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Optimal Feedback Control of Vehicle Vibration with Eight Degrees of Freedom

Ali Hemati*[1], Mehdi Tajdari[2], Ahmadreza Khoogar[3]
Department of Mechanical Research, Maleke-e-Ashtar University of Technology, Tehran, Iran
Corresponding author, e-mail: a_hemati65@yahoo.com[1], Tajdari@ yahoo.com[2], khoogar@ yahoo.com[3]

Abstract

In this paper the unfavorable body vibration of vehicles is compensated using direct feedback signals in an optimized manner. The parameter - optimization problem was gained through the second method of Liapunov. Moving vehicles bear unfavorable body vibration, especially in bumpy roads. Therefore, active control of this vibration seems attractive. Vertical vibration of the vehicle affects the driver's performance. In this paper a seven degree of freedom dynamic vibration model of a general vehicle is developed through the designation of a closed-loop control system. A control system has been designed with efficient response which computes the feedback gains using the second method of Liapunov and it has been proven that appropriate comparison is obtained with this controller.

***Keywords**: LQR, active suspension, 8 degrees of freedom*

1. Introduction

One can simulate a vehicle vibration in three ways. The first method is studying 1/4 vehicle vibration. The second method is studying 1/2 vehicle vibration and the third way would be studying the full model of vehicle vibration. In this study the full mode vehicle vibration through state feedback and Parameter-optimization have been achieved through the second method of Liapunov. The suspension system of the vehicle simulation justifies the amount of unfavorable vehicle body movement. The periodical non-linear dynamic of the suspension model changes based on the Newton and Lagrange's formulas. Exposure to whole-body vibration (WBV) associated with a prolonged seating is an important risk factor for low back pain (LBP) among drivers [1]. Both vehicle suspension system and driver's seat cushion designs have attracted significant interest over the last several decades with a significant effort being directed towards their improvements. Vibration attenuation through the suspension and seat will not only provide riding comfort but also reduce the risk of LBP due to driving. One can simulate a vehicle vibration in three ways. The first method is studying 1/4 vehicle vibration. The second method is studying 1/2 vehicle vibration, and the third way would be studying the full models of vehicle vibration. In this study, the full mode vehicle vibrations through PID controller and Parameter-optimization have been achieved through genetic algorithm. Suspension system during the vehicle simulation justifies the amount of unfavorable vehicle body movements. The periodical non-linear dynamic of the suspension model changes based on the Newton and Lagrange's formulas. In order to be able to use the state space formulations, as well as enjoying such advantages like applications on multi variable systems, and ease of operations and manipulations - in the next stage the nonlinear system has been replaced by an equivalent linear system. In this paper, the simulations have been done by the MATLAB and the SIMULINK toolboxes. Recently, passive vehicle suspension with regard to affective factors on system parameters such as; spring constant coefficients and damping coefficients, as well as the external force, have attracted a lot of researcher attentions. Thus far, different methods have been used to control 1/4 body vibration [2]. The results obtained after the suspension system analysis which is referred to, as mass-spring-damper system and proved to have initiated modes of excitation, is presented in the reference [3]. In [4] genetic algorithm (GA) method is applied to the optimization problem of a linear one degree-of-freedom (1-DOF) vibration isolator mount, and the method is extended to the optimization of a linear quarter car suspension model Neural network based robust control system is designed to control vibration of vehicle

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suspensions for full-suspension system [5]. The design of an adaptive active suspension system, in order to simultaneously improve ride comfort and travel suspension under various traffic conditions, is addressed in [6]. Vehicle ride comfort is a function of several parameters, including human sensitivity to transmit vibrations, body posture and the direction of the transmitted vibrations from road irregularities. Human sensitivity to transmitted vibrations in the objective ride comfort evaluation is usually formulated as a standard Ride Index (RI) obtained by applying frequency filters to the transmitted vibrations and combining the weighted accelerations [7]. It presents a parameter-dependent controller design approach for vehicle active suspensions to deal with changes in the vehicle inertial properties and existence of actuator time delays [8]. [9] Proposes a non-linear pitch-plane model, to be used for the gradient information, when optimizing ride comfort. [10] Presents an electromechanical wheel suspension, where the upper arm of the suspension has been provided with an electric leveling and a damper actuator, both are allowed to work in a fully active mode. In [11] to provide a systematic probe into the necessity of the active suspension based on LQG control for supplying some reference to optimal design of the suspension based on LQG control. Semi-active control of vehicle suspension with magneto-theological (MR) damper is studied in [12]. In this paper, the researcher studies and optimizes the full model vehicle performance. The controller system applied here is a feedback state. The feedback state coefficient is specified to be the second method of Liapunov (LQR). This method is done at 8 degrees of freedom vehicle which proved to give more favorable results to control the vehicle vibrations.

