})], a_{30} = N_{vrr}^{'} / [VL(I_{zz}^{'} + J_{zz}^{'})]$,

$b_{32} = -(1 + a_H) x_R^{'} V_R^2 A_R f_a / (L^3 d) / (I_{zz}^{'} + J_{zz}^{'}), b_{31} = \pm 2V^2 A_F C_{La} l_f l \cos\beta_f / [L^4 d(I_{zz}^{'} + J_{zz}^{'})$

$a_{27} = -2K_{pp} / [\rho L^4 d(I_{xx}^{'} + J_{xx}^{'})], a_{28} = -z_H^{'} Y_{rr}^{'} / (I_{xx}^{'} + J_{xx}^{'}), a_{29} = -2W \cdot GM / \rho L 4 d \varphi_v^2 / (I_{xx}^{'} + J_{xx}^{'})$,

Then the form of the affine nonlinear system (14) is $\begin{cases} \dot{x} = f(x) + g(x)u + w \\ y = h(x) \end{cases}$.

Where, $x = [x_1, x_2, ..., x_5]^T$, $u = [u_1, u_2]^T$, $y = [y_1, y_2]^T$, $g(x) = [g_1(x), g_2(x)]$, $h(x) = [h_1(x), h_2(x)]^T$ w is disturbance term.

$f_1(x) = a_{11}x_1 + a_{12}x_3 + a_{13}x_1|x_1| + a_{14}x_1|x_3|$, $f_2(x) = a_{22}x_2 + a_{24}x_4 + a_{29}x_4^3$,
$f_3(x) = a_{31}x_1 + a_{32}x_3 + a_{39}x_1x_3^2$, $f_4(x) = x_2$, $f_5(x) = x_3$, $g_1(x) = [b_{11}, b_{21}, b_{31}, 0, 0]^T$,
$g_2(x) = [b_{12}, b_{22}, b_{32}, 0, 0]^T$, $h_1(x) = x_4$, $h_2(x) = x_5$.

Take no account of disturbance term, and then the state function is:

$$\dot{x} = f(x) + g(x)u \tag{15}$$

4. Simulation Research

According to parameters of "YuKun"training ship [13]:

$a_{11} = -0.0833, a_{12} = -1.6355, a_{13} = -0.0215, a_{14} = -0.6048, b_{11} = 0.1874, b_{12} = -0.2121, a_{22} = -0.0763,$
$a_{24} = -0.3588, a_{29} = 0.7363, b_{21} = -0.0774, b_{22} = 0.0182, a_{31} = -0.0028, a_{32} = -0.2706, a_{39} = -0.3091,$
$b_{31} = -0.0014, b_{32} = 0.0166.$

Design fast terminal sliding mode switching function.

$$\sigma_s = e_s + \int_0^t (a_s e_s + b_s e_s^{q_s/p_s})dt \qquad (16)$$

Where $e_s = \Omega - \Omega_c$, $a_s = diag(a_{s1}, a_{s2})$, $b_s = diag(b_{s1}, b_{s2})$ $a_{s1} > 0$, $a_{s2} > 0$, $b_{s1} > 0$, $b_{s2} > 0$, q_s, p_s respectively are positive odd number and satisfying $1/2 < q_s / p_s < 1$.

Design a new sliding mode function.

$$\Sigma_s = \dot{\sigma}_s + c_s \sigma_s + d_s \sigma_s^{j_s/k_s} \qquad (17)$$

Where the choice of c_s, d_s and j_s, k_s respectively are similar with a_s, b_s and q_s, p_s.

Calculate derivative of formula (17), and take it to formula (15) and (16):

$$\dot{\Sigma}_s = \dot{f} + g\dot{\omega}_c + \dot{g}\omega_c - \Omega_c'' + a_s \dot{e}_s + b_s (e_s^{q_s/p_s})' + c_s \dot{\sigma}_s + d_s (\sigma_s^{js/ks})' \qquad (18)$$

Adopt exponential approach law to guarantee the system entering into sliding mode manifold in finite time.

$$\Sigma_s' = -\lambda_{s1}\Sigma_s - \lambda_{s2} \operatorname{sgn}(\Sigma_s) \qquad (19)$$

Where $\lambda_{si} = diag(\lambda_{si1}, \lambda_{si2}), \lambda_{sij} > 0, i = 1,2, j = 1,2$.

