Inventory model having preservation technology with fix lifetime under two level trade credit policy

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ABSTRACT

Supply-chain management involves moving storage supplies from origin to consumption, with manufacturers running production based on quadratic demand, distributors and retailers monitoring inventory. When a new product is released, demand often rises linearly and then declines dramatically when an alternative becomes available. Shortages are not allowed. Players' inventory will decrease at a rate of (1/(1+m-t)), where m is fixed lifetime, greater than the replenishment time. Deteriorating goods experience constant mass loss or usefulness, but preservation technology can help the damaged item to be consumed. Retailers with direct customer relationships can reduce stock spoilage through good warehouses. Manufacturers' storage systems have a higher deterioration rate. Two-tier trade credit financing is examined in this model. Distributors offer specific credit terms to stores, while manufacturers provide a grace period for invoicing. Distributors and retailers must pay interest on unsold inventories if invoices aren't settled on time. An integrated storage system reduces costs by minimizing costs through multiple shipments from manufacturers to distributors and retailers, and by adjusting replenishment times for each player. The resolution process is designed so that the supply chain operator gets the best possible decision. Therefore, results are authorized using mathematical examples for different scenarios. Management decisions are suggested.

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1. INTRODUCTION

Preservation of a product is an important issue in the inventory control system. It prevents the deterioration effect of the products while these are stored in the warehouse/showroom. Considering deterioration effect of the product and preservation technology, an inventory model of non-instantaneous deteriorating items is developed with the demand dependent on the selling price of the product. There is a fine line between replenishment lead times, carrying costs, asset management, inventory forecasting, inventory valuation, visibility, future inventory price forecasting, physical inventory, space availability, quality management, replenishment, returns, defective goods, and demand forecasting that play a crucial role in inventory management. As the business needs shift and reaction to the wider environment, the balance between these competing requirements leads to optimal inventory levels. In the business world, permissible payment delays are popular and important. By using this strategy, the supplier attracts a larger number of customers, and therefore increases sales. Due to this approach, businesses are more competitive and challenging in present scenario.

The suppliers of a two-level trade credit policy offer retailers a delay in payment (credit time) under certain terms and conditions. It is then the retailers' responsibility to accept this and offer the customer a credit period (delay in payment). In order to attract more customers and sell more products, all of this is done. Buyers are given the option of paying for the purchase amount up to a certain period through the delay in payment facility (credit time). A vendor will not charge his/her buyer interest if the purchase is completed within the credit period. Therefore, if the credit period has expired, the vendor charges interest to his/her buyer. To create an inventory model with the longest possible lifetime for declining items, this article took into consideration the demand rate as a quadratic function of time. The paper presents the formulation of the supply chain inventory model, whereas a producer runs the production proportional to the quadratic demand, while distributors and retailers evaluate quadratic demand. There is a two-level trade credit financing structure considered. Retailers receive a certain amount of credit from distributors, who in turn receive a delay time from manufacturers to pay accounts. If the account is not settled within the allotted time frame, both the distributor and the retailer will be responsible for paying interest charges for the unsold product. The goal of this paper is to provide a conceptual technique for related issues while providing an outline of inventory management and the current inventory-related issues in a modern firm. The inventory serves as both a source of information for the approach and an output, or benefit, for the technique.

2. LITERATURE REVIEW

The concept of inventory control was introduced by Haris-Wilson in the year 1913 under the assumptions (i) demand of the production is constant, (ii) shortages are not permitted, and (iii) the product has no deterioration effect. However, every product loses their freshness and utility after certain time. After that this effect is continued with the passing of time. This natural phenomenon is called deterioration. So, it has an impact in inventory analysis and it cannot be ignored. A review of the literature is done on inventory management, control, and industry-related challenges, as well as the many aspects of inventory. A theoretical approach to current company inventory problems. Various supply chain management (SCM) techniques have been the subject of numerous investigations. According to Becker-Peth [1], if a corporation holds on to SCM before fully understanding what is required to make it work, they run the danger of placing more money at risk if their understanding is incorrect.

