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Minitab 20 and Python based-the forecasting of demand and optimal inventory of liquid aluminum sulfate supplies

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ABSTRACT

In a company, inventory management is crucial due to the significant impact on various aspects of the business. Similarly, the Indonesian water supply company (PDAM) requires effective inventory management to ensure the supply of liquid aluminum sulfate chemicals. The probabilistic statistical inventory control (SIC) model is commonly used for inventory management. However, previous research on chemical inventory models in PDAMs often relied on simple linear regression to forecast demand data, which fails to capture the inherent volatility in demand. Therefore, this research aimed to predict demand data using the seasonal autoregressive integrated moving average (SARIMA) method and determine the optimal policy for supplying liquid aluminum sulfate chemicals. The results showed that the best demand forecasting model was SARIMA (2,1,2) (1,1,0)12 with a mean absolute percentage error (MAPE) value of 8.19%. The finding of the optimal inventory policy showed a safety stock value of 11,922.35 kg, a reorder point value of 49,511.20 kg, and an order quantity of 21,526.59 kg, leading to a total cost of IDR 11,132,034,145.45. The sensitivity test also showed that variations in lead time, price, μ, and σ parameters directly influence changes in total cost, reorder point, and safety stock. These calculations were conducted using Minitab and Python software.

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1. INTRODUCTION

Inventory management includes predicting the amount of stock to order, determining safety stock levels, and establishing the optimal reorder point. Effective inventory management is significant to all manufacturers including Indonesian water supply company (PDAM) as it directly impacts production efficiency, cost control, and customer satisfaction [1]–[3]. PDAM also requires a steady supply of liquid aluminum sulfate to provide clean water for the community. This chemical is essential for water purification, playing a key role in maintaining the stability and effectiveness of the clean water production process. Consequently, effectively managing the inventory of liquid aluminum sulfate to meet demand without excess stockpiling is crucial.

The appropriate supply level of the chemical is closely related to the demand, which represents the quantity of goods requested or used. Demand is often characterized by uncertainty but with a specific probability distribution. When dealing with uncertain demand, fuzzy inventory management [4] or the probabilistic statistical inventory control (SIC) method can be adopted [5].

Previous research primarily relied on historical or numerical data to formulate inventory models such as the articles conducted by [6], [7]. However, obtaining demand level parameters from forecasted data for several future periods is essential. Recognizing the inherent instability and fluctuations in demand, it becomes crucial to use forecasting methods capable of accommodating volatility.

The problem of demand data volatility has been studied by [8], using the autoregressive integrated moving average (ARIMA) method to predict instability. ARIMA has been used to forecast demand for medicine [9], electricity and power cables [10], public transportation [11], as well as birth control pills [12]. Besides ARIMA, seasonal ARIMA method (SARIMA) has been used in several forecasting research, namely electricity demand [13], electric vehicle power requirements [14], ecotourism demand [15], and model performance research [16].

The obtained demand forecast plays a crucial role in developing an inventory model, a widely observed practice in various research endeavors. For instance, ARIMA has been used to formulate deterministic inventory models in the demand for raw materials [17]. Additionally, research has explored fuzzy POQ inventory modeling, integrating demand forecasting through ARIMA and holt-winter models [18]. Resky *et al.* [19] adopted simple linear regression analysis to forecast demand and incorporated probabilistic inventory in obtaining optimum inventory policy. Other research has applied methods such as the Naïve method, moving average (MA), fuzzy time series, and deterministic min max to the Q back order inventory model [20]–[23]. However, there is a scarcity of research focusing on probabilistic inventory models with demand parameters estimated from forecasting data using the SARIMA method based on Minitab 20 and Python software. Previous research has further failed to compare the methods used with alternative approaches. This research aims to evaluate the efficacy of the forecasting model in comparison with other forecasting methods.

Previous research on chemical inventory modeling in PDAM has been conducted at PDAM Tirta Kencana Samarinda City and PDAM Nganjuk Regency [24], [25]. However, this research uses the deterministic economic order quantity (EOQ) inventory method, relying on historical demand data instead of adopting future forecast information. Dewi [26] further conducted research at PDAM Tirta Mayang Jambi City, using the probabilistic EOQ method and simple linear regression analysis for forecasting demand. However, this method is less appropriate for forecasting because the linear regression model is represented by a straight line with a continuously decreasing or increasing trend, rendering the ability to capture the volatility in the actual data futile. Previous research in PDAM has not used SARIMA and has not examined the effect of parameter changes on the optimum change of the inventory model. Therefore, this research adopts a sensitivity analysis to determine the effect of parameter changes on the optimal solution.

The optimum supply policy for liquid aluminum sulfate in PDAM Tirta Musi Palembang is determined using the hadley within algorithm for the probabilistic fixed order quantity (Q, r) model [27], [28]. In this research, demand for liquid aluminum sulfate is assumed to be a normally distributed probability. Some research has used the mathematics application package for learning and education (MAPLE) software [29], [30]. Furthermore, the probabilistic inventory model formulation is explained mathematically using Python software in this research.