Theorem 1: With regard to formula (15), to design control law described in formula (20), then the system is stable and tracking error can converge to zero in finite time.

$$\omega_c = \omega_0 + \omega_1 \qquad (20)$$

Where $\omega_0 = (g)^{-1}(-f - \dot{g}\omega_c + \Omega_c' - a_s e_s - b_s e_s^{q_s/p_s} - c_s \sigma_s - d_s \sigma_s^{j_s/k_s})$, $\omega_1 = \int_0^t v_s dt$, $v_s = -\lambda_{s1}\Sigma_s - \lambda_{s2} \operatorname{sgn}(\Sigma_s)$.

Proof: choose Lyapunov function $V = 0.5\Sigma_s^T \Sigma_s$

Then $\dot{V} = \Sigma_s^T \dot{\Sigma}_s$, take it to formula (18) and (20):

$$\dot{V} = \Sigma^T(-\lambda_{s1}\Sigma_s - \lambda_{s2} \operatorname{sgn}(\Sigma_s)) \le -\lambda_{s1} \|\Sigma_s\|^2 - \lambda_{s2} \|\Sigma_s\| < 0 \quad (when \ \ \Sigma_s \ne 0).$$

Wave disturbance is main reason causing rolling and off-course while navigating on the sea. Its influence must be considered in simulation experiment. This study employs white noise to drive a typical second order oscillation element to represent wave disturbance [14, 15] and take it into the nonlinear model of rudder-fin joint system. Wave disturbance transfer function is:

$$h(s) = \frac{2\xi\omega_0\sigma_w s}{s^2 + 2\xi\omega_0 s + \omega_0^2} \qquad (21)$$

Where, σ_w is constant describing wave intensity, ξ is damping coefficient, ω_0 is leading wave frequency. Generally, $\omega_0 = 4.85/T_w$, $\sigma_w = \sqrt{0.0185 T_w h_{1/3}}$, T_w is the average wave period and $h_{1/3}$ is significant wave height.

Considering the mechanical system of rudder and fin: $T_E\dot{\delta} + \delta = \delta_c$, $T_F\dot{\alpha} + \alpha = \alpha_c$, where δ_c is command rudder angle, α_c is command fin angle. $T_E = 2.5s$, $T_F = 0.5s$ and has the bound $\delta_{max} = 20^\circ$, $\alpha_{max} = 20^\circ$。

Under grade 6 wave, take $T_w = 8s$, $h_{1/3} = 3m$, $\xi = 0.3$, then $\omega_0 = 0.60624$, $\sigma_w = 1.1541$, the wave model is $h(s) = \frac{0.4198s}{s^2 + 0.3638s + 0.3675}$. Suppose the given yaw angle is 15°and the given rolling angle is 0°, then the simulation curve of yaw angle and rolling angle is as Figure 1. As is shown in Figure 1, the average rolling angle is within ±2°and the actual output yaw angle can track the given yaw angle appreciatively.

Figure 2 shows the fin angle and rudder angle under grade 6 wave. Fin angle and rudder angle are the actual control input. As is shown in the partial enlarged detail, the control inputs are continuous and the chattering is greatly reduced.

