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A time-efficient nonlinear control method for the hyperchaotic finance system synchronization

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ABSTRACT

Irregular and complex behavior in the financial system can disrupt stability and smooth economic growth. It causes randomness within the system, generating chaos; hindering synchronization behaviour. Achieving smooth and rapid synchronization between two coupled hyperchaotic finance (HF) systems with lessened fluctuation of input and output signals is vital for continuing financial stability and fostering economic growth, a challenge addressed in this article. The paper proposes a novel time-efficient nonlinear control (TENLC) technique and investigates HF systems synchronization using the drive-response system (DRS) arrangement. The proposed TENLC strategy realizes fast and smooth synchronization behaviour between two coupled HF systems, reducing closed-loop state-variable trajectory oscillations. The controller is designed to retain the nonlinear components within the closed-loop system and does not depend on the system's parameters, simplifying the design and analysis process. The Lyapunov stability technique confirms the closed-loop's global stability at the origin. Proofs of mathematical analysis and computer-based simulation results validate the theoretical findings, showing that the presented TENLC strategy converges the state error trajectories to zero in a short transient time with lessened fluctuations for all signals. The comparative computer-based simulation analysis confirms that the presented TENLC approach outperforms other synchronization control techniques.

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1. INTRODUCTION

Chaotic oscillators are specific types of nonlinear dynamical systems that display unpredictable behavior and are highly sensitive to minor variations in their initial conditions and parameters [1]. These variations can be recognized by their intricate and irregular oscillation patterns, described by fractals or multi-fractal structures [2]. Chaotic phenomena can be observed in various natural and artificial systems [3]. Due to their inherent randomness, chaotic systems resist synchronization behavior. Synchronization of chaotic systems occurs when two or more initially unrelated chaotic systems behave in a coordinated and correlated manner over time under the action of a feedback control input [4]. Their chaotic trajectories become similar or identical despite having different initial conditions [5]. Chaos synchronization has shown successful applications in physical systems, robotics, secure communication systems, model calibration, space engineering, electronics, biological systems [6]-[11], for potential applications.

The nonlinear finance models exhibit complex and irregular phenomena [12]-[14]. Social and economic factors, such as political policies and news, environmental interfaces, and exchange rate fluctuations, can cause abrupt variations in stock market prices. As a result, the financial model's periodic dynamics can be transferred to chaos [15], [16]. The chaotic nature of a financial system causes complications in financial system planning [17]. The chaotic financial models require synchronization to maintain economic growth and financial stability [18]. Over the past two decades, the synchronization of chaotic financial systems has been a topic of intense research, leading to the development of diverse feedback controller methodologies for achieving stable economic dynamics through coordinated synchronization control of chaotic financial behavior. For instance, [18] realizes two identical three-dimensional chaotic finance (TDCF) systems' synchronization; the Routh-Hurwitz criterion is used to investigate closed-loop stability for known systems' parameters, while an active adaptive control strategy is applied for uncertain parameters. The synchronization of two coupled uncertain TDCF systems is achieved in [19]. Jajarmi *et al.* [20] synchronizes two identical uncertain HF systems using an adaptive control algorithm based on the Lyapunov stability analysis. Tirandaz *et al.* [21] synchronizes two identical TDFC systems using a linear feedback control method. The closed-loop global stability is analyzed using the active adaptive feedback controller. Xu *et al.* [22] proposes a nonlinear adaptive control method with a time-varying delay to synchronize two coupled TDCF systems with external disturbances. Ding and Xu *et al.* [23] explores two mixed synchronization schemes for coupled TDCF systems, comparing the closed-loop systems' performance through simulation results. Chen *et al.* [24] derives sufficient conditions and designs a linear feedback control strategy to achieve global asymptotic synchronization between two identical HF systems.