2. METHOD

The methodology in this research was structured into several steps. The data on liquid aluminum sulfate demand was initially acquired and then forecasted using the SARIMA model. Finally, the optimal inventory policy was determined using the SIC method applied to the probabilistic model (Q, r).

2.1. SARIMA method

Surveys and interviews were obtained through secondary data from PDAM Tirta Musi Palembang in November 2022. The data was in the form of a time series comprising demand for liquid aluminum sulfate by the production department over 72 months, from January 2016 to December 2021. The first 60 data sets are training data, and the next 12 data sets are out-sample data or testing data. Training data is used to build a forecasting model, and testing data is used to test the goodness of the model. Additionally, other information about inventory costs was collected. The forecast was conducted using the SARIMA $(p, d, q)\ (P, D, Q)^L$ model, with the following steps:

1. Box-Cox transformation was applied to examine the data for variance stationarity.
2. Testing data for stationarity in mean was carried out using unit root test with augmented dickey-fuller (ADF) method through the following test statistics (1):

$$\tau = \frac{\delta}{Se(\delta)} \tag{1}$$

where $Se(\delta)$ represented the standard error of δ. The proposed hypothesis was H_0 =there was a unit root, signifying that data was non-stationary and H_1 =there was no unit root, signifying data to be stationed. H_0 was rejected when ADF value of the test statistic exceeded the critical ADF value from MacKinnon's Table, showing that the data was stationary [31].

3. Autocorrelation function (ACF) and partial autocorrelation function (PACF) were plotted to estimate the value of parameters ϕ, θ, Φ, and Θ.
4. The significance of model parameters was tested, showing that the hypothesis proposed H_0=not significant model parameters, H_1=significant model parameters. The t-test was applied with $t = \frac{\theta}{SE(\theta)}$ and rejected H_0 when p_value $< \alpha$ (0.05).
5. Residual white noise was calculated using the Ljung-Box test, by (2).

$$Q = n(n+2)\sum_{k-1}^{K}\frac{\rho_k^2}{n-k} \tag{2}$$

Where n represented the total number of data points, k denoted the value of the t^{th} lag, K signified the maximum lag performed, and ρ_k represented the value of the k^{th} lag autocorrelation function. Furthermore, H_0 was rejected when the p_value $> \alpha(0.05)$, signifying that the test did not meet the white noise criteria.

6. The best SARIMA model was selected by evaluating testing data based on mean squared deviation (MSD), mean absolute deviation (MAD), and mean absolute percentage error (MAPE) values. The smallest error value was selected among the results using [32].

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\frac{|X_t-\hat{X}_t|}{X_t}\times 100\% \tag{3}$$

Where n denoted the amount of data points, X_t served as the actual data in the period t, and $\hat{X}_t$ represented the forecasted data in the period t. The criteria for MAPE scores showed four results, (1) MAPE $< 10\%$ signifying excellent forecasting ability, (2) $10\% \leq$ MAPE $< 20\%$ representing good forecasting ability, (3) $20\% \leq$ MAPE $< 50\%$ sufficient forecasting ability, and (4) MAPE $\geq 50\%$ poor forecasting ability.

7. The model obtained in step 6 was subsequently applied to the initial 72 data points to forecast the 2022 and 2023 data. Using the model obtained in step 6 on the 72 initial data to forecast the 2022 and 2023 data.

The calculations were assisted by using Minitab [33] in the menu stat>time series>ARIMA through the statsmodels library in Python software [34]. Essential pieces of The Python syntax in SARIMA modeling were as in Algorithm 1:

Algorithm 1. The Python syntax in SARIMA modeling

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.arima.model import ARIMA
from statsmodels.tsa.stattools import adfuller

# Display ADF Test
result = adfuller(df['Time_Series'])
print('ADF Statistic:', result[0])
print('p-value:', result[1])
print('Critical Values:', result[4])
# ARIMA((2,1,2)(1,1,0)_6)Models
model = ARIMA(df['Data'], order=(2, 1, 2), seasonal_order=(1, 1, 0,6), trend='n')
fit_model = model.fit()
df['Predictions'] = fit_model.predict(start=1, end=60, typ='levels')
# Display the estimated parameters
print(fit_model.summary())
print(df['Data'])
print(df['Predictions'])
# Calculating MAPE
subset_data_1 = df['Data'][2:61]
subset_data_2 = df['Predictions'][2:61]
from sklearn.metrics import mean_squared_error
def calculate_mape(y_true, y_pred):
    return np.mean(np.abs((y_true - y_pred) / y_true)) * 100
mape = calculate_mape(subset_data_1, subset_data_2)
# Obtaining predictions and confidence intervals
```