Since Costco is the biggest retailer, it provides the proper training to its employees, which is essential for business success. Costco consistently works with their supplier to expedite material delivery, so they may showcase and market the product as soon as possible. In commercial dealings, the proposal to pay supplier income for purchases made without adding interest, then earn interest by depositing the money into a bank or other financial institution is appealing to the shopkeeper. The retailer may sell the goods, earn money, and accrue interest by depositing it with a bank or financial institution within this acceptable wait time. Wholesalers and retailers were supported by the cooperation of three supply chain participants, including the manufacturer [2]. In the context of perishability and partial backordering, Abad [3] approach was more useful and feasible. An economic order quantity (EOQ) model for deteriorating items with time-varying demand and partial backlog was examined by [4]. The topic of inventory rules for a single manufacturer and several buyers was covered by [5]. An approach for calculating joint economic lot-size in a distribution system with several shipments was presented by [6]. In any system, the impact of deterioration is crucial. Fruits and vegetables deteriorate over time and must be managed to prevent further damage. In 1963, Ghare and Schrader looked at an exponentially declining inventory model. The ideal cycle time for deteriorating commodities under trade credit policy was established by [7]. An EOQ inventory model for Weibull-distributed degrading commodities under ramp-type demand and shortages was provided by [8].

An inventory model for deteriorating items with time-dependent demand and time-varying holding costs under partial backlog was created [9]. An order level inventory model for degrading items with quadratic time-varying demand, shortages, and partial backlog was covered [10]. Optimal policies for deteriorating items with maximum lifetime and two-level trade credits were examined [11]. Sarkar *et al.* [12] examined the best store strategy for products with set lifetimes. Shah *et al.* [13] discussed the optimal policies for time-varying deteriorating item with preservation technology under selling price and trade credit dependent quadratic demand in a supply chain. Sarkar [14] studied the supply chain coordination with variable backorder, inspections, and discount policy for fixed lifetime products. Zhang *et al.* [15] studied buyer–vendor coordination for fixed lifetime product with quantity discount under finite production rate. In classical inventory models, the demand rate and holding cost are assumed constant. The demand and holding cost for physical goods may be time dependent. One of the assumptions in the traditional inventory model was that the items preserved their physical characteristics while they were kept stored in the inventory. This assumption is evidently true for most items, but not for all. However, the deteriorating items are subject to a continuous loss in their masses or utility throughout their lifetime due to decay, damage, spoilage, and penalty of other reasons. Owing to this fact, controlling, and maintaining the inventory of deteriorating items becomes a challenging problem for decision makers. Automation is one of the best policies of this smart manufacturing system through which all things of a production system done by machine not by human being. As a result, the chance of error in the production process is reduced. When a production process gone through a long run process it may shifted to 'out-of-control' state from 'in-control' state and may produce defective items. To detect those imperfect products an inspection is needed. By using the machine, the chance for detecting the faulty and imperfect items is reduced. Sarkar and Chung [16] developed a production model with the flexible work-in-process in supply chain management with the improvement in quality.

Nobil *et al.* [17] developed a multiproduct single machine economic production model with discrete delivery order and also with joint production policy and budget constraints. Ullah and Sarkar [18] state a recovery-channel selection process in a hybrid manufacturing-remanufacturing production model. Vandana *et al.* [19] studied a two-echelon supply chain, which included a producer and a supplier with optimal trade-credit policies. They used the idea of volume agility to make the manufacturing process flexible. Jabbarzadeh *et al.* [20] developed a model in which credit period offered by supplier is less than or equal to the length of time in which no shortage happens under fuzzy environment. Mohanta *et al.* [21] developed a neutrosophic EOQ model under the concept that consumer demand is sensitive to retail price and marketing efforts. The retailer and the supplier.

