Adaptive Hybrid Synchronization of Lorenz-84 System with Uncertain Parameters

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Abstract

    This paper presents the adaptive control and hybrid synchronization of Lorenz-84 chaotic system using a master-slave topology. The Lorenz-84 is an 11-term dissipative system that possessed four quadratic nonlinearities in its coupled algebraic structure which results to the evolution of a dense chaotic attractors in both 2-D and 3-D spaces. Firstly, an adaptive nonlinear feedback controller was designed to suppress the chaotic dynamics of the system. By using Lyapunov stability criterion, the asymptotic stability of the error states was guaranteed and the state dynamics were stabilized. Secondly, adaptive nonlinear feedback controllers were designed to guarantee the co-existence of synchronization and anti-synchronization of the system. By suitable selection of feedback coefficients and Lyapunov function candidate, the uncertain parameters of the slave system were estimated. Numerical simulations via MATLAB show the convergence of the uncertain parameters to their true values after a transient time while the two systems synchronized completely.

Keywords: adaptive control, hybrid synchronization, Lorenz-84, Lyapunov stability theory

1. Introduction

    Research interest in chaotic phenomena has risen astronomically during the last decades. This is owing to the fact that inquisition into chaotic phenomena continues to reveal new ways that chaos is embedded in man-made and natural systems, leading to better understandings of their usefulness in solving non-trivial challenges in engineering and non-engineering sciences. The breakthrough by Ott, Grebogi and Yorke [1] gave impetus to chaos control resulting in diverse methods of suppressing chaos in experimental and real-life scenarios. With the successful coupling of two chaotic systems by Pecora and Caroll [2], there has been a convergence of multidisciplinary approaches on studying methods of coupling almost all evolved chaotic systems.

    Synchronization is a process whereby the trajectories of two identical or non-identical systems are coupled unidirectionally or bidirectionally using suitably designed linear and nonlinear controllers. In the literature, most synchronization schemes falls into two classes, viz. master-slave type and mutual synchronization. In master-slave type, an original chaotic system serves as the drive system to provide coupling dynamics to regulate the state trajectories of another system termed the response system into synchrony in transient time. Chaos and chaos synchronization of chaos have found application in different types of communications systems [3], power systems [4], biological systems [5] and oscillators [6] amongst others. Different types of synchronization schemes have been proposed in the literature such as generalized synchronization [7], hybrid synchronization [8], generalized projective synchronization [9], and hybrid function synchronization [10].

    Methods of synchronization include adaptive control [11], sliding mode control [12], fuzzy control [13], adaptive feedback [14], observer-based control [15], backstepping design [16], and impulsive synchronization [17] among others. Adaptive methods of synchronization have gained acceptance due to their practical relevance in real-life system where most or all the system parameters may be unknown or uncertain. Unlike most other synchronization schemes where the control objectives are contingent upon the availability of all state parameters, the adaptive methods can be used to estimate unknown parameters of the system. The objective of

this work is to design adaptive controllers via feedback control techniques to control and hybrid-synchronize the complex dynamics of the Lorenz-84 system.

2. The Lorenz-84 Chaotic System

The Lorenz-84 system [18] is an 11-term dissipative system that possessed four quadratic nonlinearities in its coupled algebraic structure which results to the evolution of a dense chaotic attractors in both 2-D and 3-D spaces. The Lorenz-84 is topologically non-equivalent to the Lorenz-63 [19] which evolves the well-known butterfly attractor. However, unlike the Lorenz-63 system which is arguably one of the most studied chaotic system, the Lorenz-84 system has received very scanty interest in the literature even though it dynamics and properties have tremendous applications in engineering and non-engineering systems design. Thus, the motivation for this study is to study the controllability and synchronizability of the system. The governing equations of the Lorenz-84 system is given by:

$$x_1^{'} = -x_2^2 - x_3^2 - \Upsilon x_1 + \Upsilon \varpi$$
$$x_2^{'} = x_1 x_2 - \pi x_1 x_3 - x_2 + \chi \qquad\qquad (1)$$
$$x_3^{'} = \pi x_1 x_2 + x_1 x_3 - x_3$$