The challenges and motivations are given:

- Existing feedback control methodologies [18]-[24] rely on canceling nonlinear terms in the closed-loop system for synchronization. However, these methods demand precise measurement of parameters and state variables, which can be challenging due to technological limitations. Furthermore, nonlinearities can contain valuable information about the financial system. Eliminating nonlinear components can lead to losing valuable information about the financial system, potentially hindering the understanding and ability to manage the system effectively.
- The control schemes [18]-[24] consume huge amounts of energy and can cause significant synchronization errors and control signals' fluctuations. This can destabilize the financial system and may even result in a complete loss of economic stability. The precision of HF system synchronization is vital to promoting economic growth and ensuring financial stability.
- The proposed control strategies [18]-[24] can furnish a smaller synchronization error convergence gradient, potentially leading to increased investor uncertainty and prolonged periods of instability.

The abovementioned challenges develop motivations for designing a state-feedback control strategy synchronizing two identical HF systems, realizing smoother and faster synchronization error convergence, and reducing fluctuation for all signals. The developed controller should achieve the following objectives:

- The control methodology should elude the cancellation of nonlinear terms within the closed-loop system.
- It should reduce fluctuations of control signals and state error vector trajectories.
- It should demonstrate faster convergence of error vector trajectories to zero to improve the efficiency and accuracy of the system.

This work designs a new TENLC technique that accomplishes smooth and rapid synchronization behaviour between two identical HF systems. The controller retains closed-loop nonlinear components and achieves global stability at the origin based on the Lyapunov stability analysis [25]. Computer-based simulations and theoretical analysis confirm the effectiveness of the proposed synchronization approach. The contributions of the presented synchronization control approach are as follows:

- The input signals exhibited by the proposed control effort are free from the nonlinear terms of the closed-loop; it means that the stability of the closed-loop performance is not affected by any variations within the system. This controller feature simplifies the design and analysis process, ensures smooth control operations, and consumes less energy.
- The presented control law enhances the stability of the financial system by reducing sudden fluctuations and crash risks; it leads to increased efficiency, productivity, and reduced investment demands.
- Computer simulation results are compared with the feedback control scheme presented in [24]. This comparative analysis further validates the performance proposed synchronization control approach.

Section 2 analyzes the dynamics of the HF system and formulates the synchronization problem between two identical HF systems. Section 3 proposes a new TENLC technique synchronizing two identical HF systems. Section 4 presents comprehensive numerical simulations and comparative analysis to validate the proposed technique's efficiency. Section 5 concludes the article by summarizing key findings and giving future directions.

2. PROBLEM FORMULATION

2.1. Chaotic dynamics of the hyperchaotic finance system

The HF system dynamics model [16] is described in (1):

$$\dot{x}(t) = \begin{bmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -d \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} x_1(t)x_2(t) \\ -x_1^2(t) \\ 0 \\ -\vartheta x_1(t)x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

where $\mathbf{x}(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^T \in R^{4\times1}$ is the state variables vector, and a, b, c, d, and ϑ denote system parameters. The HF system (1) parameters for all simulations are set as $a = 0.9$, $b = 0.2$, $c = 1.5$, $d = 0.2$, and $\vartheta = 0.17$ [16]. The HF system exhibits chaos, as evident from Figures 1-4. Figures 1(a) and 1(b) depict the 3D chaotic attractors of system (1), in which Figure 1(a) shows the attractor along $(x_1(t) - x_2(t) - x_3(t))$ and Figure 1(b) along $(x_2(t) - x_3(t) - x_4(t))$ state variables. Figures 2(a)-2(d) show the 2D chaotic attractors of system (1), in which Figure 2(a) illustrates chaotic attractor between (a) $x_1(t)$ and $x_2(t)$, (b) $x_1(t)$ and $x_3(t)$, (c) $x_1(t)$ and $x_4(t)$, and (d) $x_2(t)$ and $x_3(t)$ state variables. Figures 3(a-d) illustrates the HF system (1) state variables chaotic response for (a) $x_1(t)$, (b) $x_2(t)$, (c) $x_3(t)$, and (d) $x_4(t)$ state variables. Figures 4(a) and 4(b) depict the HF system (1) bifurcation plots for (a) a_3 vs x_1, and (b) a_3 vs x_2.