Manna and Bhunia [22] developed a model with green sensitive demand and green production. The related differential equations of inventory levels in the proposed model are shown in interval form since interval-valued demand and faulty rates were taken into account. Giri and Dash [23] studied a single retailer single manufacturer model for imperfect production with green sensitive and advertisement dependent demand. After receiving each shipment from the manufacturer, the store conducts a faultless screening procedure to identify the defective goods. Ummeferva *et al.* [24] develop a two-level trade credit policy approach for a production inventory model under greening degree dependent demand and reliability. Garai *et al.* [25] derived the possibility, necessity, and credibility measures to determine the chances of occurrence of fuzzy events while modeling inventory for multiple items. Nagpal *et al.* [26] fuzzified the demand rate and the deterioration rate as trapezoidal membership function and used the Centroid method and signed distance method to de-fuzzify the cost function. Garai *et al.* [27] used trapezoidal fuzzy numbers to define the time varying inventory holding cost and the price-dependent demand and developed a fully fuzzy inventory model, treating all the input parameters and decision variables as imprecise. Ohmori and Yoshimoto [28] handled the demand uncertainty in inventory control problems using used robust optimization. De and Mahata [29] worked on multiple items with imperfect quality that can be sold as a single batch with a proportionate discount rate. One of the most recent works on substitutable items is that of [30] who formulated the fuzzy inventory model for multiple substitutable items being sold at multiple outlets managed by a single entity. Adak and Mahapatra [31] developed an inventory model for items whose demand and deterioration are dependent upon reliability as well as time.

In this work, considering preservation technology with the fix-life time under the two-level trade policy, an inventory model is formulated with quadratic demand. In model formulation, A manufacturer runs production according to the quadratic demand, while distributors and retailers monitor quadratic demand in the supply chain inventory model. When a new product is released, demand often rises linearly and then declines dramatically when an alternative becomes available. Shortages are not allowed. Each player's inventory is going to decrease at a rate of $\left(\frac{1}{1+m-t}\right)$, where m is the fixed lifetime. The value of m is greater than the replenishment time. Throughout their lifespan, the deteriorating goods are liable to a constant loss in mass or usefulness. However, after the application of preservation technology, these damaged items can still be consumed for some time. A retailer dealing directly with the customer implements a good warehouse that reduces spoilage of products in stock.

On the other hand, the rate of deterioration is higher in the manufacturer's storage system. Two-tier trade credit financing is examined in the model. The distributor provides a specific credit term to the store, while the manufacturer provides a grace period for invoicing. Unless the invoice is settled within a certain period, distributors and retailers are responsible for making interest payments on unsold inventory. The costs of an integrated storage system are minimized because of multiple shipments from the manufacturer to the distributor and from the distributor to the retailer, as well as the replenishment times of each player. The resolution process is designed so that the supply chain operator gets the best possible decision. Therefore, results are authorized using mathematical example for different example scenarios. Management decisions are suggested.

3. NOTATION AND ASSUMPTIONS

In order to proceed with the process of developing this mathematical model, it is essential to make use of the assumptions and notations that have been developed. Table 1 contains all of the necessary details. Table 1 contains a listing of the notations that are utilized in the model that is being proposed.

Table 1. Notation table

Notation	Explanation
D(t)	a(1+bt-ct^2)>0, Yearly demand rate, a>0 denotes the scale demand, 0<b, c<1
P(t)	λR(t), λ>1, Rate of production.
u_0	Per unit time invested in preservation technologies (in USD)
f(u_0)	$1-\frac{1}{1+\delta u_0}$ reduced deterioration item' proportion (in year), δ>0
Manufacturer parameters	
A	Set-up cost per set-up
h_{m_0}	Annual holding cost per unit
CT_{m_0}	Cost of transportation from the manufacturer to the distributor of a shipment.
$I_{m_0'}$(t)	During production, inventory levels are determined at any given time t; [0,T'].
$I_{m_0''}$(t)	During non-production, inventory levels are determined at any given time t; [0,T'']
T	($T' + T''$); time for replenishing (Decision variable).
Pm_0	Per unit selling price of the item.
m_0	Fixed lifetime
$I_o m_0$	Due to offering trade credit, the manufacturer's opportunity interest loss.
TCm_0	The manufacturer's total cost per unit time.
Distributor parameters	
B_d	Ordering cost per order.
hd	The annual holding cost per unit excluding interest.
CT_{d1}	Transportation costs while receiving a shipment from a manufacturer.
CT_{d2}	Costs associated with delivering a shipment to the retailer.
N	Manufacturer's credit period (in years) offered to distributors
n	A decision variable (≥ 1); No. of shipments to the distributor from the manufacturer.
T_2	$\frac{T}{n}$ Distributor's replenishment time (Decision variable).
I_d(t)	The inventory level at any given time t during the period; [0, T_2].
TC_d	Distributor's total cost per unit time
I_{od}	Offering trade credit results in a loss of opportunity interest for the distributor.
Q_d	Distributors ordered quantity from manufacturers for each shipment.
P_d	Selling price of an item per unit.
Retailer parameters	
G_r	Ordering cost per order.
M	Distributor credit period (in years) offered to the retailer.
I_e	Per unit, interest is earned (In USD)
I_c	Per unit per year interest charged on unsold stock. (In USD)
m	m≥1 is the No. of shipments from the distributor to the retailer (Decision variable).
I_r(t)	The inventory level at any given time t during the period [0, T_2].
Q_r	Retailers order sizes from distributors in each shipment.
T_2	$\frac{T}{mn}$; Retailer's replenishment time (Decision variable)
P_r	An item's selling price per unit.
TC_r	Retailer's total cost per unit time
TC	TC_{m_0}+TC_d+TC_r; A supply chain's combined total cost per unit time.
Relations between parameters	
$P_r \geq P_d \geq Pm_0$	
$I_c \geq I_e$	
$N \geq M$	
Objective	
Minimize	
TC=CT_{m_0}+TC_d+TC_r	
subject to constraints	
$T > 0\,, n \geq 1\,, m \geq 1$	