2. Mathematical Modeling

The full model 8 degree of freedom is illustrated in Figure 1. The vehicle degrees of freedom include roll vibration, pitching and vertical vibration, and vertical motion of four wheels. These vibrations cause the suspension system fatigue, the addition of dynamic force on the body besides lowering the driver comfort. Controlling systems considerably decrease unfavorable vibrations. The spring and damper of the linear suspension system are considered. The tire is modeled as a linear spring with a high spring constant. The vehicle parameters are given Table 1.

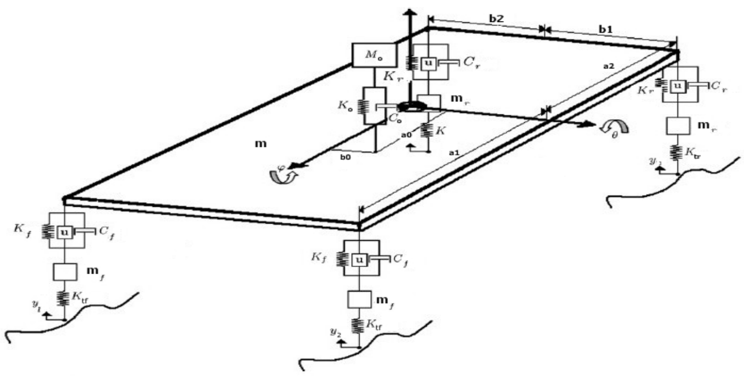

Figure 1. Full-vehicle Model [6]

$$m[\ddot{x}]+C[\dot{x}]+K[x]=f \quad (1)$$

$$\begin{aligned}&m\ddot{x}+c_f\left(\dot{x}-\dot{x}_1-a_1\dot{\theta}+b_1\dot{\varphi}\right)+c_f\left(\dot{x}-\dot{x}_2-a_1\dot{\theta}-b_2\dot{\varphi}\right)+c_r\left(\dot{x}-\dot{x}_3+a_2\dot{\theta}+b_1\dot{\varphi}\right)+\\&c_r\left(\dot{x}-\dot{x}_4+a_2\dot{\theta}-b_2\dot{\varphi}\right)+c_0\left(\dot{x}-\dot{x}_0+a_2\dot{\theta}-b_2\dot{\varphi}\right)+k_f\left(x-x_1-a_1\theta+b_1\varphi\right)+\\&k_f\left(x-x_2-a_1\theta-b_2\varphi\right)+k_r\left(x-x_3+a_2\theta+b_1\varphi\right)+k_r\left(x-x_4+a_2\theta-b_2\varphi\right)+\\&k_0\left(x-x_0+a_2\theta-b_2\varphi\right)=f_f+f_f+f_r+f_r\end{aligned} \quad (2)$$

$$I_Y\ddot{\theta}-c_f.a_1\left(\dot{x}-\dot{x}_1-a_1\dot{\theta}+b_1\dot{\varphi}\right)-c_f.a_1\left(\dot{x}-\dot{x}_2-a_1\dot{\theta}-b_2\dot{\varphi}\right)+c_r.a_2\left(\dot{x}-\dot{x}_3+a_2\dot{\theta}+b_1\dot{\varphi}\right)+$$
$$c_r.a_2\left(\dot{x}-\dot{x}_4+a_2\dot{\theta}-b_2\dot{\varphi}\right)+c_0.a_2\left(\dot{x}-\dot{x}_0+a_2\dot{\theta}-b_2\dot{\varphi}\right)-k_f.a_1\left(x-x_1-a_1\theta+b_1\varphi\right)- \quad (3)$$
$$k_f.a_1\left(x-x_2-a_1\theta-b_2\varphi\right)+k_r.a_2\left(x-x_3+a_2\theta+b_1\varphi\right)+k_r.a_2\left(x-x_4+a_2\theta-b_2\varphi\right)+$$
$$k_0.a_2\left(x-x_0+a_2\theta-b_2\varphi\right)=-f_f.a_1-f_f.a_1+f_r.a_2+f_r.a_2$$