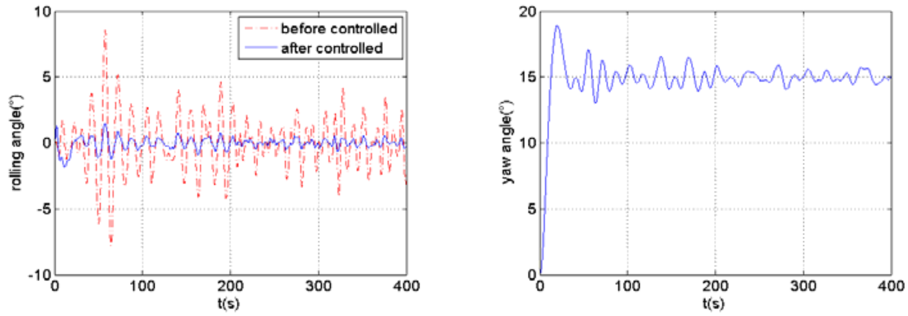

Figure 1. Rolling Angle and Yaw Angle under Grade 6 Wave

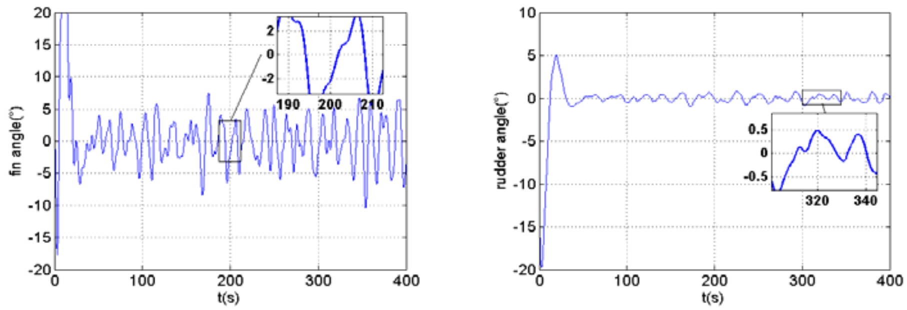

Figure 2. Fin Angle and Rudder Angle under Grade 6 Wave

Suppose the wave changing to grade 8, choose $T_w = 12s$, $h_{1/3} = 8m$, $\xi = 0.5$, then $\omega_0 = 0.40417$, $\sigma_w = 3.76935$, $h(s) = \frac{1.52436s}{s^2 + 0.40417s + 0.1634}$, Set the given course angle be 15°and the given rolling angle is 0°. After simulation at fixed step time 0.01s, the rolling angle and yaw angle are shown as Figure 3.

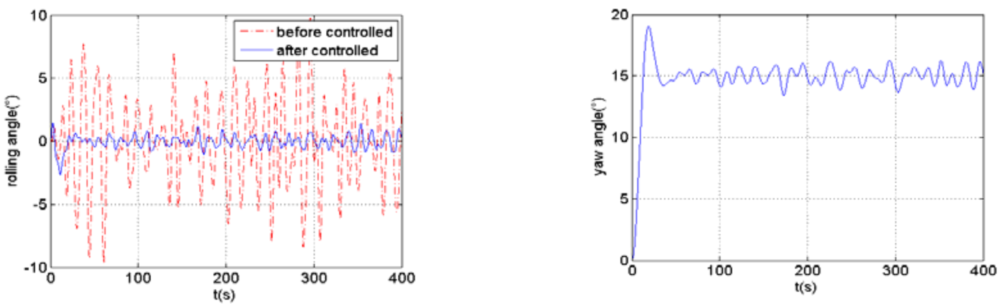

Figure 3. Rolling Angle and Yaw Angle under Grade 8 Wave

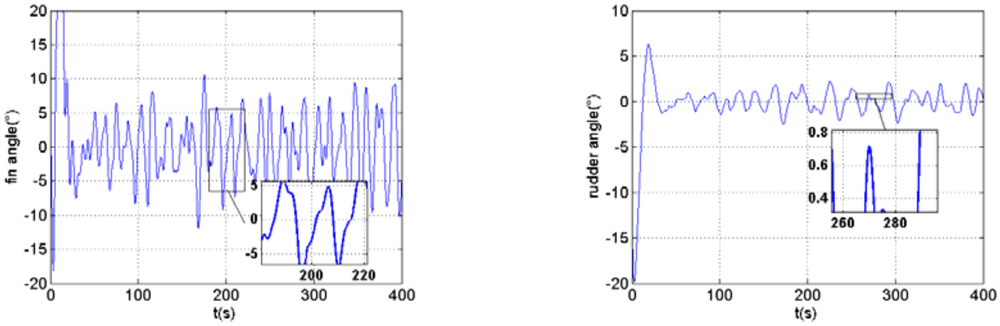

Figure 4. Fin Angle and Rudder Angle under Grade 8 Wave

The rolling angle before and after controlled can be obviously distinguished from Figure 3. The final rolling angle can also be reduced to the range±2°. Yaw angle output is relatively smooth and can not move far away from the given course angle.