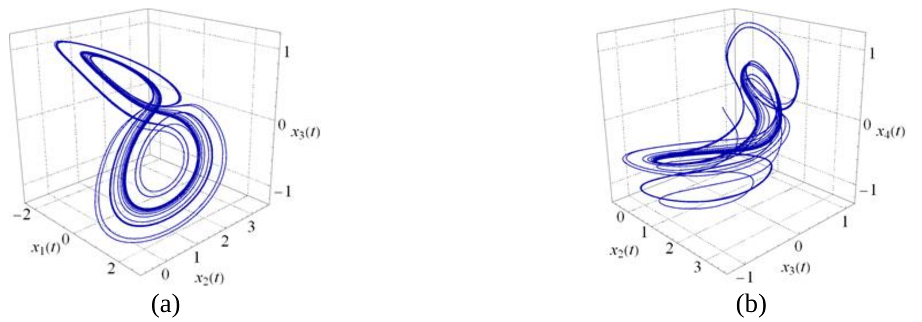

Figure 1. Three-dimensional chaotic attractors, (a) $(x_1(t) - x_2(t) - x_3(t))$ and (b) $(x_2(t) - x_3(t) - x_4(t))$

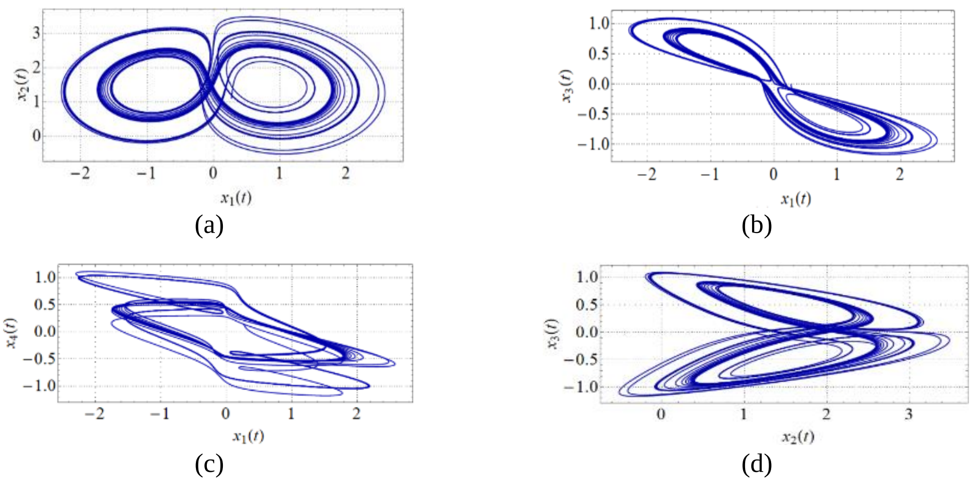

Figure 2. Two-dimensional chaotic attractors, (a) $x_1(t)\ vs\ x_2(t)$, (b) $x_1(t)\ vs\ x_3(t)$, (c) $x_1(t)\ vs\ x_4(t)$, and (d) $x_2(t)\ vs\ x_3(t)$

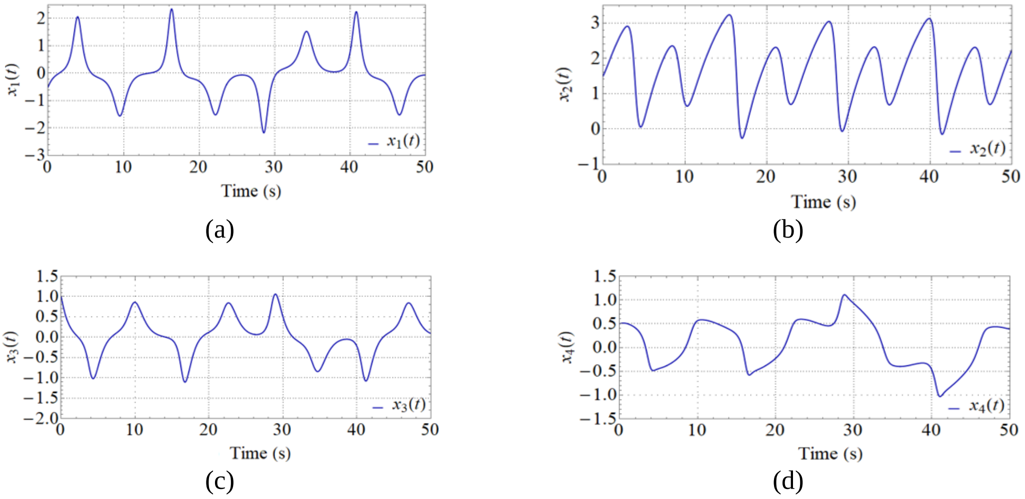

Figure 3. Chaotic response of HF system (1) state variables, (a) $x_1(t)$, (b) $x_2(t)$, (c) $x_3(t)$ and (d) $x_4(t)$