3.1. Assumptions

The assumptions: i) There is just one producer, one distributor, and one retailer in the supply chain under examination. It only addresses one thing; ii) In order to establish a long-term collaboration, the planning horizon must be infinite; iii) There is a quadratic demand rate. A proportional relationship exists between production and demand; iv) Not allow of Shortages. Lead-time is tending to zero; v) This policy takes into account the two levels of trade credit: the distributor gives the retailer a certain amount of credit time, and the manufacturer gives the distributor a delay period to clear the accounts. If the account is not cleared within the allotted time frame, both the distributor and the retailer will be responsible for paying interest charges for the unsold products; and vi) A deterioration process occurs in the inventory units of the supply chain. There is a time-dependent relationship between the deterioration rate as $\theta(t) = \frac{1}{1+m-t}$, where m is the maximum product lifetime and m>t.

4. MATHEMATICAL MODEL

In every replenishment cycle, the distributor receives ships n-orders from the manufacturer. The size of every order is Q_d- unit, therefore, during the production time interval $[0, T']$ the manufacturer produces nQ_d −units at a time-dependent production rate. At T', manufacturing stops and the manufacturer inventory deplete to zero during $[0, T'']$. Therefore, the cycle time for the manufacturer is $T=(T' + T'')$.Q_d − units are shipped to retailers in p-shipments of size Q_r − units by the distributor.

4.1. Manufacturer modelling

The manufacturing cycle begins at $t=0$ and lasts until $t=T'$, when the maximum inventory level is reached. Due to demand and the depreciation of inventory goods, manufacturing ceases at $t=T'$ and inventory eventually reaches 0 at $t=T''$ the differential equations describe. Stock levels change both in production and nonproduction processes at the rates (1) and (2).

$$\frac{dI_{m_0'}(t)}{dt} = P(t) - D(t) - \left(\frac{1}{1+m-t}\right) \cdot I_{m_0'}(t), 0 \le t \le T' \quad (1)$$

$$\frac{dI_{m_0''}(t)}{dt} = -D(t) - \left(\frac{1}{1+m-t}\right) \cdot I_{m_0''}(\mathrm{t}), 0 \le t \le T'' \quad (2)$$

with boundary conditions $I_{m_0'}(0) = 0$ and $I_{m_0''}(T'') = 0$. The differential equation mentioned in (1) and (2) has the solutions as (3) and (4).