```
    forecast_steps = 36
    forecast = fit_model.get_forecast(steps=forecast_steps)
    forecast_values = forecast.predicted_mean
confidence_intervals = forecast.conf_int()
```

2.2. Optimizing probabilistic inventory model

After estimating the demand for liquid aluminum sulfate using the SARIMA method, the optimization of probabilistic inventory modeling was conducted. This research assumed that liquid aluminum sulfate demand was a normal probability distribution. The data used ranged from 2016 to 2021, inclusive of forecasted data for 2022 to 2023. A normality test was performed using the Kolmogorov-Smirnov (*KS*) test with hypothesis H_0 asserting that demand data was a normal probability distribution and the alternative $H_1 = d$emand data was not a normal probability distribution through the (4) [35]:

$$D_{count} = sup_x |F_n(x) - F_0(x)| \tag{4}$$

where $F_n(x)$ represented the cumulative probability of a particular distribution and $F_0(x)$ denoted the cumulative probability of empirical data being tested. H_0 was rejected when $D_{statistic} > D_{(\alpha,n)}$ or the p_value $< \alpha(0.05)$ by using the Fitter library in Python [36]. The definitions of variables and parameters of the probabilistic *(Q,r)* model are found in Table 1.

Table 1. Variables and parameters

Variables and Parameters	Variables and Parameters
η	Service level
N	Expected amount of inventory shortage each cycle (unfulfilled demand)
D_L	Demand expectations during the lead-time period
α	Probability of inventory shortage, where $\eta = 1 - \alpha$
r	The amount of inventory at the time the order was placed (reorder point)
X	The random variable of demand for goods during a lead-time period
$f(x)$	Demand opportunity density function at lead time (x)
μ	The mean of demand data
σ	The standard deviation of demand data
D	Demand expectations over the planning horizon (kg/year)
S	The standard deviation of demand over the planning horizon (kg/year)
L	Lead time (year)
q	Order lot size for each order (kg)
p	Price of liquid aluminum sulfate per kg
A_2	Cost of contracting (rupiah per year)
A_1	Message fee (rupiah per message)
h	Holding cost per unit (% unit per year) of the price of liquid aluminum sulfate per unit, proportional to the number of liquid aluminum sulfate and storage time
C_u	Proportional to the number of liquid aluminum sulfate that cannot be fulfilled.
Tc	Total cost
ss	The number of goods in the warehouse (safety stock)

The optimal solution of r, q, ss, and η was obtained by minimizing Tc as (5):

$$Tc = Dp + (A_1 + \frac{A_2 D}{q}) + h\left(\frac{1}{2}q + r - D_L\right) + \frac{C_u D}{q}\int_r^\infty (x-r)f(x)dx \tag{5}$$

where $f(x)$ represented the normal probability density function with the (6).

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \text{ for } -\infty < x < \infty \tag{6}$$

The hadley within algorithm [27], [28], [37], [38], through iteration was used to obtain optimal values of q and r as follows:

1. Using Wilson's formula, the initial value of q_1 was formulated by $q_1 = \sqrt{\frac{2A_1 D}{h}}$
2. Determined α and r through the following exact method optimization principle. Where $\frac{\partial Tc}{\partial r} = 0$ and $\frac{\partial^2 Tc}{\partial r^2} > 0$, to obtain,

$$\alpha = \int_r^\infty f(x)dx = \frac{q_0 h}{C_u D} \quad (7)$$

3. Demand during lead time was a normal probability distribution. Therefore, in (6) and (7) were used to determine the value of r as (8).

$$r = \{[erfinv(1-2\alpha)]\sqrt{2}\sigma\} + \mu \quad (8)$$

4. Subsequently, determine q through the following exact method optimization principle, where $\frac{\partial Tc}{\partial q} = 0$, and $\frac{\partial^2 Tc}{\partial q^2} > 0$. The optimal q value was obtained as (9):

$$q = \sqrt{\frac{2D(A_2 + C_u N)}{h}} \quad (9)$$

where $N = \int_r^\infty (x-r)f(x)dx$ represented the number of inventory shortages. Subsequently, the *r* value was substituted from step 2 to (9).

5. The α and new r were recalculated using the same formula by going back to numbers 2 and 3 until a relatively similar r value was obtained signifying that the iteration was complete. The optimal safety reserve *ss* for the Back Order was $ss = r - D_L$, with optimal r and q. The service level in backorder cases for liquid aluminum sulfate chemicals was $\eta = \frac{D_L - N}{D_L} \times 100\%$.

Essential components of the mathematical Python syntax for implementing the hadley within algorithm with normal probability distribution demand were crucial. Additionally, the algorithm 2 adopted specific mathematical expressions to optimize the inventory policy under the assumption of normal demand distribution as can be seen in Algorithm 2.

Algorithm 2. Specific mathematical the assumption of normal

```
import math
from sympy import *
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import erfinv, erf