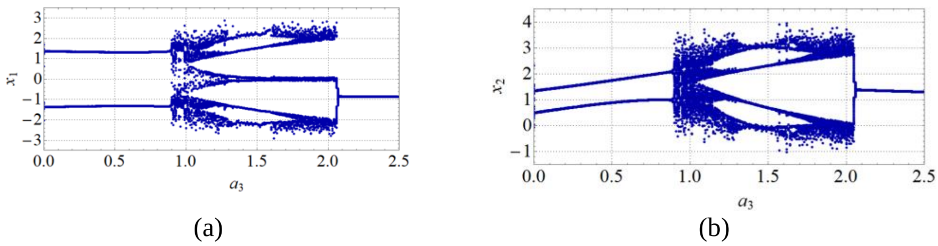

Figure 4. HF system (1) bifurcation plots for the parameter a_3, (a) a_3 vs x_1 and (b) a_3 vs x_2

To realize two identical HF systems synchronization behaviour, in (2) and (3) show the drive-response system (DRS) arrangement; the drive HF system (2) is represented by the state vector $\mathrm{x}(t)$, and $\mathrm{y}(t)$ denotes the response system state vector given in (3).

Drive HF system:

$$\dot{\mathrm{x}}(t) = \mathrm{A}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} x_1(t)x_2(t) \\ -x_1^2(t) \\ 0 \\ -\vartheta x_1(t)x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (2)$$

Response HF system:

$$\dot{\mathrm{y}}(t) = \mathrm{A}\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix} + \begin{bmatrix} y_1(t)y_2(t) \\ -y_1^2(t) \\ 0 \\ -\vartheta y_1(t)y_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \mathrm{u}(t), \quad (3)$$

Where:

$$\mathrm{A} = \begin{bmatrix} -a & 0 & 1 & 1 \\ 0 & -b & 0 & 0 \\ -1 & 0 & -c & 0 \\ 0 & 0 & 0 & -d \end{bmatrix}, \text{ and}$$

$\mathrm{u}(t) = [u_1(t) \quad u_2(t) \quad u_3(t) \quad u_4(t)]^T \in R^{4\times 1}$ is the feedback controller.

The DRS error vector is given in (4).

$$\dot{\mathrm{e}}(t) = \mathrm{Ae}(t) + \mathrm{F}\big(\mathrm{x}(t), \mathrm{y}(t), \mathrm{e}(t)\big) + \mathrm{u}(t), \quad (4)$$

Where:

$$\mathrm{F}(\mathrm{x}(t),\mathrm{y}(t),\mathrm{e}(t)) = \begin{bmatrix} y_1(t)y_2(t) \\ -y_1^2(t) \\ 0 \\ -\vartheta y_1(t)y_2(t) \end{bmatrix} - \begin{bmatrix} x_1(t)x_2(t) \\ -x_1^2(t) \\ 0 \\ -\vartheta x_1(t)x_2(t) \end{bmatrix} = \begin{bmatrix} y_2(t)e_1(t) + x_1(t)e_2(t) \\ -e_1(t)\big(x_1(t)+y_1(t)\big) \\ 0 \\ -\vartheta\big(y_2(t)e_1(t)+x_1(t)e_2(t)\big) \end{bmatrix}, \tag{5}$$