Table 1. Full-vehicle Model Characteristics

Parameter	***Symbol***	***Value***
Mass of the vehicle body	m	840 kg
Unsprung mass in front left/right side	m_f	53 kg
Unsprung mass at rear side	m_r	76 kg
Distance between front wheel and full-vehicle model at its mass center	a_1	1.4 m
Distance between rear wheel and full-vehicle model at its mass center	a_2	1.47 m
Spring constant of front suspension	k_f	10000 N/m
Spring constant of rear suspension	k_r	10000 N/m
Spring constant of front tire	k_{tf}	200000 N/m
Spring constant of rear tire	k_{tr}	200000 N/m
Fixed damping coefficient of the front suspension damper	c_f	2000 N.s/m
Fixed damping coefficient of the rear suspension damper	c_r	2000 N.s/m
Distance between front and rear right side wheel and full-vehicle model at its mass center	b_1	0.7 m
The distance between front and rear left side wheel and full-vehicle model at its mass center	b_2	0.75 m
Roll moment of inertia of the vehicle body	Ix	820
Pitch moment of inertia of the vehicle body	Iy	1100
Pitch angle of the vehicle body	θ	Variable
Roll angle of the vehicle body	φ	Variable
Spring constant of Driver's seat	k_o	1200 N/m
Fixed damping coefficient of the constant of the Driver's seat	c_o	400N.s/m
Mass of the Driver's seat	m_o	80 kg

$$I_x\ddot{\varphi}+c_f.b_1\left(\dot{x}-\dot{x}_1-a_1\dot{\theta}+b_1\dot{\varphi}\right)+c_f.b_1\left(\dot{x}-\dot{x}_1-a_1\dot{\theta}+b_1\dot{\varphi}\right)-$$
$$c_f.b_2\left(\dot{x}-\dot{x}_2-a_1\dot{\theta}-b_2\dot{\varphi}\right)+c_r.b_1\left(\dot{x}-\dot{x}_3+a_2\dot{\theta}+b_1\dot{\varphi}\right)-c_f.b_2\left(\dot{x}-\dot{x}_4+a_2\dot{\theta}-b_2\dot{\varphi}\right)- \quad (4)$$
$$c_0.b_2\left(\dot{x}-\dot{x}_0+a_2\dot{\theta}-b_2\dot{\varphi}\right)k_f.b_1\left(x-x_1-a_1\theta+b_1\varphi\right)-k_f.b_2\left(x-x_2-a_1\theta-b_2\varphi\right)+$$
$$k_r.b_1\left(x-x_3+a_2\theta+b_1\varphi\right)-k_r.b_2\left(x-x_4+a_2\theta-b_2\varphi\right)-k_0.b_2\left(x-x_0+a_2\theta-b_2\varphi\right)=$$
$$f_f.b_1-f_f.b_2+f_r.b_1-f_r.b_2$$

$$m_1\ddot{x}_1-c_f\left(\dot{x}-\dot{x}_1-a_1\dot{\theta}+b_1\dot{\varphi}\right)-k_f\left(x-x_1-a_1\theta+b_1\varphi\right)+k_{tf}\left(y_1-x_1\right)=f_1 \quad (5)$$

$$m_2\ddot{x}_2-c_f\left(\dot{x}-\dot{x}_2-a_1\dot{\theta}-b_2\dot{\varphi}\right)-k_f\left(x-x_2-a_1\theta-b_2\varphi\right)+k_{tf}\left(y_2-x_2\right)=f_2 \quad (6)$$

$$m_3\ddot{x}_3 - c_r\left(\dot{x} - \dot{x}_3 + a_2\dot{\theta} + b_1\dot{\varphi}\right) - k_r\left(x - x_3 + a_2\theta + b_1\varphi\right) + k_{tr}\left(y_3 - x_3\right) = f_3 \quad (7)$$