MU_L = MU*L
STD_L = STD*L
DL = D*L
q1 = math.sqrt((2*D*A2)/h)
a = (q1*h)/(CU*D)
r = (erfinv(1-2*a)*STD_L*math.sqrt(2))+MU_L
n = 7
for i in range(1,n):
    E = (math.sqrt(2*pi))*STD_L
    N=integrate((x-r)*(1/(E))*exp(-((x-MU_L)**(2)/(2*(STD_L)**2))), (x,r,oo))
    q = math.sqrt((2*D*(A2+(CU*N)))/h)
    a = (q*h)/(CU*D)
    r = (erfinv(1-2*a)*STD_L*math.sqrt(2))+MU_L
DL = D*L
r_optimal = r
q_optimal = q
SS = r_optimal-(D*L)
N_optimal = N
Tc     =     D*P     +     A1+((A2*D)/Q_optimal)+     h*((0.5*q_optimal)+r_optimal-(D*L))+
   ((CU*D)/q_optimal)*N_optimal
```

A summary of the methods used was depicted in the chart in Figure 1, showing the step-by-step process undertaken in the optimization of the inventory policy. This visual representation aided in understanding the sequential method adopted in the research method.

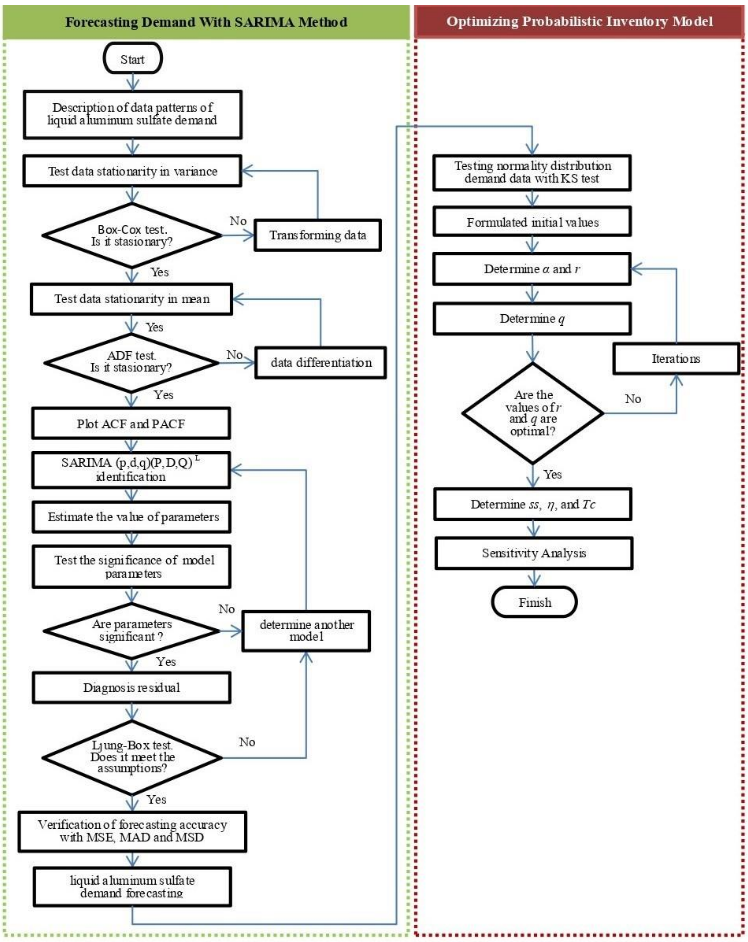


Figure 1. Flowchart diagram of research method

3. RESULTS AND DISCUSSION

3.1. Forecasting the liquid aluminum sulfate demand

Liquid aluminum sulfate data was a time series from January 2016 to December 2021. The plotting of the data is depicted in Figure 2. Volatility was observed in demand data in Figure 2, where demand fluctuates intermittently. As the condition of stationarity in variance was unmet, Box-Cox transformation was conducted. The transformation was executed using Minitab software through the menu Stat > Control Chart > Box-Cox transformation, and the results are depicted in Figure 3.

Figure 3(a) showed Box-Cox plot of liquid aluminum sulfate demand producing a rounded value (λ) of -1.00, signifying that the data was not stationary in variance. Furthermore, Figure 3(b) depicted the results of Box-Cox transformation using the $1/X_t$ form with a value of λ = 1.00. This suggested that the transformation data could be considered stationary in variance.

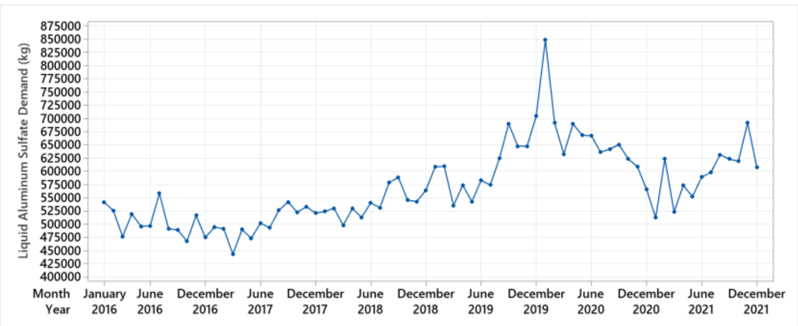
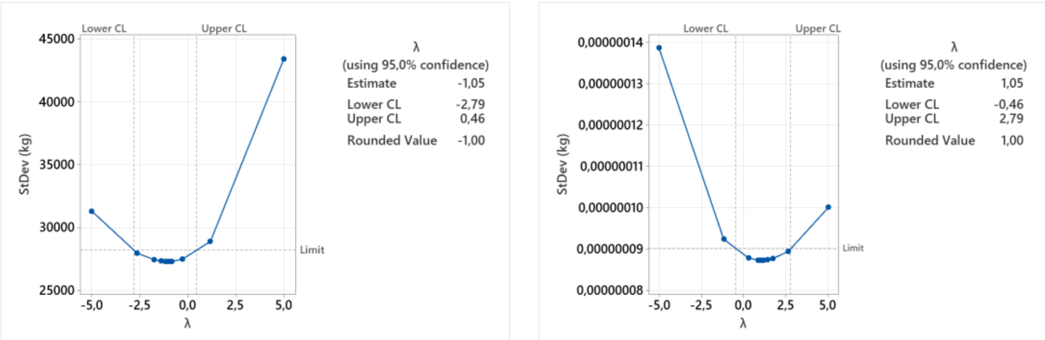


Figure 2. Time series plot of liquid aluminum sulfate

ADF test was conducted on the training data and resulted in an absolute ADF statistic value of 0.2069. This signified that the test was lower than an absolute ADF statistic table value of 2.9253 at a significance level of 5%. Therefore, the decision was made to accept H_0, showing that the data remained non-stationary in the mean. The data was subjected to differentiation and after the second differencing, the absolute ADF statistic reached 4.3029, exceeding the critical absolute ADF statistic table value of 2.9253. The decision was further made to reject H_0, signifying that the second differencing data became stationary in the mean.

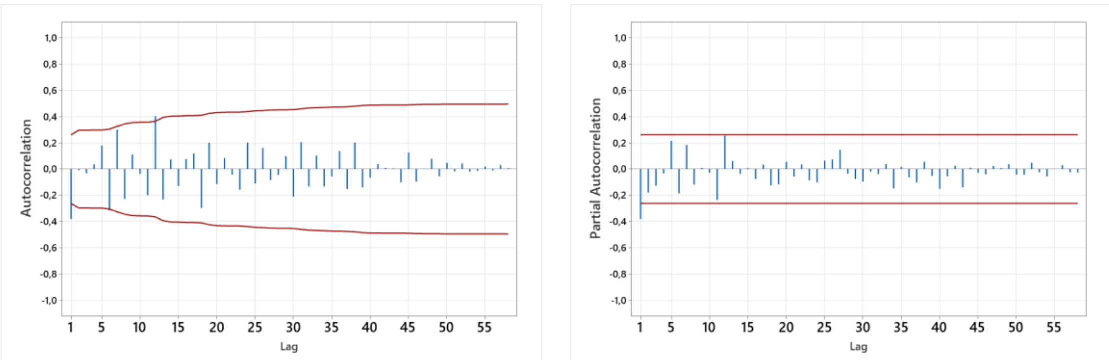


(a) (b)

Figure 3. Box-cox plot on (a) original data and (b) first transformation data