And,

$$\begin{cases} \mathbf{e}(t) = \mathbf{y}(t) - \mathbf{x}(t), \\ y_1(t)y_2(t) - x_1(t)x_2(t) = y_2(t)e_1(t) + x_1(t)e_2(t), x_1^2(t) - y_1^2(t) = -e_1(t)\big(x_1(t)+y_1(t)\big), \\ -\vartheta y_1(t)y_2(t) + \vartheta x_1(t)x_2(t) = -\vartheta\big(y_2(t)e_1(t) + x_1(t)e_2(t)\big). \end{cases} \tag{6}$$

Remark 1: conventional synchronization control methods often rely on discontinuous signum (sign) functions within their control protocols to rapidly converge state errors to zero. Although these methods effectively speed up error convergence, it can lead to chattering, a high-frequency oscillation in the control input. This undesirable behavior can cause increased actuator wear and tear, unwanted noise and vibrations, and potential instability in the system dynamics. Further, the TDCF/HF systems synchronization feedback control strategies [18]-[24] involve cancelling the nonlinear terms. However, this procedure can have a great impact on the economy. For instance, if the nonlinear components are removed through the feedback controller, it is likely to reduce investment and economic activity.

To address these challenges, the present article replaces the sign function with a smoother hyperbolic tangent function to avoid the chattering behaviour and make the closed-loop energy-efficient. This controller avoids the closed-loop's nonlinear terms cancellation to enhance the proposed synchronization strategy efficiency. The introduced control law enhances the stability of the financial system by mitigating unexpected fluctuations and minimizing the risk of crashes. The reduction in active synchronization fluctuations can offer several benefits, including high productivity and efficiency and diminished demands on investment.

3. HYPERCHAOTIC FINANCE SYSTEMS SYNCHRONIZATION

3.1. Controller design

This subsection proposes the design of a time-efficient nonlinear feedback controller that addresses the challenges discussed in Subsection 1.3 and achieves the objectives given in Subsection 1.4. In (7) gives the design of the feedback control input vector $\mathrm{u}(t) \in R^{4\times 1}$ that realizes the globally stable synchronization between two identical HF systems (2-3) such that $\lim_{t\to\infty} \|\mathrm{e}(t)\| = 0$.

$$\mathrm{u}(t) = -\alpha\mathrm{e}(t) - \beta\big(\mathrm{I} - \rho\delta(t)\big)\tanh\big(\mathrm{e}(t)\big), \tag{7}$$

Where $\delta(t) = [\delta_{ii}(t), i = 1, 2, 3, 4]_{4\times 4}$ is a diagonal matrix with $\delta_{ii}(t) = \mathrm{e}^{-\varrho|e_i(t)|}$, $\mathrm{I} = \mathrm{I}_{4\times 4}$ is the identity matrix, and e denotes the logarithmic base. $\alpha = [\alpha_{ii}, i = 1, 2, 3, 4]_{4\times 4}$ and $\beta = [\beta_{ii}, i = 1, 2, 3, 4]_{4\times 4}$ are the diagonal matrices of the feedback gains, and $0 < \rho < 1$ and $0 < \varrho < 1$ are any real constants.

Remark 2. The proposed TENLC design structure has two parts, each serving a specific role:
- The closed-loop dynamical system's linear dynamics stability is guaranteed by a suitable choice of the diagonal entries in matrix α.
- The mathematical analysis shows that $\beta\big(\mathrm{I} - \rho\delta(t)\big)\tanh\big(\mathrm{e}(t)\big)$ effectively manages the system's inherent nonlinearities; it translates to rapid and smooth convergence of error signals toward zero, significantly reducing unwanted oscillations.

Using the control law (7) in (4) yields:

$$\dot{\mathrm{e}}(t) = (\mathrm{A} - \alpha)\mathrm{e}(t) + \mathrm{F}\big(\mathrm{x}(t),\mathrm{y}(t),\mathrm{e}(t)\big) - \beta\big(\mathrm{I} - \rho\delta(t)\big)\tanh\big(\mathrm{e}(t)\big). \tag{8}$$

3.2. Closed-loop stability analysis

This subsection presents the theoretical analysis describing the closed-loop dynamic system (8) global stability. Theorem: the feedback control strategy (7) added to the response HF system (3) realizes globally stable synchronization. Proof: consider the Lyapunov function candidate in (9).

