# The 3D Modeling of Cutting Force in High-speed Milling for Flat Mill

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#### Abstract

Aiming at the varying cutting features of depth and thickness in high-speed milling, using mathematical methods to model the theoretical three-dimensional model of calculating cutting forces based on the machining principle. First of all, according to the oblique cutting model, a cutting force model of flank edge was presented. The differential method was used in this process. The model was approached with calculating instantaneous chip thickness based on real tooth trajectory. Secondly, the chisel edge for differential along the vertical direction of cutting edges according to the orthogonal cutting model, calculating the cutting force of the infinitesimal. The cutting force model of chisel edge was constructed by the integral method. Merging the both upon, then the three-dimensional cutting force model is established. In the end, the model was programmed by means of the software Matlab. The result indicates that the numerical results agree well with experimental data, and the foundation of the cutter's stress field can be laid by this model.

Keywords: high-speed flat mill, cutting force, model

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#### 1. Introduction

The metal cutting process is a process of interaction between tool and job, Milling is a cutting process with discontinuous contact between tool and job and variation in cutting thickness. Its mechanism is very complicated. Cutting force affects directly the tool wear, breakage and the stability of the processing system; it also influences the processing precision of the job. As flat milling belongs to the complex three-dimensional cutting, involving many cutting parameters, it is difficult to conform the prediction of cutting force correspond to reality. Qinglong A [1], Altintas Y etc [2] many scholars have a lot of research in the modeling of the milling force. Among the numerous cutting force modeling, the instantaneous rigid force model established by Yusuf A [3] has a more widely application.

The thinking of discretization process makes the model has a high degree of accuracy and practicability in the simulation of instantaneous milling force. In view of the deformation feature of the process system with low stiffness, Houjun Q [4] set up a elastic milling force model of low stiffness milling process system. Zhihuan Zhang [5] established the mathematical model of milling based on the characteristics of discontinuous contact and shifty cutting thickness in CNC milling. ZHAO Yongjuan [6] analyzed the effects of processing parameters on tangential cutting force of vertical milling on Ni - base superalloy GH4169 based on the finite element software ABAQUS. Considering the change of cutting thickness and cutting width, Zhijie X [7] established dynamic cutting force model of spiral flat mill. Analyze the regulation of the processing parameters effect on the tangential and radial cutting force coefficient when flat milling the 4Cr16Mo die steel. Min Liu etc [8] put forward the simple and effective way to reduce the cutting force. But with few direct calculation models, most of models based on the identification of the cutting force coefficient, its experimental process is very complicated. Besides, it considered few about the cutting force of the cross cutter's cutting edge, which is not conform to the three-dimensional milling practical situation of mill. In this paper, we focused on the stress analysis of the cutting mill when cutting, using the micro cutting unit along the cutting edge axial integral, set up mechanical model on flank edge and chisel edge of 4 cutting edge carbide flat mill.

## 5045

## 2. The Establishment of the Milling Cutting Force Coordinate System

The experiment machine is V850 made in Jie-yongda, z-axis is its principal axis, vertical upward as positive. the local coordinate system of cutting tool is established as shown in Figure 1 with the x axis of the machine tool for the x-axis of the coordinate system, the left for positive, the y-axis according to the right-hand rule, the center of the circle in the mill's base plane is determined as the origin of coordinates. In the process of milling, the instantaneous cutting force which effects on the mill is a space force with continuous variation along the cutting edge. The research method is mainly to divided the mill along its axis direction into very thin infinitesimal, each infinitesimal can be regarded as a cutting process of single cutting edge knife's oblique cutting, we modeled the infinitesimal cutting force model by using different theories and methods. To get the total milling force, we must integrate the cutting force on the infinitesimal along the cutter axis. The value range of integral calculating is the key to calculate correctly the milling force. Assuming that the cutting force on the infinitesimal with height of dz are the tangential force dFc, the radial force dFf and the axial force dFp.

The position of cutting infinitesimal in the cutter's local coordinate system is shown in Figure 1. (X '-axis is in the cutting plane that perpendicular to the cutting edge, Y '-axis coincide with the cutting edge and Z 'axis is perpendicular to the plane x'o'y'). The angle between the two coordinate system is the cutter spiral angle  $\beta$ .