The number of parameters for the model was identified based on the lags observed in ACF and PACF diagrams, as shown in Figure 4. Based on Figures 4(a) and 4(b), ACF and PACF plots exhibited a pattern that quickly approached zero with a cutoff after the first lag. Significantly, lines evolved from the horizontal line as seen in lag 1, 6, and 12. This observation led to the prediction of maximum values for the non-seasonal model (p=2, d=2, q=3) and the seasonal model (P=1, D=2, Q=2, and L=6 or 12). The result obtained was SARIMA $(2, 1, 2)\ (1, 1, 0)^{12}$ using Minitab 20 as the best model with all coefficients $(\phi_1, \phi_2, \Phi_1, \theta_1, \theta_2)$ significant at the 5% level. Additionally, Table 2 showed that the residual white noise test did not reject H_0 because the value of the Q-statistic was 24.89 and the p-value was 0.773, exceeding the significant level α of 0.05, signifying that the test met the white noise criteria, or the model captures temporal patterns.

The goodness of SARIMA models was assessed using the MAPE value from the 2021 testing data, comparing it with the predicted data and calculating the error value. SARIMA $(2,1,2)\ (1,1,0)^{12}$ model in Minitab 20 software achieved a MAPE value of 8.10%, signifying excellent data forecasting. SARIMA $(2,1,2)\ (1,1,0)^{12}$ in Minitab 20 software exhibited the best performance compared to other forecasting models such as multiplicative holt-winter and additive holt-winter [18], MA [21], simple linear regression [19]. Similarly, when compared to SARIMA $(2,1,2)\ (1,1,0)^{6}$ and SARIMA $(2,1,2)\ (0,1,1)^{6}$ using Python software, SARIMA $(2,1,2)\ (1,1,0)^{12}$ in Minitab 20 software was superior.



(a) (b)

Figure 4. The number of parameters in ACF and PACF diagrams (a) ACF and (b) PACF plot data transformation with the first difference

Table 2. Significance test of the coefficients and Ljung-Box of SARIMA model

Model	Parameters	Coefficient	t-value	p-value	Explanation
$(2,1,2)(1,1,0)^{12}$	AR 1 (ϕ_1)	0.646	4.86	0.000	Significant
	AR 2 (ϕ_2)	-0.692	-5.12	0.000	Significant
	MA 1 (θ_1)	1.121	9.05	0.000	Significant
	MA 2 (θ_2)	-0.9493	-10.27	0.000	Significant
	SAR 1 (Φ_1)	-0.484	-2.14	0.038	Significant
Ljung-Box (Q)			24.89	0.773	The model captures temporal patterns

The comparison of the forecasting model goodness was based on the values of MAPE, MSD, and MAD, as shown in Table 3 and Figure 5. Table 3 showed evidence that the multiplicative holt-winter model and additive holt-winter model produced nearly identical MAPE, MAD, and MSD values when the exponential (α), trend (β), and seasonality (γ) smoothing constants were set to $\alpha = 0.3, \beta = 0.2, \gamma = 0.2$. MAPE value of the MA (4) model with 4 periods produced the smallest MAPE, MAD, and