Where x_1, x_2, x_3 are states of the system. $\Upsilon, \varpi, \chi, \pi$ are positive constants. For values of $\Upsilon = 0.25, \varpi = 8, \chi = 1, \pi = 4$, the system evolves the state dynamics in Figure 1.
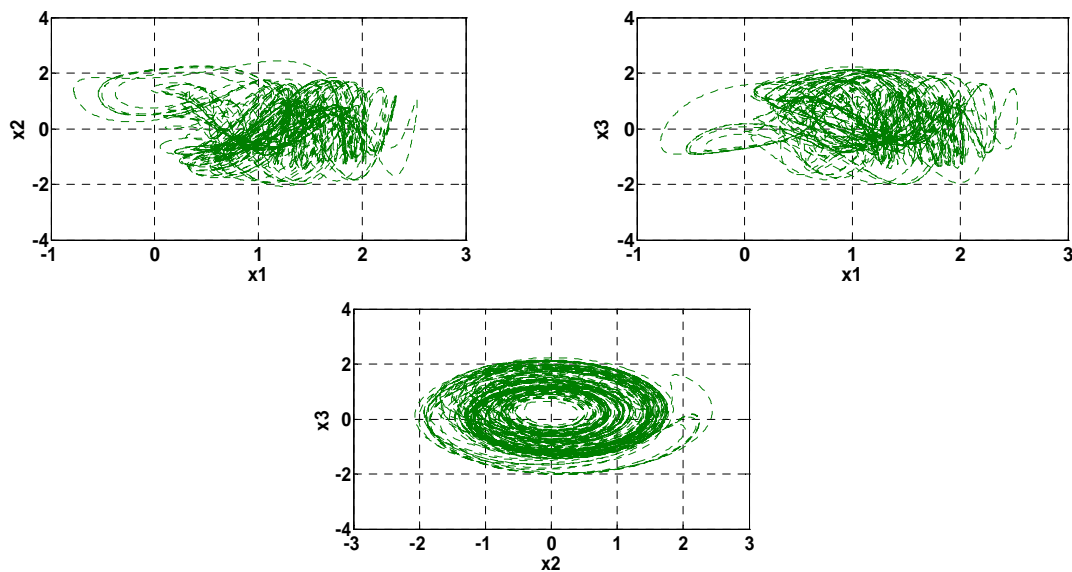

Figure 1. 2-D Portraits of the Open-loop Lorenz-84 Chaotic System

By linearizing (1) at the point $E(0,0,0)$, we obtained the Jacobian given by:

$$J\big|_{E(0,0,0)} = \begin{bmatrix} -\Upsilon & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -0.25 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad\qquad (2)$$

The characteristic equation is $\lambda^3 - 1.75\lambda^2 + 0.5\lambda + 0.25$. This gives the following eigenvalues $\lambda_{1,2,3} = (1,1,-0.25)$.

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3. Adaptive Control of the Lorenz-84 System with Uncertain Parameters

In order to asymptotically y stabilize the dynamics of the Lorenz-84 systems with uncertain parameters at equillibrium point $x_e = 0$, we add adaptive feedback controllers and the controlled system (1) becomes:

$$
\begin{aligned}
x_1^{'} &= -x_2^2 - x_3^2 - \Upsilon x_1 + \Upsilon \varpi + u_1 \\
x_2^{'} &= x_1 x_2 - \pi x_1 x_3 - x_2 + \chi + u_2 \\
x_3^{'} &= \pi x_1 x_2 + x_1 x_3 - x_3 + u_3
\end{aligned}
\tag{3}
$$

Where $u_i, i = 1,2,3$ are adaptive feedback controllers to be designed using the states of the system and $\hat{\Upsilon}, \hat{\pi}, \hat{\varpi}, \hat{\chi}$ are estimated parameters of $\Upsilon, \varpi, \chi, \pi$. The adaptive feedback controllers can be represented as:

$$
\begin{aligned}
u_1 &= x_2^2 + x_3^2 + \hat{\Upsilon} x_1 - \hat{\Upsilon}\hat{\varpi} - \kappa^1 \\
u_2 &= -x_1 x_2 + \hat{\pi} x_1 x_3 + x_2 - \hat{\chi} - \kappa^2 \\
u_3 &= -\hat{\pi} x_1 x_2 - x_1 x_3 + x_3 - \kappa^3
\end{aligned}
\tag{4}
$$