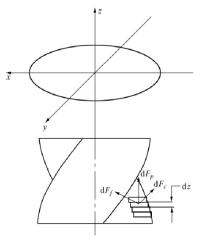


Figure 1. The Analysis of the Force in the Infinitesimal of the Flat Mill

## 3. The Model of the Oblique Cutting Force

During the process of actual cutting, milling is three-dimensional cutting process. The oblique cutting is the situation when the cutting edge and cutting direction isn't vertical in common three-dimensional cutting's.

Flat end mill commonly has two or more cutting edges that evenly distributed round its circumference, a few cutting edge cutting the job at the same time may appear during the milling process. The rotation angle of every point on the spiral mill's cutting edge changed with the mill's rotation direction, it has a lag angle or advance angle compare with the rotation angle in the basis plane. The value of the lag angle or advance angle  $\psi$  is the function of the value of the z- coordinate point.

$$\psi_i(z) = \frac{z \tan \beta}{\beta}$$

The lag angle at the position is  $a_p$ .

$$\psi_{\alpha} = \frac{a_p \tan \beta}{r}$$

We choose the rotation angle at the base plane as the mill's rotation angle, then the expression of the infinitesimal position angle at the same height, at the time of t is:

 $\theta = \theta_0 + \psi_i(z) - 2\pi nt$ 

Now we choose the 4 blade right-lateral flat mill as the research object, we established the milling model, assuming that the radius of the mill is r, spiral angle is  $\beta$ , gear number is z, depth of cut is ap, the radial cutting width is ae.

## 3.1. The Model of Flank Edge Cutting Force

The cutting of the flat mill's flank edge belongs to oblique cutting, its state coordinate system of cutting is shown in Figure 2.

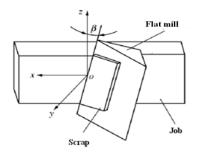


Figure 2. The Oblique Cutting State of the Flank Edge

The component of x-axis direction of the tangential force dFc is:

 $dF_c = dF\cos\theta_i\cos\theta_i\cos\theta$ 

The component of y-axis direction of the tangential force dFc is:

 $dF_c = dF \sin \theta_i \sin i$ 

In the expression:  $\theta$ n is the angle between the x axis and the resultant force F's projection in the normal plane,  $\theta$ i is the inclined angle of the cutting speed, i is the angle between the shear plane and the plane xoy.

$dF_c = dF(\cos\theta_i \cos\theta_n \cos i + \sin\theta_i \sin i)$	(1)

Also, we can get that:

 $dF_f = dF\cos\theta_i\sin\theta_n \tag{2}$ 

 $dF_p = dF(\sin\theta_i \cos i - \cos\theta_i \cos\theta_n \sin i)$ (3)

The shear force can be express as the resultant force F's projection in the shear direction according to the geometry

$$\mathbf{F}_{s} = F\left[\cos(\theta_{n} + \phi_{n})\cos\theta_{i}\cos\phi_{i} + \sin\theta_{i}\sin\phi_{i}\right]$$

It can be expressed as the product of the shear stress and shearing area.

 $\mathbf{F}_{s} = \tau_{s} \mathbf{A}_{s} = \tau \left( \frac{a_{p}}{\cos i} \right) \left( \frac{h}{\sin \phi_{n}} \right)$ 

From the two expressions above:

$$F = \frac{\tau_s a_p h}{\left[\cos(\phi_n + \theta_n)\cos\theta_i\cos\phi_i + \sin\theta_i\sin\phi_i\right]\cos i\sin\phi_n}$$

In the expression: h is the depth of cut in the axial direction, the cutting force of each infinitesimal is:

$$dF = \frac{\tau_s a_p dz}{\left[\cos(\phi_n + \theta_n)\cos\theta_i\cos\phi_i + \sin\theta_i\sin\phi_i\right]\cos i\sin\phi_n}$$
(4)

In the expression: ap is the instantaneous cutting thickness during milling

## 3.1.1. The Calculation of the Instantaneous Cutting Thickness

Set up the calculation coordinate system of cutting thickness as shown in Figure 3 (the center of the cutter starts at the point O, feed direction is to the right, clockwise rotation). The track of the cutting edge has been simplified as circle in many researches, thus the instantaneous cutting thickness  $a_p = f_z \sin \theta$ , in the expression: fz is the feed rate for each tooth,  $\theta$  is the angle position for mill's tool nose. Without considering the deformation and the eccentricity of the cutter, this assumption can meet the calculation requirements in some extent when calculating the instantaneous cutting thickness with the certain feed rate.

