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Identification of Nonlinear System Based on Fuzzy Model with Enhanced Gradient Search

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Abstract

The identification and modeling theory of nonlinear systems has always been challenging to researchers. Fuzzy system due to its language descriptive way similar to human brain and deal with qualitative information intelligently proves better choice for nonlinear system modeling over last few decades. The fuzzy system theory itself also has nonlinear characteristics therefore when establishing the fuzzy model of nonlinear system; it should be able to well describe the nonlinear characteristics. Takagi-Sugeno (TS) fuzzy systems are not only suitable for modeling the nonlinear system due to combination of the good performance with the simple linear expressions, but also useful to design the fuzzy controller. This paper proposed a new optimization algorithm named as Enhanced Gradient Search (EGS) for identification of nonlinear system based on TS fuzzy system. In proposed EGS, parameters of membership functions are trained adaptively so as to calculate the gradient of cost function which is necessary for minimizing the error. Using gradient information of cost function, EGS applies in an innovative way such that it keeps and updates the best search results at every training step during the optimization process. The applicability of EGS for TS fuzzy model shows splendid performance especially in modeling of nonlinear system.

Keywords: *enhanced gradient search, parameter estimation, Gaussian membership function, nonlinear system*

1. Introduction

Dynamics modeling for unknown nonlinear plant becomes challenge during recent years. Many methods have been proposed and implemented to cope with this demanding issue. Adaption of accurate parameters for nonlinear model through training is one of the most common techniques. Among all, fuzzy modeling proves as a dominant modeling tool for nonlinear system identification [1-4]. However, the nonlinear systems are mostly complex enough to be identified; therefore the accurate modeling has to done by estimating the parameters of membership functions and fuzzy rules. The estimation of such parameters is an important task because these parameters show nonlinearity in the output of fuzzy model. Two major steps are considered when design a fuzzy model from I/O (input and output) data e.g. structure identification (estimation of number of required fuzzy rules and membership function with centers and widths) and parameter identification (learning process of the consequent) [5-6]. The number of rules in the fuzzy system can be determined by dividing the input and output data space into many partitions. This step is known as structure identification. After the selection of structure of fuzzy system, the estimation of parameters for both membership function and rules are need to be determined under parameter estimation step. This is considered as an essential part for fuzzy modeling since the structure is normally selected based upon a priori system knowledge and accuracy of fuzzy model mainly depends on both the fuzzy rules and membership function. Parameter estimation can be considered as an optimization problem, which is used for finding the proper parameters so as to reduce the cost function that directly describes the accuracy of model. Several methods are being used for estimating the parameters such as genetic algorithms [7-9], least mean square (LMS) and evolutionary algorithm [10-11]. Separated estimation stages are required to find the parameters optimal values and that is the main drawback of these methods. Also guaranty about the convergence of optimization process could not be given.

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This paper proposed an enhanced optimization algorithm using gradient search in such a way that it keeps the best tracks of many searches and also updates the best search with fast convergence. The incorporation of enhancing concept in the gradient search bounds the optimization process to keep the best results and at the same time ignore the unsatisfied solution. At first, TS fuzzy model is built with the initially guessed structure and parameters for a certain nonlinear system. After that, EGS is applied to find out the best parameters for nonlinear system based on its input-output data. The proposed EGS proves better choice for finding the optimal parameters of the TS fuzzy models with better accuracy.

The paper is organized as follows: Section-2 described fuzzy modeling of nonlinear system and the appropriate cost function used for identification. Section-3 introduced EGS algorithm for tuning the unknown parameters of nonlinear system. Section-4 shows the simulation results of parameter estimation using EGS. Finally, section-5 discussed conclusion of this research.