Where $\kappa^i, i = 1,2,3$ is given as:

$$
\begin{bmatrix} \kappa^1 \\ \kappa^2 \\ \kappa^3 \end{bmatrix} = \aleph \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\tag{5}
$$

And $\aleph$ is a diagonal matrix whose diagonals elements $diag[\Lambda_{11}, \Lambda_{22}, \Lambda_{33}]$ constitutes the feedback coefficients of the controllers, such that:

$$
\begin{bmatrix} \kappa^1 \\ \kappa^2 \\ \kappa^3 \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 & 0 \\ 0 & \Lambda_2 & 0 \\ 0 & 0 & \Lambda_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\tag{6}
$$

By inserting (6) into (4), Equation (3) becomes:

$$
\begin{aligned}
x_1^{'} &= -\Upsilon x_1 + \Upsilon \varpi + \hat{\Upsilon} x_1 - \hat{\Upsilon}\hat{\varpi} - \Lambda_1 x_1 \\
x_2^{'} &= -\pi x_1 x_3 + \chi + \hat{\pi} x_1 x_3 - \hat{\chi} - \Lambda_2 x_2 \\
x_3^{'} &= \pi x_1 x_2 - \hat{\pi} x_1 x_2 - \Lambda_3 x_3
\end{aligned}
\tag{7}
$$

After expanding, (7) becomes:

$$x_1^{'} = -(\Upsilon - \hat{\Upsilon})x_1 + (\Upsilon - \hat{\Upsilon})(\varpi - \hat{\varpi}) + \hat{\varpi}(\Upsilon - \hat{\Upsilon}) + \hat{\Upsilon}(\varpi - \hat{\varpi}) - \Lambda_1 x_1$$
$$x_2^{'} = -x_1 x_3 (\pi - \hat{\pi}) + \chi - \hat{\chi} - \Lambda_2 x_2 \tag{8}$$
$$x_3^{'} = -x_1 x_2 (\pi - \hat{\pi}) - \Lambda_3 x_3$$

Let,

$$\tilde{\Upsilon} = \Upsilon - \hat{\Upsilon}$$
$$\tilde{\pi} = \pi - \hat{\pi}$$
$$\tilde{\varpi} = \varpi - \hat{\varpi} \tag{9}$$
$$\tilde{\chi} = \chi - \hat{\chi}$$

By using (9) in (8), the equation reduces to:

$$x_1^{'} = -\tilde{\Upsilon}x_1 + 3\tilde{\Upsilon}\tilde{\varpi} - \Lambda_1 x_1$$
$$x_2^{'} = \tilde{\pi}x_1 x_3 + \tilde{\chi} - \Lambda_1 x_2 \tag{10}$$
$$x_3^{'} = \tilde{\pi}x_1 x_2 - \Lambda_3 x_3$$

Inorder to derive the relationship for the parameter update law, we choose a Lyapunov function candidate [20]:

$$V(x_1, x_2, x_3, \Upsilon, \chi, \varpi, \pi) = \frac{\varpi}{2\pi}(x_1^2 + x_2^2 + x_3^2 + \tilde{\Upsilon}^2 + \tilde{\chi}^2 + \tilde{\varpi}^2 + \tilde{\pi}^2) \tag{11}$$

For asymptotic stabilization of the system, $V(0) = 0; \dot{V}(.) \le 0;$

$$\dot{V}(.) = \frac{\varpi}{\pi}(x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 + \tilde{\Upsilon}\dot{\tilde{\Upsilon}} + \tilde{\chi}\dot{\tilde{\chi}} + \tilde{\varpi}\dot{\tilde{\varpi}} + \tilde{\pi}\dot{\tilde{\pi}}) \tag{12}$$

From (9), it is noted that:

$$\dot{\tilde{\Upsilon}} = -\dot{\hat{\Upsilon}}; \dot{\tilde{\varpi}} = -\dot{\hat{\varpi}}; \dot{\tilde{\pi}} = -\dot{\hat{\pi}}; \dot{\tilde{\chi}} = -\dot{\hat{\chi}} \tag{13}$$