MSD values. However, the forecast results did not show volatility because the outcomes were constant, rendering it unsuitable for forecasting demand for liquid aluminum sulfate. Linear regression further produced a linear increasing forecast value, as shown in Figure 5. The forecasting result for the next 24 months with SARIMA $(2,1,2)(1,1,0)^{12}$ model in Minitab 20 from January 2022 to December 2023, based on baseline data from 2016 to 2021 was depicted in Figure 6. The graph showed that demand for liquid aluminum sulfate over the next 2 years would fluctuate monthly, following the pattern observed in historical data.

Table 3. Comparison of the results of liquid aluminum sulfate demand forecasting models

	SARIMA $(2,1,2)(1,1,0)^6$ (in Python)	SARIMA $(2,1,2)(0,1,1)^6$ (in Python	SARIMA $(2,1,2)(1,1,0)^{12}$ (in Minitab)	Multiplicative Holt-Winter (0.3, 0.2, 0.2) (in Minitab)	Aditif Holt-Winter (0.3, 0.2, 0.2) (in Minitab)	MA (4) (in Minitab)	Linear regression (in Minitab)
MAPE	16.68	11.34	8.10	13.15	13.25	7.16	16.42
MAD	102116.14	69094.28	48211.53	80518.94	81113.22	40284.21	93906.03
MSD	14480131482.72	6909087412.01	4432581478.80	9180484092.67	9260742231.87	2985657247.01	10443353091.63

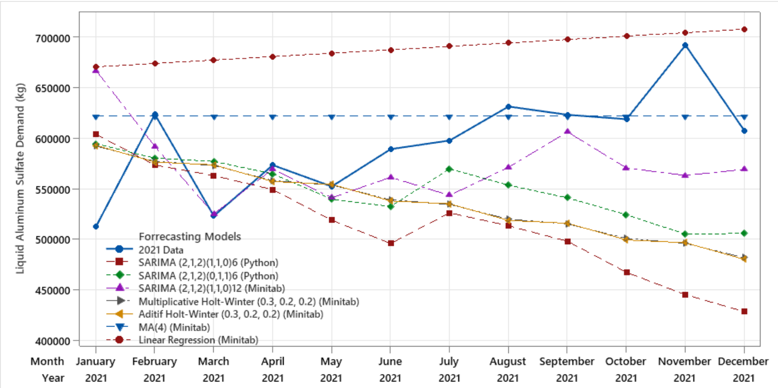


Figure 5. Comparison of forecasting results with several methods using testing data

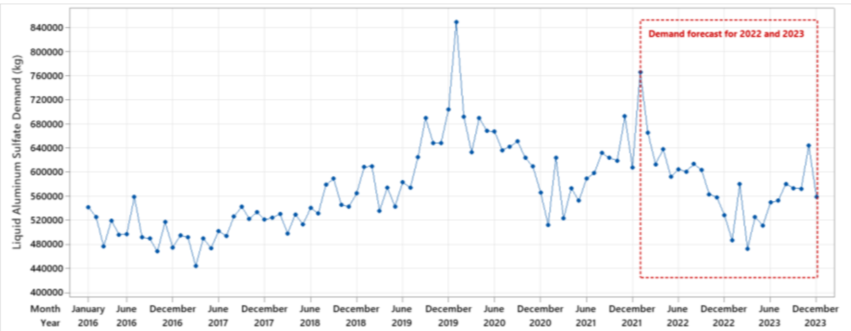


Figure 6. Forecast graph for January 2022 to December 2023

3.2. Optimizing liquid aluminum sulfate supplies

After obtaining forecast data on demand for liquid aluminum sulfate, optimizing liquid aluminum sulfate supplies was carried out through the formulation of a probabilistic inventory model under the assumption of normally distributed demand. The normality assumption for demand data was tested using KS in (4). This resulted in a D-statistic of 0.169 and $D_{(0.05,8)}$ of 0.454, with a p-value of 0.948 exceeding the alpha critical value of 0.05. Therefore, the decision was made to accept H_0 or normal distribution demand data. The average parameter of μ was 6,867,264.18 kg with a standard deviation of σ=633,810.14 kg.

Secondary data on liquid aluminum sulfate obtained from PDAM Tirta Musi Palembang provided additional information, including the average storage cost of IDR 161.5/kg/year, order contract cost of IDR 36,000,000/year, phone ordering costs of IDR 5,000/order, the average purchase cost of IDR 1,615/kg/year, and the average shortage cost of IDR 1,695.75/kg/year. The average lead time was further obtained as 2 days or 0.005479 years. The limitation of the research was not giving attention to the capacity of the warehouse. The optimal result was obtained in the seventh iteration, with parameters of r=49,511.20 kg, q=21,526.59 kg, N=0.26 kg, ss=11,922.35 kg, Tc=IDR 11,132,034,145.45, and η=99.99%.