$$\mathrm{I}_{\mathrm{m}_0'}(\mathrm{t}) = \mathrm{a} \cdot (\lambda - 1) \cdot (1 + \mathrm{m} - \mathrm{t}) \left\{\left(1 + \mathrm{b} \cdot (1 + \mathrm{m}) + \mathrm{c} \cdot (1 + \mathrm{m})^2 \cdot \ln\left(\frac{1+m}{1+m-t}\right) - (b + c) \cdot t - c \cdot t \cdot \left(m + \frac{t}{2}\right)\right)\right\}, 0 \le t \le T' \quad (3)$$

$$I_{m_0''}(t) = a \cdot (1 + m - t) \left\{\begin{matrix} \ln\left(\frac{1+m-t}{1+m-T''}\right)\begin{pmatrix} 1 + b \cdot (1 + m) \\ -c(1 + m)^2 \end{pmatrix} \\ +b \cdot (t - T'') \\ +\frac{1}{2} \cdot c(t - T'') \cdot (2 + 2m + t + T'') \end{matrix}\right\}, 0 \le t \le T'' \quad (4)$$

The manufacturer's replenishment time of cycle is $\mathrm{T}=(\mathrm{T}' + \mathrm{T}'')$. The manufacturer's cost components per unit time as: i) Set-up cost; $OC_m = \frac{A}{T}$; ii) Cost of transportation; $TCT_m = \frac{(n \cdot C_m)}{T}$; iii) Holding cost and deterioration cost: $HC_m = \frac{h_m + \left(\frac{1}{1+m-t}\right) \cdot C_m}{T}\left[\int_0^{T'} I_{m_0'}(t)dt + \int_0^{T''} I_{m_0''}(t)dt\right]$; and iv) Loss of opportunity interest throughout the allowable credit term N in n-shipments: $OL_m = \frac{n \cdot P_m \cdot I_{om}}{T}\int_0^N D(t)dt$. Hunce, the manufacturer's yearly total cost per unit time is: $TC_m = OC_m + TCT_m + HC_m + OL_m$.

4.2. Distributor modelling

Due to deterioration and quadratic demand, the distributor's inventory level runs out because it has a finite lifespan. The (5) can be used to explain this variation:

$$\frac{dI_d(t)}{dt} = -D(t) - \left(\frac{1}{1+m-t}\right) \cdot I_d(t), 0 \le t \le T_2 = \frac{T}{n} \quad (5)$$

with condition $I_d(T_2) = 0$ the solution of the (5) is (6).

$$I_d(t) = a \cdot (1 + m - t)\{\ln\left(\frac{1+m-t}{1+m-T_2}\right) \left(1 + b \cdot (1 + m) - c \cdot (1 + m)^2\right) + b \cdot (t - T_2) + \frac{1}{2} \cdot c \cdot (t - T_2) \cdot (2 + 2m + t + T_2)\} \quad 0 \le t \le T_2 \quad (6)$$

Each shipment consists of a quantity of orders $Q_d = I_d(0) =$

$$a \cdot (1+m)\begin{bmatrix}\ln\left(\frac{1+m}{1+m-T_2}\right)\cdot\{1+b(1+m)-c(1+m)^2\}\\ +b\cdot(-T_2)+\frac{1}{2}\cdot c\cdot(-T_2)\cdot(2+2m+T_2)\end{bmatrix} \quad (7)$$

in terms of costs per unit time, the distributor's n-shipments have the components:

i. Ordering cost: $OC_d = \frac{B_d}{T_2} = \frac{nB_d}{T}$

ii. Holding cost and deterioration cost: $HC_d = \frac{n\cdot\left(h_d+\left(\frac{1}{1+m-t}\right)\cdot P_m\right)}{T}\int_0^{T_2} I_d(t)dt$

iii. Two shipment costs are incurred by the distributor, namely a manufacturer's shipment will be received by one department, while the retailer's order will be delivered to the second.
- The cost of receiving an order via shipping: $CT_{d1=\frac{nC_r}{T}}$, and
- Cost of shipment for delivering the order; $CT_{d2=\frac{npC_d}{T}}$

iv. During the permitted credit period, chance interest loss is allowed Min number of pn –shipments: $OL_d = \frac{pnP_dI_{od}}{T}\int_0^M R(t)dt.$

Based on the lengths of T' and N, we will discuss the two possible cases that are calculate payable and earned interest.