Putting (10) and (13) into (12) and solving gives the following:

$$\dot{V}(.) = \frac{\varpi}{\pi}(x_1(-\tilde{\Upsilon}x_1 + 3\tilde{\Upsilon}\tilde{\varpi} - \Lambda_1 x_1) + x_2(\tilde{\pi}x_1 x_3 + \tilde{\chi} - \Lambda_1 x_2) + x_3(\tilde{\pi}x_1 x_2 - \Lambda_3 x_3)$$
$$+ \tilde{\Upsilon}(-\dot{\hat{\Upsilon}}) + \tilde{\chi}(-\dot{\hat{\chi}}) + \tilde{\varpi}(-\dot{\hat{\varpi}}) + \tilde{\pi}(-\dot{\hat{\pi}})) \tag{14}$$

Rearranging (14) gives:

$$\dot{V}(.) = \frac{\varpi}{\pi}(-\Lambda_1 x_1^2 - \Lambda_1 x_2^2 - \Lambda_3 x_3^2 + \tilde{\Upsilon}(x_1^2 - \dot{\hat{\Upsilon}}) + \tilde{\chi}(x_2 - \dot{\hat{\chi}}) + \tilde{\varpi}(3\tilde{\Upsilon}x_1 - \dot{\hat{\varpi}}) +$$
$$\tilde{\pi}(2x_1 x_2 x_3 - \dot{\hat{\pi}})) \tag{15}$$

From (15), the parameter update laws becomes:

$$\dot{\tilde{\Upsilon}} = x_1^2 + \Lambda_4 \tilde{\Upsilon}$$

$$\dot{\tilde{\pi}} = 2x_1 x_2 x_3 + \Lambda_5 \tilde{\pi}$$

$$\dot{\hat{\varpi}} = 3\tilde{\Upsilon} x_1 + \Lambda_6 \tilde{\varpi} \qquad (16)$$

$$\dot{\hat{\chi}} = x_2 + \Lambda_7 \tilde{\chi}$$

Where $\Lambda_i, i = 4, 5, 6, 7$ are positive constants.

Theorem 1: The controlled Lorenz-84 system (3) with uncertain parameters is asymptotically stabilized in the sense of Lyapunov for all initial conditions by the adaptive feedback control law (4) where the parameter update law is given by (16).

Proof: By inserting (16) into (15), it is observed that:

$$\dot{V}(.) = \frac{\varpi}{\pi}(-\Lambda_1 x_1^2 - \Lambda_1 x_2^2 - \Lambda_3 x_3^2 - \Lambda_4 \tilde{\Upsilon}^2 - \Lambda_5 \tilde{\pi}^2 - \Lambda_6 \tilde{\varpi}^2 - \Lambda_7 \tilde{\chi}^2) \qquad (17)$$
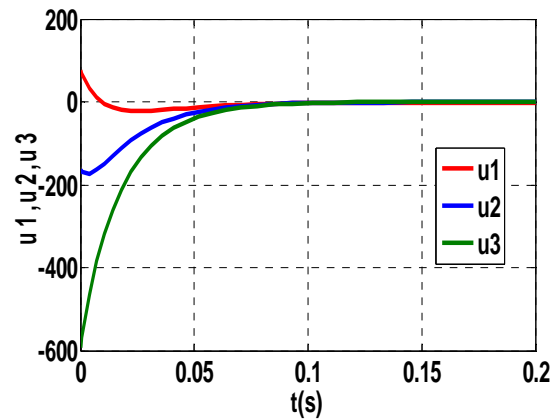
Which is negative definite function on $\Re^7$. Therefore, the parameter estimation errors would converge exponentially to zero as $t \to 0$.