$$V(t) = \frac{1}{2}\mathrm{e}^T(t)\mathrm{e}(t) \geq 0. \tag{9}$$

Now:

$$\dot{V}(t) = \mathrm{e}^T(t)\dot{\mathrm{e}}(t) \quad (10)$$

Using (8) into (9) implies:

$$\begin{aligned}\dot{V}(t) &= \mathrm{e}^T(t)(\mathrm{A}-\alpha)\mathrm{e}(t) + \mathrm{e}^T(t)\mathrm{F}\big(\mathrm{x}(t),\mathrm{y}(t),\mathrm{e}(t)\big) + \mathrm{e}^T(t)\Big(-\beta\big(\mathrm{I}-\rho\delta(t)\big)\tanh\big(\mathrm{e}(t)\big)\Big)\\ &= -\mathrm{e}^T(t)(\alpha-\mathrm{A})\mathrm{e}(t) + \mathrm{e}^T(t)\mathrm{F}\big(\mathrm{x}(t),\mathrm{y}(t),\mathrm{e}(t)\big) - \mathrm{e}^T(t)\Big(\beta\big(\mathrm{I}-\rho\delta(t)\big)\tanh\big(\mathrm{e}(t)\big)\Big)\\ &= -\mathrm{e}^T(t)\mathrm{P}_1(t)\mathrm{e}(t) + \mathrm{e}^T(t)\mathrm{P}_2(t)\mathrm{e}(t) - \mathrm{e}^T(t)\beta\big(\mathrm{I}-\rho\delta(t)\big)\tanh\big(\mathrm{e}(t)\big),\\ &= -\mathrm{e}^T(t)\mathrm{Q}(t)\mathrm{e}(t) - \mathrm{e}^T(t)\beta\big(\mathrm{I}-\rho\delta(t)\big)\tanh\big(\mathrm{e}(t)\big), \end{aligned} \quad (11)$$

Where:

$$-\mathrm{e}^T(t)\mathrm{Q}(t)\mathrm{e}(t) = -\mathrm{e}^T(t)\big(\mathrm{P}_1(t)-\mathrm{P}_2(t)\big)\mathrm{e}(t), \quad (12)$$

$$\mathrm{P}_1(t) = \begin{bmatrix}\alpha_1+a & 0 & 0 & 1\\ 0 & \alpha_2+b & 0 & 0\\ 0 & 0 & \alpha_3+c & 0\\ 0 & 0 & 0 & \alpha_4+d\end{bmatrix}, \quad (13)$$

$$\begin{aligned}\mathrm{e}^T(t)\mathrm{F}\big(\mathrm{x}(t),\mathrm{y}(t),\mathrm{e}(t)\big) &= y_2(t)e_1^2(t) + x_1(t)e_1(t)e_2(t) - \big(x_1(t)+y_1(t)\big)e_1(t)e_2(t)\\ &\quad -\vartheta y_2(t)e_1(t)e_4(t) - \vartheta x_1(t)e_2(t)e_4(t)\\ &= \mathrm{e}^T(t)\mathrm{P}_2(t)\mathrm{e}(t),\end{aligned} \quad (14)$$

$$\mathrm{P}_2(t) = \begin{bmatrix} y_2(t) & -y_1(t) & 0 & -\vartheta y_2(t)\\ 0 & 0 & 0 & -\vartheta x_1(t)\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \quad (15)$$

And,

$$\mathrm{Q}(t) = \mathrm{P}_1(t) - \mathrm{P}_2(t) = \begin{bmatrix}\alpha_1+a-y_2(t) & y_1(t) & 0 & 1+\vartheta y_2(t)\\ 0 & \alpha_2+b & 0 & \vartheta x_1(t)\\ 0 & 0 & \alpha_3+c & 0\\ 0 & 0 & 0 & \alpha_4+d\end{bmatrix}. \quad (16)$$

Remark 3. Since $\mathrm{e}^T(t)\beta\big(\mathrm{I}-\rho\delta(t)\big)\tanh\big(\mathrm{e}(t)\big) \geq 0$ for $\beta_{ii} \geq 1$ and selecting $\alpha_1 \geq y_2(t) - a$, $\alpha_2, \alpha_3 \geq 0$ confirms that $\mathrm{e}^T(t)\mathrm{Q}(t)\mathrm{e}(t) \geq 0$. Hence, it is established that $\dot{V}(t) \leq 0$. Thus, closed-loop system (8) remains stable at the origin globally. Therefore, $\lim_{t\to\infty}\|\mathrm{e}(t)\| = 0$.