To make the cutting force with more accuracy, the more accuracy of instantaneous cutting thickness should be get [9]. The compositions of actual process of mill are the translation of the mill and the rotation of its own axis, the track of the mill is a trochoid compounded by the motion above.

As is shown in Figure 3, when the point M on the cutting edge move to the point E which on the j-1 rotation track of the cutter tooth, its rotation angle is  $\theta$ . When the rotation angle of the cutter tooth is  $\theta 1 = \theta$ , point M move to point D, |ED| = fz, the link between the point B and D intersect with the tack of the j-1 rotation at the point C To simplify the calculation, assuming that the center of mill is O and remain the same during the cutter tooth's rotation movement track from E to C, prolonging the line BC and make line OA meet at right angles with the line BC at A,  $\angle AOB=\theta$ .

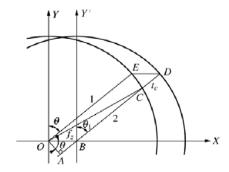


Figure 3. The Instantaneous Cutting Thickness of the Single Cutting Edge

So the instantaneous cutting thickness is:

 $a_{p} = |CD| = |BD| - |BC| = R - |BC|$ 

According to law of cosines:

 $|BC|^{2} = |OC|^{2} + |OB|^{2} - 2|OC||OB|\cos \angle BOC = R^{2}f_{z}^{2} - 2Rf_{z}\angle BOC$ 

 $\angle BOC = \pi/2 - \theta - \angle EOC = \pi/2 - \theta - [\pi/2 - \arccos(f_z \cos \theta/R)] = -\theta + \arccos(f_z \cos \theta/R)$ 

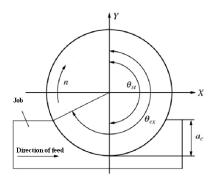
$$a_{p} = R - \sqrt{R^{2} + f_{z}^{2} - 2Rf\left[-\theta + \arccos\left(f_{z}\cos\theta/R\right)\right]}$$

In the end, putting ap into expression (4), we can get the cutting force of infinitesimal dF, putting it into expression (1) to (3), we can get the cutting force from the three direction. If you want to get the total cutting force of the flat mill, you have to integrate the cutting force on the infinitesimal along the cutter axis. The value range of integral calculating is the key to correctly calculate the milling force. The value range of integral is often get by calculating the angle.

Take right-hand mill for example to discuss the climb milling and up milling, for the convenient of calculating, we give some angles as are shown in the Figure 4 and 5, the minimum cut-in angle is  $\theta$ st, the maximum cut-away angle is  $\theta$ ex, stipulating the starting point of each angle is the forward direction of y-axis, clockwise is positive.

## 3.1.2. Determining the Integrating Range of Up Milling

As is shown in the Figure 4, during the up milling, the minimum cut-in angle is  $\theta$ st= $\pi$ , the maximum cut-away angle is  $\theta$ ex has to be calculated by the milling width and mill's size under the 4 conditions below:



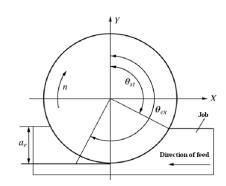


Figure 4. Up Milling

Figure 5. Climb Milling

- (1) when ae< r,  $\theta_{ex} = 3\pi/2 \arcsin(1 a_e/r) + \psi_a$
- (2) when ae= r,  $\theta_{ex} = 3\pi/2 + \psi_a$
- (3) when r < ae< 2r,  $\theta_{ex} = 3\pi/2 + \arcsin(1 a_e/r) + \psi_a$
- (4) when ae= 2r,  $\theta_{ex} = 2\pi + \psi_a$

The size of  $\theta$ ex- $\theta$ st and  $\psi$ a determine whether the cutting edge which its the axial depth of cut is ap can all involved in cutting or not, so we discuss the bound of the integration under the following  $\theta$ ex- $\theta$ st $\geq$  $\psi$ a and  $\theta$ ex- $\theta$ st $<\psi$ a two conditions. If the intersection angle in the number i edge line's end face is  $\theta$ , the position angle at the bottom of the adjacent edge line is  $\theta$ +  $2\pi/N$ .