- Case1: When $N \le T_1$
  i. The earned interest per unit time for pn- shipments is given as: $IEd_1 = \frac{nP_dI_e}{T}\int_0^T D(t)dt.$
  ii. The charged interest per unit time for pn- shipments is given as: $ICd_1 = \frac{nP_{m0}I_c}{T}\int_N^{T_2} I_d(t)dt.$
- Case 2: When $N \ge T_1$
  i. The earned interest per unit time for pn- shipments is given as: $IEd_2 = \frac{nP_dI_e}{T}\left[\int_0^{T_1} D(t)dt + (N-T_1)Q_d\right].$
  ii. The charged interest per unit time for n- shipments is given as: $ICd_2 = 0.$

Hunce, the distributor's yearly total cost per unit time is $TCd_1 = \begin{cases}TCd_1, N \le T_1\\ TCd_2, N \ge T_1\end{cases}$. Where $TCd_1 = OC_d + HC_d + CT_{d1} + CT_{d2} + OL_d + IC_{d1} - IE_{d1}$. And $TCd_2 = C_d + HC_d + CT_{d1} + CT_{d2} + OL_d + IC_{d2} - IE_{d2}$.

4.3. Retailer modelling

Retailer inventory decreases due to quadratic demand, deterioration, and fixed life. In terms of differential equation, this variation can be described as (8).

$$\frac{dI_r(t)}{dt} = -D(t) - \left(\frac{1}{1+m-t}\right)\cdot\left(1-f(u_0)\right)I_r(t), 0 \le t \le T_2 = \frac{T}{pn} \quad (8)$$

With boundary conditions: $I_r(T_2)$= 0. The differential equation's solution is (9).

$$I_r(t) = a\cdot(1+m-t)\begin{Bmatrix}\ln\left(\frac{1+m-t}{1+m-T_2}\right)(1+b\cdot(1+m)-c\cdot(1+m)^2)\\ +b\cdot(t-T_2)+\frac{1}{2}\cdot c\cdot(t-T_1)\cdot(2+2m+t+T_2)\end{Bmatrix} 0 \le t \le T_2 \quad (9)$$

Each shipment contains an order quantity of $Q_r = I_r(0) =$ as (10).

$$a \cdot (1+m)\begin{bmatrix}\ln\left(\frac{1+m}{1+m-T_2}\right)\cdot\{1+b(1+m)-c(1+m)^2\}\\ +b\cdot(-T_2)+\frac{1}{2}\cdot c\cdot(-T_2)\cdot(2+2m+T_2)\end{bmatrix} \quad (10)$$

Distributor’s cost components for *pn*-shipments per unit time as:

i. Ordering cost: $OC_r = \frac{G_r}{T_1} = \frac{p\cdot n\cdot G_r}{T}$.

ii. Costs associated with holding and deterioration: $HC_r = \frac{n\cdot\left(h_r+\left(\frac{1}{1+m-t}\right)\cdot P_d\right)}{T}\int_0^{T_2} I_r(t)dt.$

iii. Shipment cost acquired for m- shipments: $CT_r = \frac{p\cdot n\cdot C_t}{T}$.

iv. Investment for preservation technology; PTI = u_0T.

Now, considered the two situations based on the lengths of T_2 and M for computing interest earned and payable.

- Case 1: When $M \leq T_2$
  i. The earned interest per unit time for pn- shipments is given as: $IE_{r1} = \frac{pnP_rI_e}{T}\int_0^M D(t)tdt$.
  ii. The charged interest per unit time for pn- shipments is given as: $IC_{d1} = \frac{nP_dI_c}{T}\int_M^{T_2} I_r(t)dt$.
- Case 2: When $M \geq T_2$
  i. The earned interest per unit time for pn- shipments is given as: $IE_{r2} = \frac{pnP_rI_e}{T}\left[\int_0^{T_2} D(t)tdt + (M - T_2)Q_r\right]$.
  ii. The charged interest per unit time for pn- shipments is given as: $IC_{r_2} = 0$.

Hunce, the retailer's yearly total cost per unit time is: $TC_r = \begin{cases} TC_{r1}, N \leq T_2 \\ TC_{r2}, N \geq T_2 \end{cases}$. Where: $TC_{r1} = OC_r + HC_r + CT_r + IC_{r_1} - IC_{r_{1+P}}$ and $TC_{r2} = OC_r + HC_r + CT_r + IC_{r_2} - IE_{r_2}$. The objective is to optimize the Combined total cost, that's sum of manufacturer's cost, distributor's cost, and retailer's cost.