$$m_4\ddot{x}_4 - c_r\left(\dot{x} - \dot{x}_4 + a_2\dot{\theta} - b_2\dot{\varphi}\right) - k_r\left(x - x_4 + a_2\theta - b_2\varphi\right) + k_{tr}\left(y_4 - x_4\right) = f_4 \quad (8)$$

$$m_0\ddot{x}_0 - c_0\left(\dot{x} - \dot{x}_0 + a_2\dot{\theta} - b_2\dot{\varphi}\right) - k_0\left(x - x_0 + a_2\theta - b_2\varphi\right) = 0 \quad (9)$$

The vehicle parameters are given Table 1.

3. Designing of Controlling System in State Space through Closed-loop Method [14]

Designing the controlling system through the closed - loop method shows the designing of controlling in state space. In this research, the efficiency coefficient of feedback has determined the second method of Liapunov (LQR). By considering LQR and having the Equation (13), the following steps will be:

$$\dot{\mathrm{x}} = \mathrm{Ax} + \mathrm{Bu} \quad (13)$$

Determine the matrix K of the optimal control vector:

$$U(t) = -k\,\mathrm{x}(t) \quad (14)$$

So as to minimize the performance index:

$$J = \int_0^\infty \left(x^* Qx + u^* Ru\right) dt \quad (15)$$

Where Q is a positive-definite (or semi positive-definite) Hermitian or real symmetric matrix and R is a positive-definite Hermitian real symmetric matrix. Note that the second term on the right-hand side of the Equation (15) accounts for the expenditure of the energy of the control signals. The matrices Q and R determine the relative importance of the error and the expenditure of this energy. In this problem, It is assumed that the control vector U (t) unconstrained. As will be seen later, the linear control law given by Equation (14) is the optimal control law. Therefore, if the unknown element matrix K is determined so as to minimize the performance index, then U (t) = -k x (t) is optimal for any initial state x (0). The block diagram showing the optimal configuration is shown in Figure 2

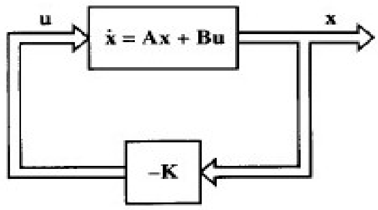

Figure 2. Optimal Control System

Solve optimization problem substituting Equation (14) into Equation (13).

$$\dot{\mathrm{x}} = \mathrm{Ax} - \mathrm{B}Kx = \left(\mathrm{A} - \mathrm{B}K\right)x \quad (16)$$

In following derivations, we assume that the matrix A-BK is stable, or that the eigenvalues of A-BK have negative real parts. Substituting Equation (14) into Equation (16) yields:

$$J = \int_0^\infty \left(x * Q + x * k * Rkx \right) dt$$

Following the discussion given in solving the parameter-optimizing problem, it is set as follows:

$$x * \left(Q + K * RK \right) x = -\frac{d\left(x * px \right)}{dt}$$

Where P is positive-definite Hermitian or real symmetric matrix. Then it is concluded that:

$$x * \left(Q + K * RK \right) x = -\dot{x} * Px - x * P\dot{x} = -x * \left[\left(A - BK \right) * P + P\left(A - BK \right) \right] x$$

Comparing both sides of this last equation and noting that this equation must hold true for any x, so it requires:

$$\left(A - BK \right) * P + P\left(A - BK \right) = -\left(Q + K * RK \right) \quad (17)$$

By the second method of Liapunov, if A-BK is stable matrix, there exists a positive definite matrix P that satisfies Equation (17). The P variables are all extracted from the Equation (17). By determining P, the index J will be obtained as follows:

$$J = \int_0^\infty x * \left(Q + u * k * Rk \right) x\, dt = -x * Px = -x * \left(\infty \right) Px\left(\infty \right) + x * \left(0 \right) Px\left(0 \right) \quad (18)$$