4. Numerical Simulations

The Lorenz-84 system (3), adaptive feedback control laws (4) and the parameter update laws (16) were simulated in MATLAB environment for the following parameters $\Upsilon = 0.25, \varpi = 8, \chi = 1, \pi = 4$ and initial conditions for system $[x_1(0), x_2(0), x_3(0)] = [2, 6, 10]$, parameter estimates $[\hat{\Upsilon}(0), \hat{\pi}(0), \hat{\varpi}(0), \hat{\chi}(0)] = [-4, 7, 12, -2]$. The resultant plots are given in the following figures.
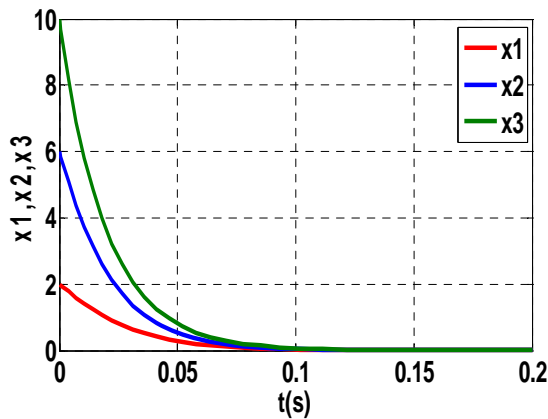
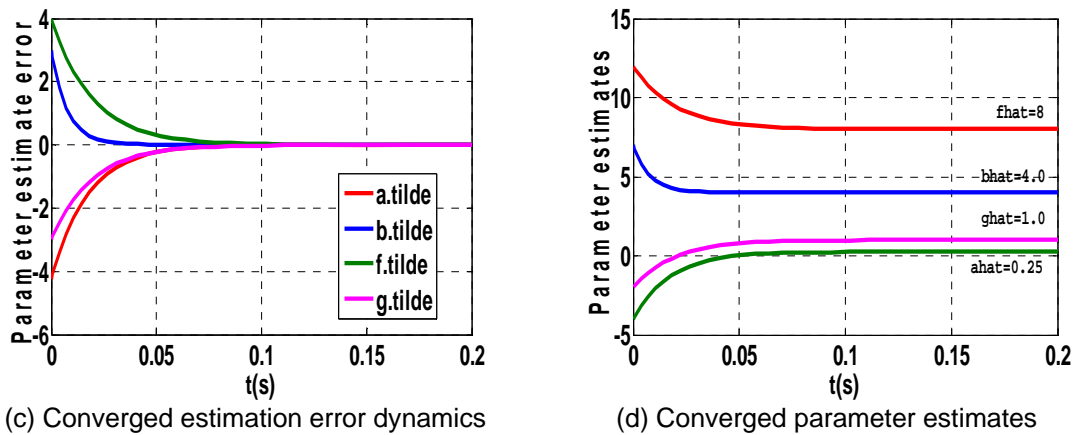(a) Stabilized state dynamics   (b) Converged control laws

(c) Converged estimation error dynamics   (d) Converged parameter estimates

Figure 2. Simulated Results of the Lorenz-84 System

5. Adaptive Hybrid Synchronization of the Lorenz-84 Chaotic System

In this section, the objective of complete synchronization of identical Lorenz-84 is realized via the design of linear and nonlinear controllers. In hybrid synchronization, there is a co-existence of complete synchronization and anti-synchronization [21]. In this paper, the slave system is adopted as the controlled system with uncertain parameters. Thus, the two systems are represented as follows:

$$
\begin{aligned}
x_1^{'} &= -x_2^2 - x_3^2 - \Upsilon x_1 + \Upsilon \varpi \\
x_2^{'} &= x_1 x_2 - \pi x_1 x_3 - x_2 + \chi \\
x_3^{'} &= \pi x_1 x_2 + x_1 x_3 - x_3
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
y_1^{'} &= -y_2^2 - y_3^2 - \hat{\Upsilon} y_1 + \hat{\Upsilon} \hat{\varpi} + u_L^1 \\
y_2^{'} &= y_1 y_2 - \hat{\pi} y_1 y_3 - y_2 + \hat{\chi} + u_L^2 \\
y_3^{'} &= \hat{\pi} y_1 y_2 + y_1 y_3 - y_3 + u_L^3
\end{aligned}
\tag{19}
$$

Let the hybrid synchronization error be defined as:

$$
\begin{aligned}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 - x_3
\end{aligned}
\tag{20}
$$