Figure 4 denotes the actual control inputs, fin angle and rudder angle under grade 8 waves. Chattering is contained in the differential of control input. So the fin angle and rudder angle are smooth.

3. Conclusion

A dynamic sliding mode controller for MIMO nonlinear affine system is designed and the mathematical model of rudder-fin joint system is stated. The proposed sliding mode controller is applied to the rudder-fin joint system with 4 freedoms. Simulation research is implemented under grade 6 and 8 wind by matlab/simulink. Simulation results show the effectiveness of the designed control system. The chattering caused by sliding mode is almost avoided and such that mechanical wear can be lowered to the hilt. Next radial basis function neural network will be employed to design rolling disturbance observer and combined with dynamic sliding mode to control rudder-fin joint system.

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References

[1] Umeda N, Hashimoto H. Qualitative aspects of nonlinear ship motions in following and quartering seas with high forward velocity.*Journal of Marine Science and Technology.* 2002; 6:111-121.
[2] M Tomera, Kalman-Bucy. Filter design for multivariable ship motion control. *International Journal on Marine Navigation and Safety of Sea Transportation*. 2011; 5(3): 345-355.
[3] Xinping Wang, Xianku Zhang, Wei Guan. Overview of ship rudder/fin joint control. *Navigation of China*. 2009; 32(2): 20-26.
[4] Hongzhang Jin, Fan Wang. Rudder/fin roll stabilization with energy optimization using a lower-speed steering gear. *Journal of China Ordnance*. 2009; 30(7): 945-949.
[5] LIU Yan-wen, MIAO Qiu-yan, LIU Sheng, GAO Zhen-guo. An H_∞ control approach for rudder/fin roll stabilization. *Control Engineering of China*. 2010; 17(5): 595-599.
[6] Zhang Bing, Xu Kejian, Jiang Changsheng. Research on nonlinear rudder/fin joint control based on ruzzy variable structure control theory. *Navigation of China*. 2005; 1(4): 1-3.
[7] Ming-Chung Fang, Jhih-Hong Luo. On the track keeping and roll reduction of the ship in random waves using different sliding mode controllers. *Ocean Engineering.* 2007; 34: 479-488.
[8] Koshkouei AJ, Burnham KJ, Law Y. A comparative study between sliding mode and proportional integrative derivative controllers for ship roll stabilisation. *IET Control Theory & Applications*, 2007; 1(5): 1266-1275.

[9] Liang Li, Xie Jian, and Huang Jian Zhao. Fuzzy adaptive sliding mode control of large erecting mechanism. *TELKOMNIKA Indonesian Journal of Electrical Engineering.* 2013; 11(12): 7259-7268.
[10] Guoqin Gao, et al. Smooth sliding mode control for trajectory tracking of greenhouse spraying mobile robot. *TELKOMNIKA Indonesian Journal of Electrical Engineering* 2013; 11 (2): 642-652.
[11] GR Ansarifar, HA Talebi, H Davilu. An adaptive-dynamic sliding mode controller for non-minimum phase systems. *Commun Nonlinear Sci Numer Simulat*. 2012; 17: 414-425.
[12] PU Ming, WU Qing-xian, JIANG Chang-sheng, ZHANG Jun. Near space vehicle control based on second-order dynamic terminal sliding node. *Journal of Astronautics*. 2010; 31(4): 1056-1062.
[13] WANG Xin-ping, ZHANG Xian-ku, GUAN We. Nonlinear rudder/fin joint mathematical modeling and simulation. *Navigation of China*. 2009; 32(4): 58-66.
[14] Jian-Chuan Yin, Li-Dong Wang, Ni-Ni Wang. A variable-structure gradient RBF network with its application to predictive ship motion control. *Asian Journal of Control*. 2012; 14(3): 716-725.
[15] H Yasukawa, T Hirono, Y Nakayama, KK Koh. Course stability and yaw motion of a ship in steady wind. *Journal of Marine Science and Technology*. 2012; 17(3): 291-304.