(1) If θex -θst≥ψa

a) when  $\pi \le \theta < \pi + \psi a$ , only part of the cutting edge go into the cutting, so the lower angle limit of integral should be  $\pi$ , upper limit of integral should be  $\theta$ .

b) when  $\pi+\psi a < \theta \le \theta ex-\psi a$ , all of the cutting edge go into the cutting under the axial depth of cut, so the lower angle limit of integral should be  $\theta$ - $\psi a$ , upper limit of integral should be  $\theta$ .

c) when  $\theta ex-\psi a < \theta \le \theta ex$ , part of the cutting edge has left the cutting area, but the above part still cutting, its lower angle limit of integral is  $\theta$ -  $\psi a$ , upper limit of integral is  $\theta ex$ .

(2) If θex- θst< ψa

a) when  $\pi \le \theta \le \theta ex \cdot \psi a$ , only part of the cutting edge go into the cutting, so the lower angle limit of integral should be  $\pi$ , upper limit of integral should be  $\theta$ .

b) when  $\theta ex-\psi a < \theta \le \pi + \psi a$ , all of the cutting edge go into the cutting under the axial depth of cut, so the lower angle limit of integral should be  $\pi$ , upper limit of integral should be  $\theta ex-\psi a$ .

c) when  $\pi$ + $\psi$ a< $\theta$ ≤ $\theta$ ex, part of the cutting edge has left the cutting area, but the above part still cutting, its lower angle limit of integral is  $\theta$ - $\psi$ a, upper limit of integral is  $\theta$ ex- $\psi$ a.

## 3.1.3. Determining the Integrating Range of Climb Milling

As is shown in the Figure 5, during the climb milling, the maximum cut-away angle is  $\theta ex=\pi+\psi a$ , the minimum cut-in angle is  $\theta st$  has to be calculated by the milling width and mill's size under the 4 conditions below:

(1) when ae< r,  $\theta_{\rm st} = \pi/2 + \arcsin(1 - a_e/r)$ 

(2) when ae= r,  $\theta_{\rm st} = \pi/2$ 

(3) when r< ae<2r,  $\theta_{\rm st} = 3\pi/2 - \arcsin(1 - a_e/r)$ 

(4) when ae=2r,  $\theta st=0$ 

In a similar way, we discuss the bound of the integration under the following  $\theta ex-\theta st \ge \psi a$  and  $\theta ex-\theta st \le \psi a$  two conditions.

(1) If θex-θst≥ψa

a) When  $\theta st \le \theta < \theta st + \psi a$ , only part of the cutting edge go into the cutting, so the lower angle limit of integral should be  $\theta st$ , upper limit of integral should be  $\theta$ .

b) When  $\pi+\psi a < \theta \le \theta ex-\psi a$ , all of the cutting edge go into the cutting under the axial depth of cut, so the lower angle limit of integral should be  $\theta$ - $\psi a$ , upper limit of integral should be  $\theta$ .

c) When  $\theta ex-\psi a < \theta \le \theta ex$ , part of the cutting edge has left the cutting area, but the above part still cutting, its lower angle limit of integral is  $\theta$ - $\psi a$ , upper limit of integral is  $\pi$ .

(2) If θex- θst< ψa

a) When  $\theta st \le \theta \le \theta ex \cdot \psi a$ , only part of the cutting edge go into the cutting, so the lower angle limit of integral should be  $\theta st$ , upper limit of integral should be  $\theta$ .

b) When  $\theta ex-\psi a < \theta \le \theta st+\psi a$ , all of the cutting edge go into the cutting under the axial depth of cut, so the lower angle limit of integral should be  $\pi$ , upper limit of integral should be  $\theta ex-\psi a$ .

c) When  $\theta$ st+ $\psi$ a< $\theta$ ≤ $\theta$ ex, part of the cutting edge has left the cutting area, but the above part still cutting, its lower angle limit of integral is  $\theta$ - $\psi$ a, upper limit of integral is  $\theta$ ex- $\psi$ a.

#### 3.2. The Modeling of the Cutting Force at the Chisel Edge

The chisel edge of the flat mill is also involved in the cutting. As the chisel edge and the cutting speed always vertical, so the cutting progress of the chisel edges can be regarded as the orthogonal cutting. The resultant cutting force in the vertical cutting is the function of the shear stress ts, frictional angle  $\beta$ , shear angle  $\phi$ c, cutting width and feed rate.

$$F = \frac{t_s a_p a_e}{\sin \phi_c \cos(\phi_c + \beta - \gamma_o)}$$

In the expression: Except the frictional angle  $\beta$  and the shear angle  $\phi$ c, other parameters can be obtained through consulting the related manual.