Since all eigenvalues of A-BK are assumed to have negative real parts, if$x \to 0$. Therefore, it is:

$$J = x * \left(0 \right) Px\left(0 \right) \quad (19)$$

Since R has been assumed to be a positive – definite Hermitian or real symmetric matrix, it can be written as:

$$R = T * T$$

Where T is a nonsingular matrix. Then Equation (17) can be written as:

$$\left(A - BK \right) * P + P\left(A - BK \right) + \left(Q + K * T * T \right)$$

Which of which can be replaced as:

$$A * P + PA + \left[TK - \left(T * \right)^{-1} B * P \right] * \left[TK - \left(T * \right)^{-1} B * P \right] - PBR^{-1}B * P + Q = 0$$

The minimization of J with respect to K requires the minimization of:

$$x * \left[TK - \left(T * \right)^{-1} B * P \right] * \left[TK - \left(T * \right)^{-1} B * P \right] X$$

With respect to K. Since this last expression is nonnegative, the minimum occurs when it is zero, or when:

$$K = T^{-1}\left(T^*\right)^{-1} B^* P = R^{-1} B^* P \qquad (20)$$

Equation (20) gives the optimal matrix K. Thus, the optimal control law to the quadratic optimal control problem when the performance index is given by Equation (20) is linear and is given by:

$$U(t) = -K.x(t) = -R^{-1} B^* P x(t)$$

The P matrix in the Equation (21) should be satisfied with the following equation:

$$A^* P + PA - PBR^{-1} B^* P + Q \qquad (21)$$

Equation (21) is called the reduced-matrix Riccati equation. These steps have been taken to determine the optimal K matrix. The K obtained by LQR is used for feedback. In this paper Q and R are diagonal and unique matrix For simulations simplicity

4. Results and Analysis

Regarding the above equations, optimal simulations are the result. This simulation has been designed for the vehicle vibrations with 8 degrees of freedom. Figure 3, Figure 4, Figure 5, and Figure 6 show vertical acceleration, pitching acceleration, roll acceleration and dynamic load on suspension system respectively.