2. Fuzzy Modeling

2.1. Nonlinear System

Single input single output (SISO) is used to show the nonlinear dynamic system in discrete time as:

$$y(k) = \theta(Y(k), U(k)) \quad (1)$$

Where $y(k)$ is the current output, θ shows the nonlinear mapping between output $(Y(k))$ and input$\big(U(k)\big)$:

$$Y(k) = [y(k-1) \;\; y\,(k-2) \ldots\ldots y(k-m)]$$
$$U(k) = [u(k-1) \;\; u\,(k-2) \ldots\ldots u(k-n)]$$

Nonlinear system identification can be defined as to find out the nonlinear relationship($\hat{\theta}$) between the output and input, representing as:

$$\hat{y}(k) = \hat{\theta}(Y(k), U(k)) \quad (2)$$

So that $\hat{y}(k)$ is close to $y(k)$ and could be considered as estimation of $y(k)$. This phenomenon is shown by Figure 1.

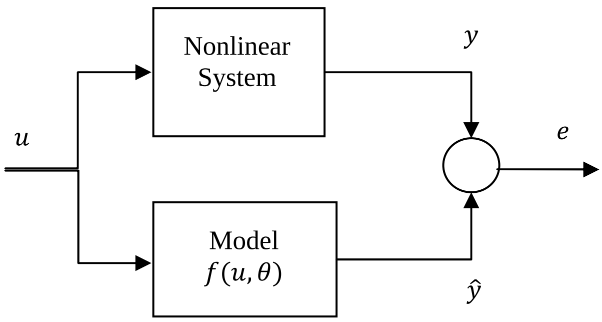

Figure 1. Nonlinear System and Model

2.2. TS Fuzzy Model

The model introduced by Takagi-Sugeno has gained interest in fuzzy modeling and control applications [12-13]. IF.....THEN rules for reasoning is the base structure of TS model which consists of antecedents as a fuzzy sets and consequents as a linear functions. Due to this arrangement, a complex affine nonlinear system could be approximated by TS fuzzy model.

For TS fuzzy modeling, choose the linear dynamic autoregressive moving average with Exogeneous input (ARMAX) model to describe the nonlinear system. For i^{th} rule:

R_i: **If** $y(k)$ is $a_{i,1}$ and $y(k-1)$ is $a_{i,2}$ and ……. and $y(k-m)$ is $a_{i,m}$
and $u(k)$ is $b_{i,1}$ and $u(k-1)$ is $b_{i,2}$ and ……. and $u(k-n)$ is $b_{i,n}$
Then $y_i(k) = a_{i,1}\, y(k) + a_{i,2}\, y(k-1) + \ldots\ldots.. + a_{i,m}\, y(k-m) + b_{i,1}\, u(k) + b_{i,2}\, u(k-1) + \ldots\ldots.. + b_{i,n}\, u(k-n)$
or $y_i(k) = \sum_{j=1}^{m} a_{i,j}\ y(k-j) + \sum_{j=1}^{n} b_{i,j}\ u(k-j)$

Where $y(k) \ldots\ldots, y(k-m)$ and $u(k), \ldots\ldots, u(k-n)$ are present and past plant outputs and inputs. After center average defuzzification, the estimated output is:

$$\hat{y}(k) = \hat{\theta}\,(k) = \frac{\sum_{i=1}^{R} y_i\,(k)\,\mu_i(y(k),\ldots..,y(k-m),u(k),\ldots,..,u(k-n)}{\sum_{i=1}^{R} \mu_i(y(k),\ldots..,y(k-m),u(k),\ldots..,u(k-n)} \quad (3)$$

Where $\hat{\theta}(x)$ is unknown parameter vector, representing as $\hat{\theta}\,(k) = [y_i\ \mu_i\ a_{i,j}\ b_{i,j}]$. The nonlinear system identification using TS fuzzy model could be done by estimating the unknown parameter vector ($\hat{\theta}\,(x)$) through the optimization of cost function.