By using (20), the error dynamics of the two systems becomes:

$$
\begin{aligned}
e_1^{'} &= -y_2^2 - y_3^2 - \hat{\Upsilon} y_1 + \hat{\Upsilon} \hat{\varpi} + x_2^2 + x_3^2 + \Upsilon x_1 - \Upsilon \varpi + u_L^1 \\
e_2^{'} &= y_1 y_2 - \hat{\pi} y_1 y_3 - y_2 + \hat{\chi} + x_1 x_2 - \pi x_1 x_3 - x_2 + \chi + u_L^2 \\
e_3^{'} &= \hat{\pi} y_1 y_2 + y_1 y_3 - y_3 - \pi x_1 x_2 - x_1 x_3 + x_3 + u_L^3
\end{aligned}
\tag{21}
$$

Equation (21) can be simplified to:

$$
\begin{aligned}
e_1^{'} &= -e_2(y_2 - x_2) - e_3(y_3 + x_3) + (\Upsilon - \hat{\Upsilon})e_1 - \Upsilon e_1 - y_1(\Upsilon - \hat{\Upsilon}) \\
&\quad + 3(\Upsilon - \hat{\Upsilon})(\varpi - \hat{\varpi}) + u_L^1 \\
e_2^{'} &= y_1 y_2 + x_1 x_2 - \hat{\pi} y_1 y_3 - \pi x_1 x_3 - e_2 + \chi + \hat{\chi} + u_L^2 \\
e_3^{'} &= \hat{\pi} y_1 y_2 - \pi x_1 x_2 + y_1 y_3 - x_1 x_3 - e_3 + u_L^3
\end{aligned}
\tag{22}
$$

Let,

$$
\begin{aligned}
\tilde{\Upsilon} &= \Upsilon - \hat{\Upsilon} \\
\tilde{\pi} &= \pi - \hat{\pi} \\
\tilde{\varpi} &= \varpi - \hat{\varpi} \\
\tilde{\chi} &= \chi + \hat{\chi}
\end{aligned}
\tag{23}
$$

By using (23) in (22), the equation reduces to:

$$
\begin{aligned}
e_1^{'} &= -e_2(y_2 - x_2) - e_3(y_3 + x_3) + \tilde{\Upsilon}e_1 - \Upsilon e_1 - y_1\tilde{\Upsilon} + 3\tilde{\Upsilon}\tilde{\varpi} + u_L^1 \\
e_2^{'} &= y_1 y_2 + x_1 x_2 - \hat{\pi} y_1 y_3 - \pi x_1 x_3 - e_2 + \chi + \hat{\chi} + u_L^2 \\
e_3^{'} &= \hat{\pi} y_1 y_2 - \pi x_1 x_2 + y_1 y_3 - x_1 x_3 - e_3 + u_L^3
\end{aligned}
\tag{24}
$$

And the adaptive control law becomes:

$$
\begin{aligned}
u_L^1 &= e_2(y_2 - x_2) + e_3(y_3 + x_3) - \tilde{\Upsilon}e_1 + \Upsilon e_1 + y_1\tilde{\Upsilon} - 3\tilde{\Upsilon}\tilde{\varpi} - \Lambda_1 e_1 \\
u_L^2 &= -y_1 y_2 - x_1 x_2 + \hat{\pi} y_1 y_3 + \pi x_1 x_3 + e_2 - \chi - \hat{\chi} - \Lambda_2 e_2 \\
u_L^3 &= -\hat{\pi} y_1 y_2 + \pi x_1 x_2 - y_1 y_3 + x_1 x_3 + e_3 - \Lambda_3 e_3
\end{aligned}
\tag{25}
$$

Then by inserting (25) in (24), we have a new relationship for the synchronization error dynamics:

$$
\begin{aligned}
e_1^{'} &= -\Lambda_1 e_1 \\
e_1^{'} &= -\Lambda_2 e_2 \\
e_1^{'} &= -\Lambda_3 e_3
\end{aligned}
\tag{26}
$$

We can also note from (23) that:

$$
\dot{\tilde{\Upsilon}} = -\dot{\hat{\Upsilon}}; \quad \dot{\tilde{\varpi}} = -\dot{\hat{\varpi}}; \quad \dot{\tilde{\pi}} = -\dot{\hat{\pi}}; \quad \dot{\tilde{\chi}} = -\dot{\hat{\chi}}
\tag{27}
$$