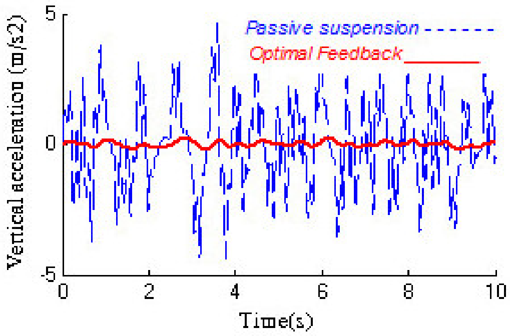

Figure 1. The Simulation Chart of Body Vertical Acceleration

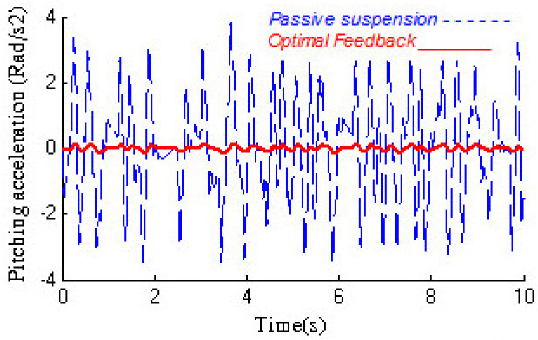

Figure 2. The Simulation Chart of Body Pitching Acceleration

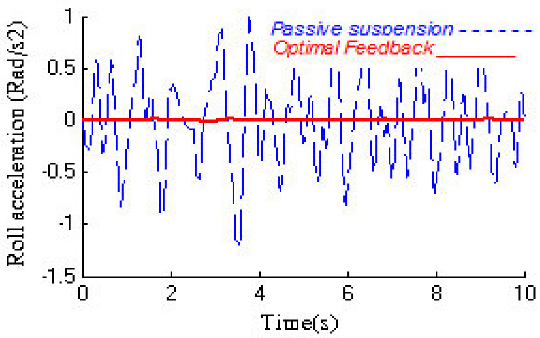

Figure 3. The Simulation Chart of Body Roll Acceleration

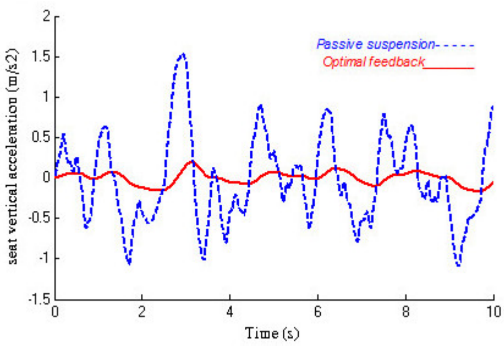

Figure 4. The Simulation Chart of the Driver's Seat Acceleration

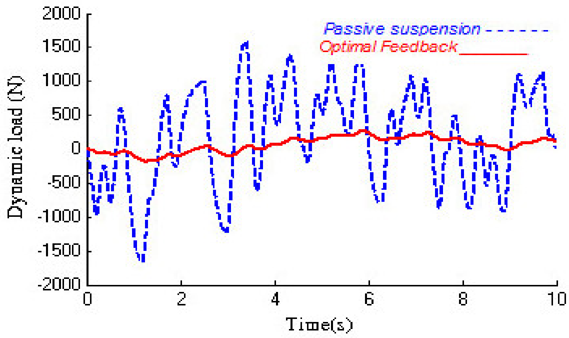

Figure 5. The Simulation Chart of Front Suspension Dynamic Load

All of suspension simulation charts, determine base input pavement show in Figure 6.

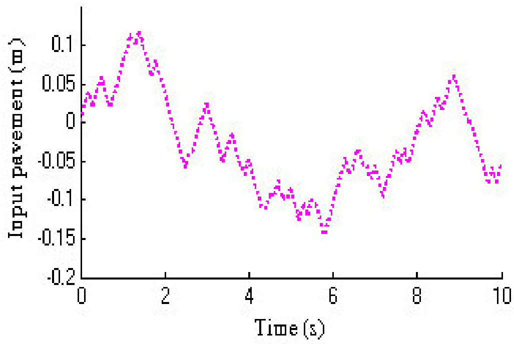

Figure 6. The Simulation Chart of Input Pavement

Figure 6 shows the flowchart of suspension system on MATLAB/SIMULINK.

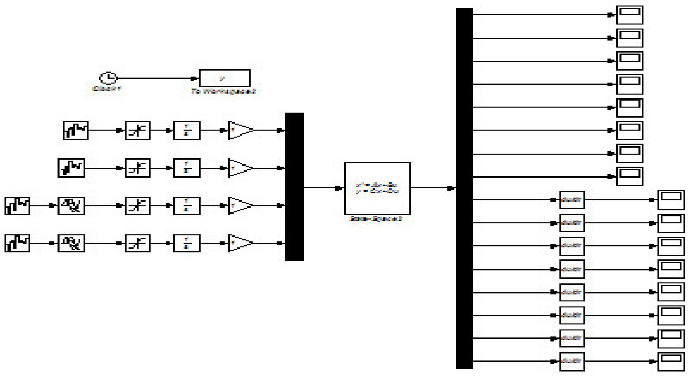

Figure 7. Simulation Flowchart of Suspension System on MATLAB/SIMULINK

A body vibration control using optimal control is an appropriate method for control of vertical, roll and pitch acceleration. The result of the simulation suspension system with optimal control show in Table 2.

Table 1. Result of Full Vehicle Model Vibration

Performance		Active suspension (Optimal feedback)	Passive suspension
Body vertical acceleration	$\frac{m}{s^2}$	0.2	5
Body pitch acceleration	$\frac{Rad}{s^2}$	0.15	4
Body pitch acceleration	$\frac{Rad}{s^2}$	0.01	1
Driver's seat vertical acceleration	$\frac{m}{s^2}$	0.2	2
Dynamic load	N	300	2000

5. Conclusion

Mathematical modeling is a base for the simulation of the suspension system. The mathematical equations are extracted based on a full model vehicle. All unfavorable vibrations of the vehicle's body have been optimized through optimal feedback control. It is concluded that utilizing control feedback is an appropriate method for optimizing the vehicle performance. The vehicle performance has been controlled by considering suspension system parameters as fixed values and utilizing optimal feedback control, therefore; an optimal result is gained in roll, pitch and vertical vibrations. By determining the optimal K efficiency of feedback through LQR, the control state feedback changes into the optimal feedback so that the vehicle performance will improve.

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