4.4. For every case, the combined total cost of the supply chain is given

- Case 1: When $N \leq T_1$
  i. The combined total cost of the system is given by: $TC_{J1} = \begin{cases} TC_1, M \leq T_2 \\ TC_2, M \geq T_2 \end{cases}$.
  ii. Where $TC_1 = TC_m + TC_{d_1} + TC_{r_1}$ and $TC_2 = TC_m + TC_{d_1} + TC_{r_2}$.
- Case 2: When $N \geq T_1$
  i. The combined total cost of the system is given by: $TC_{J2} = \begin{cases} TC_3, M \leq T_2 \\ TC_4, M \geq T_2 \end{cases}$
  ii. Where $TC_3 = TC_m + TC_{d_2} + TC_{r_1}$ and $TC_4 = TC_m + TC_{d_2} + TC_{r_2}$

The aim is to optimise TC_{j1} (TC_{j2}) in the problem based on the shipments from the manufacturer from the distributor to the retailer and cycle time.

4.5. Sensitivity analysis and mathematical examples

The information is shown in the proper units. $A_0 = 200$, $\lambda = 1$, $P_m = 10$, $h_d = 0.3$, $C_{Td_1} = 70$, $I_c = 30\ \%$, $m = 1$, $B_d = 100$, $a = 100$, $P_d = 12$, $h_r = 0.5$, $C_{Td_2} = 150$, $I_e = 20\ \%$, $G_r = 50$, $b = 15\ \%$, $c = 20\ \%$, $P_r = 15$, $M = 0.1$, $I_{om} = 10\ \%$, $CT_m = 200$, $C_m = 4$, $h_m = 0.2$, $I_{od} = 15\%$, $CT_r = 50$ and $N = 0.2$. Table 2 shows that the supply chain's minimum cost is 71933.21. when 3.1 units are shipped from the distributor to the retailer, and 6 units are shipped from the manufacturer to the distributor for $N \leq T_1$ and $M \leq T_2$. Figure 1 shows the graph of total cost with respect to time and Figure 2 shows that the convexicity graph with respect to total cost.

Table 2. Table for total cost

Decision policy	No. of shipments	Cycle time	Total cost
TC_1	p=4 n=3	0.47	71933.2
TC_2	p=6 n=1	1.78	117718.4
TC_3	p=3 n=3	0.24	139177
TC_4	p=7 n=1	1.15	146329.39

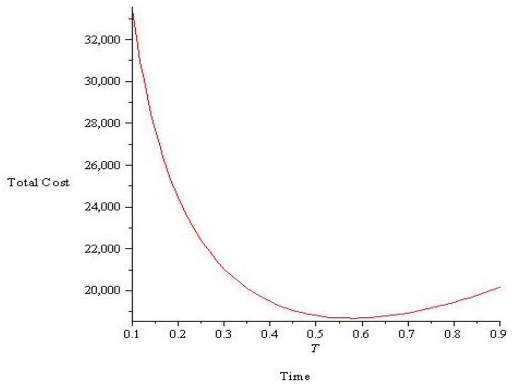

Figure 1. Total cost graph with respect to time

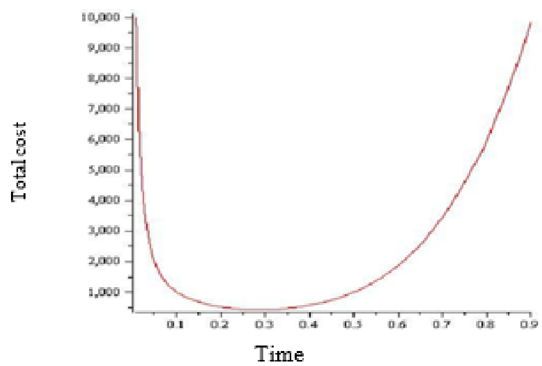

Figure 2. Graph for convexity with respect to total cost with time

4.6. Sensitivity analysis

The optimal solution is obtained using this data set, as well as a sensitivity analysis for the parameter values which can affect the model. Sensitivity processes were defined as parameter fluctuations between the ideal values as mentioned earlier. Table 3 shows the parameters that were adjusted one by one while keeping all other parameters constant throughout the analysis.