Inorder to derive the relationship for the parameter update law, we choose a Lyapunov function candidate:

$$
V(e_1, e_2, e_3, \Upsilon, \chi, \varpi, \pi) = \frac{\varpi}{2\pi}(e_1^2 + e_2^2 + e_3^2 + \hat{\Upsilon}^2 + \tilde{\chi}^2 + \tilde{\varpi}^2 + \tilde{\pi}^2)
\tag{28}
$$

The partial derivative of (28) along the trajectories of the system becomes:

$$
\dot{V}(.) = \frac{\varpi}{2\pi}(e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + \tilde{\Upsilon}\dot{\tilde{\Upsilon}} + \tilde{\chi}\dot{\tilde{\chi}} + \tilde{\varpi}\dot{\tilde{\varpi}} + \tilde{\pi}\dot{\tilde{\pi}})
\tag{29}
$$

By using (26) and (27) in (29), the relationship (29) becomes:

$$\dot{V}(.) = \frac{\varpi}{\pi}(-\Lambda_1 e_1^2 - \Lambda_1 e_2^2 - \Lambda_3 e_3^2 - \tilde{\Upsilon}\dot{\hat{\Upsilon}} - \tilde{\chi}\dot{\hat{\chi}} - \tilde{\varpi}\dot{\hat{\varpi}} - \tilde{\pi}\dot{\hat{\pi}})$$ (30)

It can also be observed from (30), that the parameter update law is given by:

$$\dot{\hat{\Upsilon}} = \Lambda_4 \tilde{\Upsilon}$$
$$\dot{\hat{\pi}} = \Lambda_5 \tilde{\pi}$$
$$\dot{\hat{\varpi}} = \Lambda_6 \tilde{\varpi}$$ (31)
$$\dot{\hat{\chi}} = \Lambda_7 \tilde{\chi}$$

For all $\Lambda_i > 0$, $i = 4, 5, 6, 7$

Theorem 2: The master Lorenz-84 system (18) and the controlled slave system with uncertain parameters are hybrid-synchronized for all initial conditions by the adaptive control law (25) where the parameter update law is given by (31) while the synchronization errors and parameter estimation errors converged asymptotically in transient time.

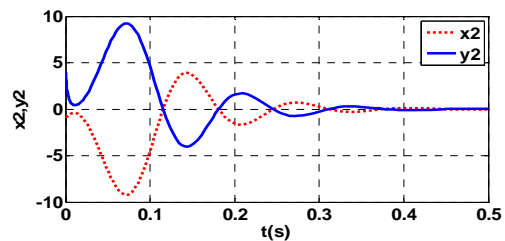**Proof:** By inserting (31) into (30), it is observed that:

$$\dot{V}(.) = \frac{\varpi}{\pi}(-\Lambda_1 e_1^2 - \Lambda_1 e_2^2 - \Lambda_3 e_3^2 - \Lambda_4 \tilde{\Upsilon}^2 - \Lambda_5 \tilde{\pi}^2 - \Lambda_6 \tilde{\varpi}^2 - \Lambda_7 \tilde{\chi}^2)$$ (32)

Which is negative definite function on $\Re^7$. Therefore, the synchronization and parameter estimation errors would converge exponentially to zero as $t \to 0$.
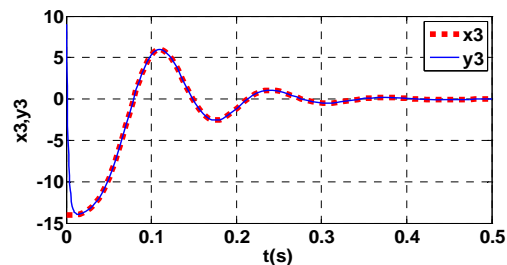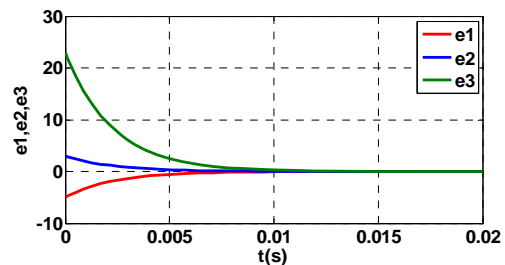
6. Numerical Simulations
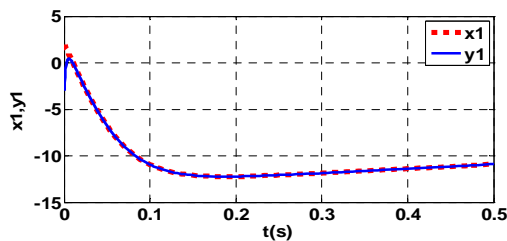
(a) Synchronized x1-y1 states