3. Enhanced Gradient Search (EGS)

Generally, gradient information of cost function is used to calculate the next update direction in gradient search without using the original function value [14-15]. The basic idea behind the gradient search is to move the parameters in such direction that it should be opposite to gradient (or slope) of the error surface. This ensures that error should always be decreased when a new parameter updates are initiated. While in proposed Enhanced Gradient Search (EGS), cost function and slope (gradient) both are considered in order to keep the best searches during optimization.

3.1. Problem Formulation

The optimization problem starts with minimization of the cost function $J(\theta) > 0$, where $\theta \in S \subseteq R^n$, is a vector of adjustable parameters. Therefore to find $\theta^* \in S$ such that:

$$\theta^* = argmin\, J(\theta) = argmin\, \{1/2\ \sum_{k=1}^{N}[\hat{y}(k) - y(k)]^2\} \quad (4)$$

To find a parameter (θ^*) that shapes a function $F(x,\theta)$ in such a way when $F(x,\theta)$ and $F(x)$ matches, then cost function can be minimized as:

$$J(\theta) = [\hat{y}(k) - y(k)]^2 \quad (5)$$

3.2. EGS with TS Fuzzy Model

The main idea of EGS in TS fuzzy modeling is that the gradient enhanced search starts from initialization, selection, calculation and updating. In first step, initialize the parameters such as learning rate for gradient search, centers and spreads of Gaussian membership function for the optimization process. In the next step, select the vectors for parameters. During calculation, crisp output of fuzzy system has to be determined. Finally updating the parameter values by applying the EGS and obtained best parameter values. The overall EGS algorithm for TS fuzzy model can be summarized by Figure 2.

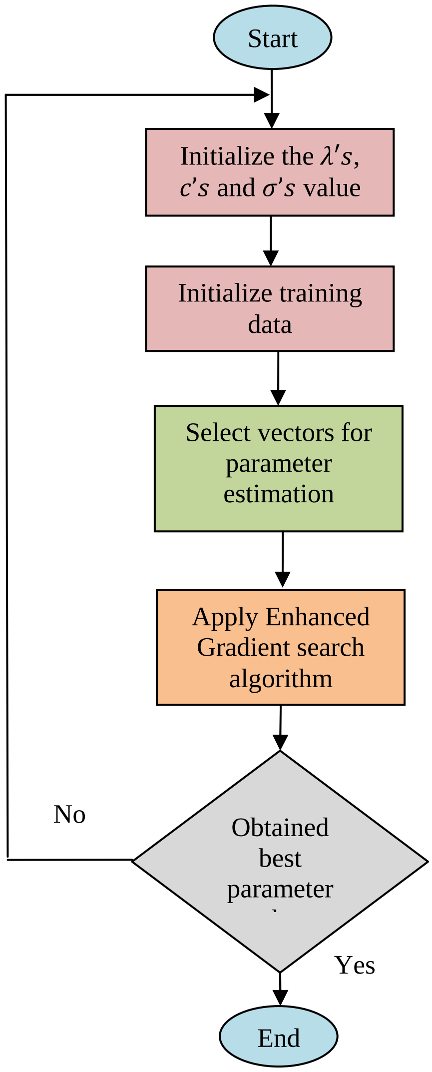

Figure 2. Proposed EGS Algorithm for TS Fuzzy Model

4. Parameter Estimation using EGS

The proposed EGS algorithm is used for parameter estimation of TS fuzzy model explained in section-2. To estimate the parameter, the following nonlinear function is used.

$$g(x) = x - \cos(1.5x) + \sin(0.5x)$$

The output (g) is a nonlinear function of input (x). To find fuzzy system $f(x|\theta\ )$ that approximates the nonlinear function over a certain time interval. Gaussian fuzzy sets are used because it ensures greatest possible generalization of the system. For Gaussian fuzzy sets, the level of contribution of nonlinear function to overall output can be determined as:

$$\mu_j^i(x_j) = exp\left(-\frac{1}{2}\left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right) \qquad (6)$$