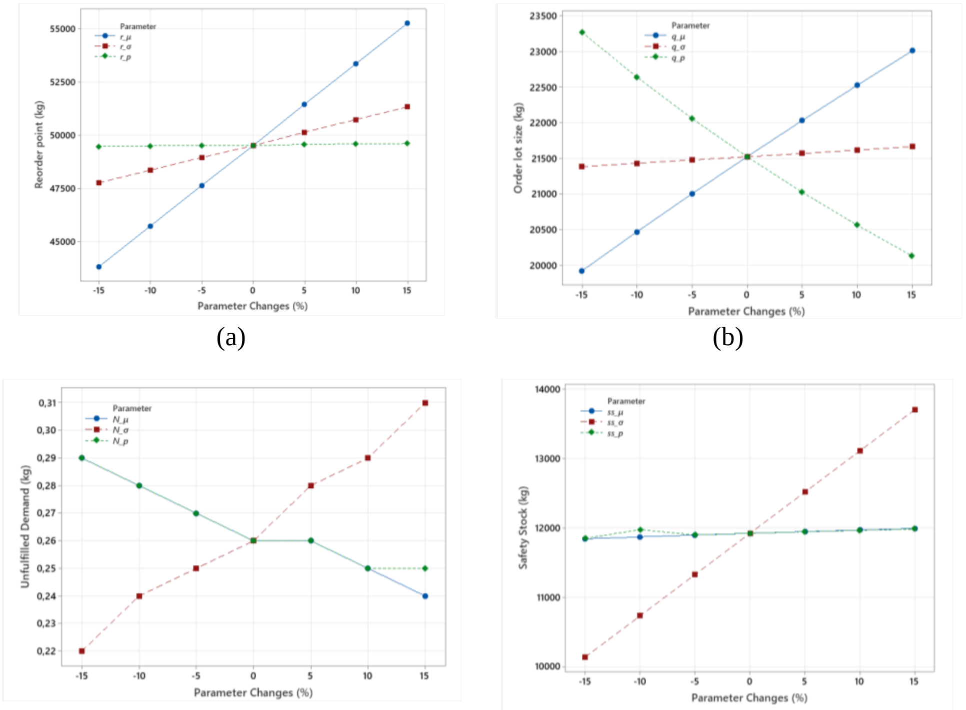
3.3. Sensitivity test

A sensitivity test was carried out to validate the optimality of the obtained inventory policy by changing the parameters referred to as scenarios. This analysis also showed the influence of parameter changes on optimal solution when the other values were fixed, with results detailed in Table 4. The most prominent was the effect of changes in the μ, σ, and P parameters on r, N, ss, and Tc. This was signified using Minitab software through the menu graph>scatter plot>connect and groups, with details in Figure 7.

As detailed in Table 4 and Figures 7(a) to 7(d), changes in μ had a positive impact on r, q, ss, and Tc while negatively affecting N. Variations in μ showed alterations in the average demand for liquid aluminum sulfate. Similarly, changes in σ positively influenced the optimal solution, showing variations in the standard deviation of liquid aluminum sulfate demand and annual change. Alterations in p positively impacted r, ss, and Tc but had negative effects on q and N. An increase in the price of liquid aluminum sulfate resulted in a reduction in the order lot size, accompanied by an increase in the reorder point and safety stock to mitigate inventory shortages. Consequently, total costs increased due to the larger inventory stock in the warehouse leading to increased holding costs. Similar to the change in σ, variations in L significantly affected the optimal solution. An increase in lead time and σ parameters raised the possibility of inventory shortages. Consequently, larger order lot sizes and reorder points anticipated longer lead times and fostered safety stock.

Table 4. Effect of parameter changes on the total cost

Parameter	Changes	r (kg)	q (kg)	N (kg)	ss (kg)	Tc (IDR)
μ	+15%	55,264.06	23,013.70	0.24	11,990.89	12,795,880,130.73
	+10%	53,360.88	22,529.34	0.25	11,969.15	12,241,266,813.51
	+5%	51,456.63	22,033.91	0.26	11,946.35	11,686,651,533.86
	+0%	49,511.20	21,526.59	0.26	11,922.35	11,132,034,145.45
	-5%	47,644.47	21,006.51	0.27	11,897.06	10,577,414,482.89
	-10%	45,736.28	20,472.63	0.28	11,870.32	10,022,792,358.17
	-15%	43,826.46	19,923.79	0.29	11,841.95	9,468,167,555.99
σ	+15%	51,332.56	21,666.18	0.31	13,703.71	11,132,344,449.58
	+10%	50,738.98	21,619.54	0.29	13,110.13	11,132,241,030.30
	+5%	50,145.19	21,573.02	0.28	12,516.35	11,132,137,595.59
	+0%	49,511.20	21,526.59	0.26	11,922.35	11,132,034,145.45
	-5%	48,957.01	21,480.28	0.25	11,328.17	11,131,930,679.88
	-10%	48,362.62	21,434.08	0.24	10,733.77	11,131,827,198.90
	-15%	47,768.02	21,387.99	0.22	10,139.17	11,131,723,702.51
p	+15%	49,614.11	20,132.33	0.25	11,985.27	12,796,192,003.89
	+10%	49,594.16	20,565.27	0.25	11,965.31	12,241,474,411.78