(b) Antisynchronized x2-y2 states

(c) Synchronized x3-y3 states

(d) Converged error dynamics

(e) Converged parameter estimates

(f) Converged estimation errors
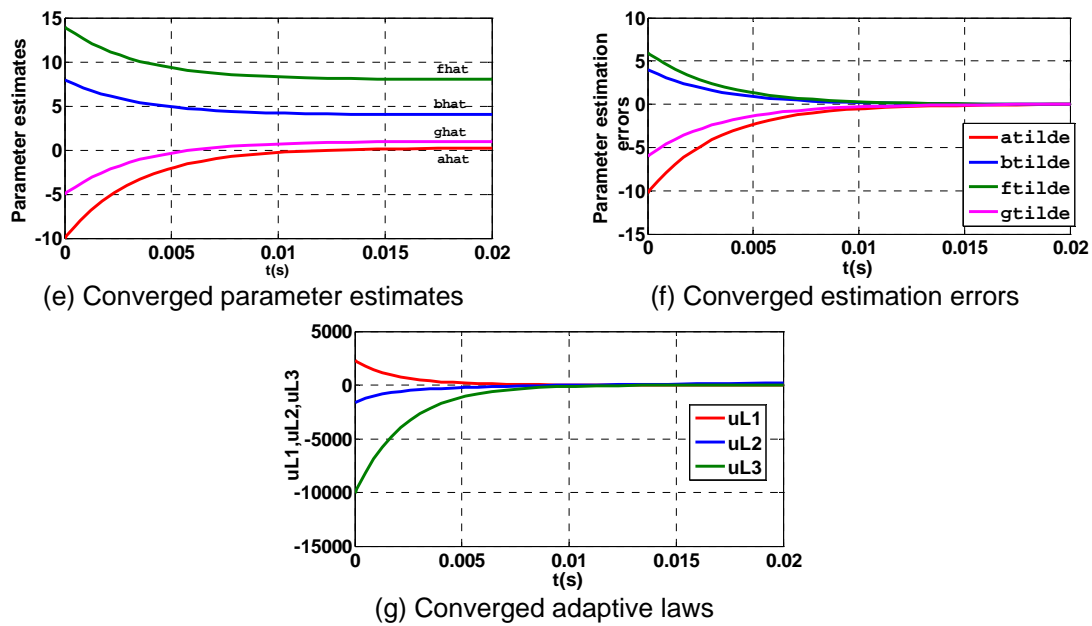

(g) Converged adaptive laws

Figure 3. Dynamics of the Synchronized Lorenz-84 Systems

The master Lorenz-84 system (18), adaptive controlled response system (19), adaptive control laws (25) and the parameter update law (31) were simulated in MATLAB environment for the following parameters $\Upsilon = 0.25, \varpi = 8, \chi = 1, \pi = 4$ and initial conditions for master system $[x_1(0), x_2(0), x_3(0)] = [2, -1, -14]$, slave system $[y_1(0), y_2(0), y_3(0)] = [-3, 4, 9]$ parameter estimates $[\hat{\Upsilon}(0), \hat{\pi}(0), \hat{\varpi}(0), \hat{\chi}(0)] = [10, 8, 14, -5]$. The initial conditions of the synchronization error dynamics becomes $[e_1(0), e_2(0), e_3(0)] = [-5, 3, 23]$. The resultant plots are given in the Figure 3.

7. Conclusion

Adaptive control and hybrid synchronization of the Lorenz-84 system with uncertain parameters is reported in this paper. By appropriately selecting the feedback coefficients of the control law, the state dynamics of the system were asymptotically stabilized in transient time and the estimated parameters converged to their true values. Appropriate control laws were equally designed for co-existent coupling of identical Lorenz-84 system in a master-slave topology via hybrid synchronization scheme. Proper selection of the feedback coefficients engendered a complete synchronization and anti-synchronization of the states of the system while the estimated parameters of the controlled slave system converged to their true values in transient time.

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