Where c_j^i and σ_j^i are center and width for j^{th} membership function (i^{th} input). The fuzzy input-output characteristic is then described by:

$$f(x|\theta) = \frac{\sum_{i=1}^{R} b^i \prod_{j=1}^{n} exp\left(-\frac{1}{2}\left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right)}{\sum_{i=1}^{R} \prod_{j=1}^{n} exp\left(-\frac{1}{2}\left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right)} \quad (7)$$

Suppose there are five rules for five Gaussian input TS fuzzy model, therefore for these arrangement, fifteen parameters need to be adjusted (five input centers and spreads and five output singleton). Initially, parameter values are chosen randomly within the range of training data. Choose five initial values randomly for centers $(c^i, i = 1 \ldots..5)$ in the ranges [0 6] and spreads $(\sigma^i\ , i = 1 \ldots.5)$ from [0 2]. Similarly, initial values are chosen randomly for singleton $(b^i$, $i = 1 \ldots..5)$ in the range [0 15]. For EGS algorithm, the required parameters are selected as $\lambda_i = 0.0001,\ i = 1 \ldots 3$. Randomly chosen initial parameters value for Gaussian membership functions are distributed equally in the range [0 6] as shown in Figure 3 and Table 1.

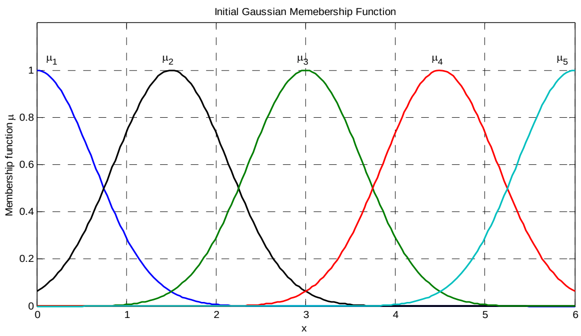

Figure 3. Initial Input Membership Function

Table 1. Parameters Initial Value

n	1	2	3	4	5
y_c^i	2.2310	0.7078	13.1423	9.4320	9.7337
μ_c^i	0	1.5	3	4.5	6
μ_σ^i	0.64	0.64	0.64	0.64	0.64

The modeling result of a certain nonlinear function after applying EGS parameters estimation algorithm are showing through Figures 4-8. From the simulated results, it is cleared that the fuzzy model which is estimated through EGS can closely match the nonlinear function very well. The value of the cost function in terms of measurement of modeling error is showing in Figure 9. The best estimated parameters values of membership function are shown in Table 2. It is observed from the simulated results that some parameters are converging to different values even though their initial starting values are same. This means that EGS optimization algorithm make partitions of the input and output domain automatically according to the desired input and output data. Estimated parameters are overlapped in the form of input membership functions as shown in Figure 5 means that proposed optimization process tends to reduce the structure complexity in a significant way and does not depend upon the input and output spaces (in this case which are divided into five partitions). The cost function value in terms of absolute error lies between 0-0.15 as shown in Figure 9 clearly depicted the excellent performance of proposed EGS.

Table 2. Parameters Final Value

n	1	2	3	4	5
y_c^i	9.3198	8.8581	9.9612	-1.5783	13.7114
μ_c^i	1.3237	1.6316	4.9389	0.5769	0.2455
μ_σ^i	0.3852	1.3457	1.2294	1.0126	1.4696