	+5%	49,573.22	21,028.76	0.26	11,944.37	11,686,755,166.64
	+0%	49,511.20	21,526.59	0.26	11,922.35	11,132,034,145.45
	-5%	49,527.99	22,063.23	0.27	11,899.15	10,577,311,209.42
	-10%	49,503.47	22,643.97	0.28	11,974.62	10,022,586,201.02
	-15%	49,477.46	23,275.22	0.29	11,848.62	9,467,858,940.24
L	+5 days	173,072.06	23,980.79	1.04	42,371.10	11,137,187,674.73
	+4 days	148,408.94	23,468.13	0.87	35,522.40	11,136,160,076.51
	+3 days	123,725.26	22,966.39	0.71	29,653.14	11,135,130,921.26
	+2 days	99,021.04	22,475.58	0.57	23,763.35	11,134,100,211.98
	+1 days	74,296.34	21,995.66	0.41	17,853.07	11,133,067,952.07
	Two days	49,511.20	21,526.59	0.26	11,922.35	11,132,034,145.45
	-1 days	24,785.72	21,068.33	0.13	5,971.30	11,130,998,796.61



(a) (b)

(c) (d)

Figure 7. Sensitivity analysis to; (a) reorder point, (b) order lot size, (c) unfulfilled demand, and (d) safety stock

This research comprehensively and systematically examined liquid aluminum sulfate demand forecast using the SARIMA method in Minitab and Python software. Subsequently, it used the forecast results to formulate a probabilistic inventory model to optimize liquid aluminum sulfate supplies. Further research on time-dynamic probabilistic inventory models was required to compare and determine which model represented liquid aluminum sulfate inventory management at PDAM Tirta Musi.

4. CONCLUSION

In conclusion, the forecasting results obtained using Minitab 20 and Python software showed that the SARIMA (2,1,2) $(1,1,0)^{12}$ model provided the best performance with a MAPE value of 8.19% when Minitab software was used. This model also outperformed the holt-winter, MA, or simple linear regression models. Demand data for liquid aluminum sulfate satisfied the assumption of data normality, allowing the use of the SIC probability model with a normal distribution. The optimal inventory policy for liquid aluminum sulfate was determined using this method and the hadley within algorithm mathematically, assisted by Python software. This suggested maintaining a safety stock of 11,922.35 kg, with a reorder of 21,526.59 kg when the inventory level of liquid aluminum sulfate in the warehouse reached 49,511.20 kg. With the optimal policy, the estimated total cost was approximately IDR 11,132,034,145.45. The sensitivity test showed that changes in the parameters μ, σ, p, and L affected variations in the optimal solution and total costs. For further research, this article was developed in terms of forecasting methods using the Robust ARIMA and wavelet ARIMA to enhance the accuracy of demand data forecasting. Additionally, the inventory model was developed into multiple items for further research to focus on a specific framework.

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