During the vertical cutting, the relationship between the cutting speed V, shearing velocity Vs and the speed of chip flow Vc is:

V = Vc -Vs

 $V \times hD = Vc \times hch$ 

According to the incompressible of the metal material, we decompose the force above into the coordinate axis, substituting into the expression above, we can get:

$$\cot\phi_c = -\tan\gamma_o + \frac{\Lambda_h}{\cos\gamma_o}$$

In the expression:

$$\Lambda_{\rm h} = \frac{h_{ch}}{h_D}$$

We calculating the cutting force choosing the value of the compression ratio of chip is 1.8, the shear angle  $\phi$ c= 30. 2°, frictional angle  $\beta$ =19. 8°, after calculating, we get the F = 68 N, Fx= 65. 6 N, Fz= 17. 5 N.

So whole cutting force of the mill is the sum of the cutting force at side-edge and chisel edge.

#### 4. The Consequence of the Experiment and the Simulation

According to the established model of the cutting force above, we can get the cutting force curve along the direction of the x-axis as is shown in Figure 6 under the following condition that the speed of main spindle is 6000r/min, feed per tooth is fz= 0.1mm / tooth, the cutting parameters are listed as follows: the axial depth of cut are ap=1mm and ap=2.5 mm, the width of mill are ae=2mm, ae=3mm, ae=6mm and ae=12mm separtely.

- 1: ap=2.5mm, ae=6mm Up milling curved
- 2: ap=1mm, ae=12mm Climb milling curved
- 3: ap=1mm, ae=3mm Up milling curved
- 4: ap=1mm, ae=2mm Up milling curved
- 5: ap=1mm, ae=2mm Climb milling curved

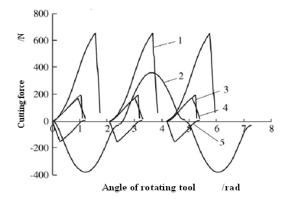


Figure 6. The Simulation Value of the X-axis's Cutting Force

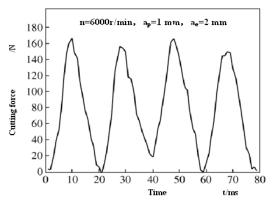


Figure 7. The Measured Value of the X-axis's Cutting Force in High Speed Milling

The Figure 7 shows the measured value of the cutting force along the one rotation of the mill. The experiment condition are: The experiment machine is machining center V850, the equipment of obtaining the value of the cutting force is the general contact-type dynamometer, the equipment of gathering the data is the Multifunctional acquisition meter, the frequency of sampling is 7000Hz, the flat mill is Kenna HA-DIN6542 flat mill with 4 cutting edge, 10mm diameter, spiral Angle is 55° and the work material is 4Cr5MoSiV heat-treated die steel with the rigidity is 42HRC. The cutting parameters corresponds with the curve 4 in Figure 6.

When the mill begin cutting, only part of the cutting edge go into the cutting, so the cutting force is very low at the beginning. After all of the cutting edge go into the cutting under the axial depth of cut and the cutting force will reach peak when the mill rotating, then the cutting force will become lower because of the decrease of thickness of cutting layer and will become 0 at last. Comparing the curve 4 in Figure 6 with the Figure 7, the change trend of the waveform and the value of the amplitude in the simulation cutting force is coincided well with the measured milling force.

## 5. The Results and Discussion

The model of flat end mill is established. The mill along the axial direction is discretized and the cutting force of the infinitesimal and the whole cutting force in the mill process are calculated by means of the numerical integration. The model of cutting force we established taking into account of the cutting in the flank edge and the chisel edge which is conformed to the truth, and can simulate the force condition of the mill during milling well. The cutting force is simulated using the model we established and the verification test of cutting force has been done. The results show that the maximum error values between the simulation and the measurement is 7.38%. As not consider the influence of the tool wear and radial run-out of the mill, the vibration in the manufacturing system and other influences, a certain error values between the simulation and the measurement still remained. The cutting force can be simulated well through this cutting force model during milling, and the foundation of the cutter's stress field can be laid by this model.

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