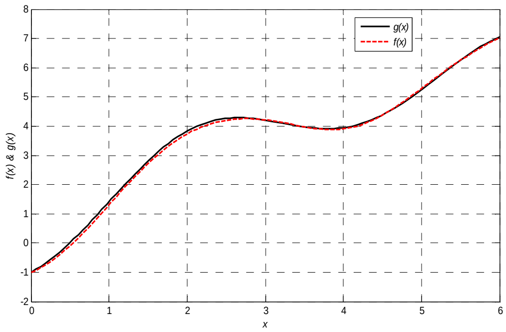

Figure 4. Nonlinear Function $g(x)$ and its Fuzzy Approximation $f(x/\theta^*)$

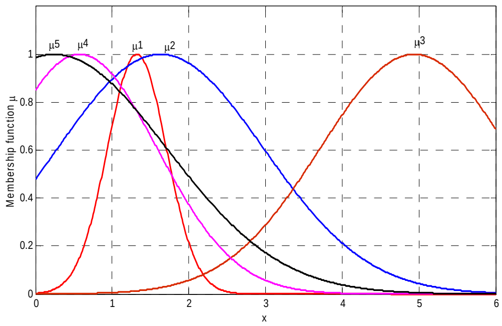

Figure 5. Input Membership after Function Approximation

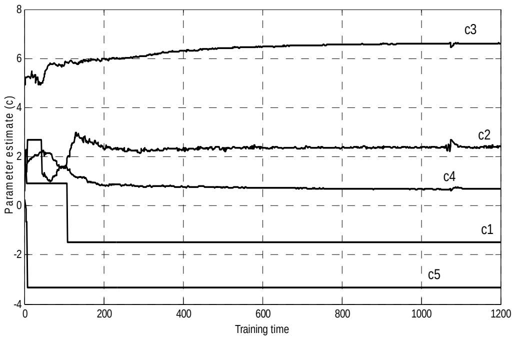

Figure 6. Estimates of Input Membership Function Center

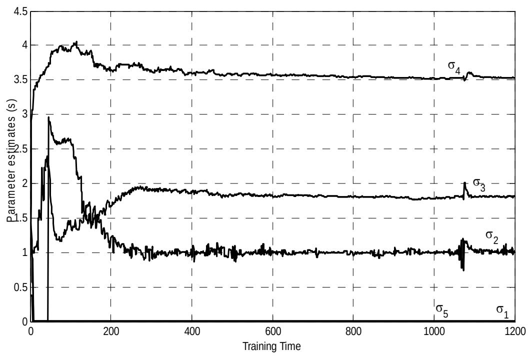

Figure 8. Estimates of Input Membership Function Spreads

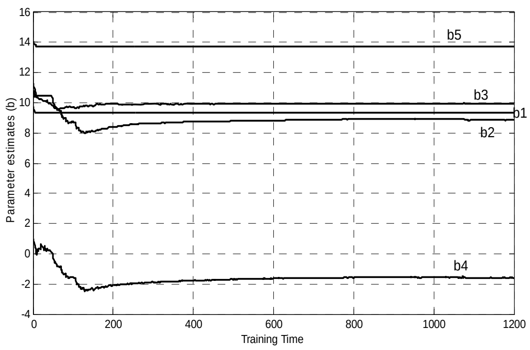

Figure 7. Estimate of Output Membership Function Centers

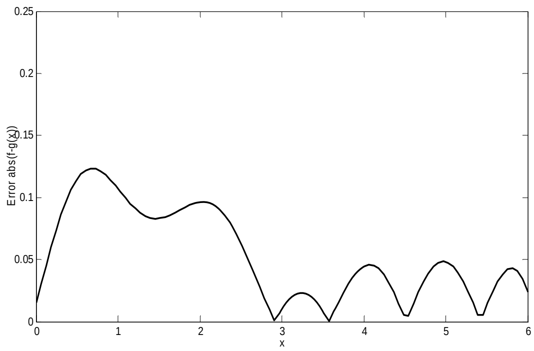

Figure 9. Error Values between Nonlinear and Fuzzy Approximated Function

5. Conclusion

This paper presents a simple and accurate method to identify a nonlinear system through parameter estimation. Enhanced Gradient Search (EGS) which is basically inferred from gradient descent method is proposed for parameter estimation of TS fuzzy modeling. The proposed EGS uses the original function value and gradient of cost function for updating it during optimization process in enhancing way, therefore the proposed identification method is a hybridization of optimization and complexity reduction. The simulated result for parameter estimation of nonlinear function showed that the proposed algorithm is capable to provide the excellent solutions.

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