Table 3. Sensitivity analysis table

	Parameters	Cycle time (TC_1)	TC_1	Cycle time (TC_3)	TC_3
M	0.05	0.22	75,019.1	0.05	157431.0
	0.09	0.30	95281.29	0.16	146535.7
	0.1	0.32	105676.4	0.22	150148.7
	0.15	0.44	155152.2	0.35	168596.6
N	0.3	0.41	87817.15	1.74	185289.3
	0.4	0.51	71820.3	0.219	220506.8
	0.5	0.58	482159	0.216	255976.6
	0.6	0.60	22851.60	0.213	290930.8
A_0	250	0.3300	108118.3	0.204	150419.8
	450	0.3315	108725.5	0.203	151355.9
	650	0.3317	109328.7	0.201	152296.2
	850	0.330	109937.1	0.200	153244.1
B_d	200	0.3315	108574.4	0.203	151121.2
	400	0.330	109785.1	0.200	153005.9
	600	0.328	111002.9	0.218	154915.5
	800	0.326	112228.2	0.215	154895.7
G_r	75	0.330	106725.5	0.203	151355.9
	100	0.328	107481.1	0.201	152533.2
	125	0.329	108240.6	0.219	153718.7
	150	0.328	111002.9	0.218	154915.5

5. RESULT AND DISCUSSION

As this model has deal with SCM model which involves the movement of storage supplies from the point of origin to the point of consumption. A supply chain inventory model is formulated in which a distributor and a retailer observe quadratic demand and a manufacturer runs the production proportional to the quadratic demand. The demand increases linearly for sometimes when a product is launched and with new substitute available demand decreases exponentially. The Table 3 of sensitivity analysis shows that:

- As per the value of M increases with its rate, both the cycle time with its cost function has increment in its value.
- The increment in the value of N discusses the decrement of the first cycle time with its decrement of the total cost function, but on the other hand the decrement of the cycle time raises its total cost function value.
- The value of increment in the setup cost per order shows the increment of the cycle time with its total cost function.
- The value increment of the order cost of the distributer directly proportional to the increment of the total cost of the distributor.
- On the other hand, if the retailer order cost also goes high then its total coat also raises.
- A retailer dealing directly with the customer implements a good warehouse that reduces spoilage of products in stock. As, the rate of deterioration is higher in the manufacturer's storage system.
- The distributor provides a specific credit term to the store, while the manufacturer provides a grace period for invoicing.
- The costs of an integrated storage system are minimized because of multiple shipments from the manufacturer to the distributor and from the distributor to the retailer, as well as the replenishment times of each player.

Time is not the only profit factor, the number of shipments ordered by the manufacturer and distributor also plays an important role in optimizing the total cost.

6. CONCLUSION

An economic production quantity model has been developed under two level trade credit policy with the effect of preservation technology. Demand is depending on the quadratic factor. This paper formulates a supply chain inventory model where the distributor and retailer observe quadratic demand and manufacture controls production relative to quadratic demand. A production has a fixed life and therefore it deteriorates after a certain period of time. For fixed-life product, the ordering strategies of distributors and retailers are investigated. The bottom line is that the distributor should order the product in time to make a profit, the same is the case with the retailer. Time is not the only profit factor, the number of shipments ordered by the manufacturer and distributor also plays an important role in optimizing the total cost. In this paper, one numerical example has been developed for the case. The minimum cost is obtained with the cycle length finally a sensitivity analysis has been made for the numerical example and some useful results have been obtained from the table of sensitivity. Two-tier trade credit financing is examined in the model. Unless the invoice is settled within a certain period, distributors and retailers are responsible for making interest payments on unsold inventory. The resolution process is designed so that the supply chain operator gets the best possible decision. Therefore, results are authorized using mathematical example for different example scenarios. For further future extension demand can be changed such price and stock dependent, and credit linked demand. In addition, to reduce the deterioration, carbon emission can be used.

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