4. NUMERICAL SIMULATIONS AND RESULTS DISCUSSION

The drive-response HF systems' (2-3) initial conditions are set as $x_1(0) = -0.5$, $x_2(0) = 1$, $x_3(0) = 1$, $x_4(0) = 2$, $y_1(0) = -1$, $y_2(0) = 2.5$, $y_3(0) = 1.7$, $y_4(0) = 1$. The controller (6) parameters are set as $\alpha_{ii} = \beta_{ii} = 1$, $\rho = 0.1$, and $\varrho = 0.01$.

4.1. Example 1

This example aims to analyze the performance of the presented TENLC approach (7). Figure 5 shows the behaviour of closed loop's state variable trajectories behaviour without and with control effort. Figure 5(a) depicts the transient behaviour of the error trajectories (8) without applying any control effort. As depicted in Figure 5(a), the error trajectories exhibit significant oscillations and diverge considerably from the origin.

Figure 5(b) depicts the transient behaviour of the error trajectories (8). The introduction of control input (6) to the response HF system (3) effectively lessens active oscillations, allowing state error trajectories to converge smoothly to zero in 2 seconds as shown in Figure 5(b). The control input signals behaviour computed by control effort (7) is illustrated in Figure 5(c), showing less oscillatory and quickly reaching zero steady-state.

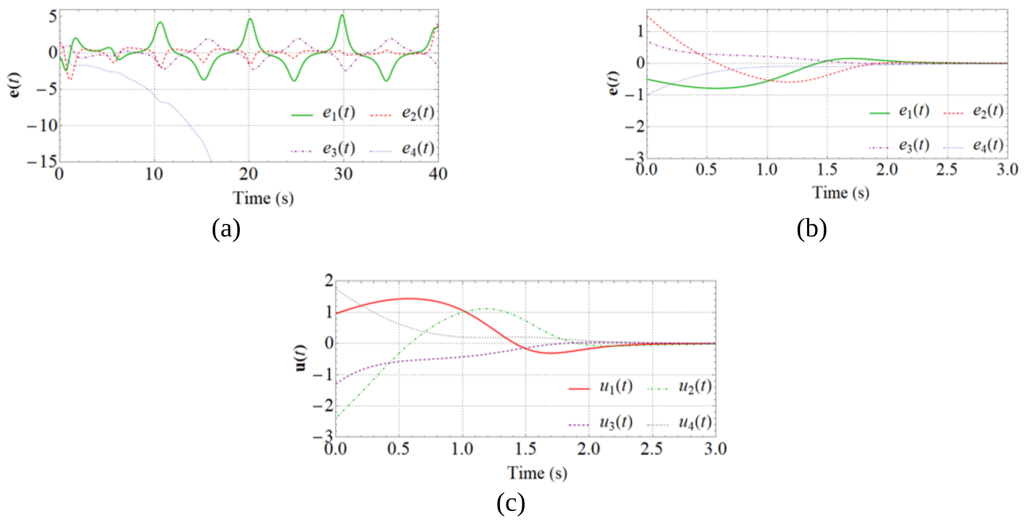

Figure 5. Transient behaviour of the error trajectories and control input signals; (a) error signals behaviour without any control effort, (b) convergence of the synchronization error using control effort (7) and (c) transient behaviour of the control input (7)

4.2. Example 2: comparative study

This example presents a comparative analysis utilizing numerical simulation results to evaluate the efficiency of the proposed TENLC approach (7) with the previously established control scheme [24] described in (17). The state-feedback control strategy $u(t) \in R^{4\times1}$ in (17) is added to the response HF system (3). Further, the controller parameters, HF system's (1) parameters, and initial conditions are set the same for the benchmark.

$$u(t) = \begin{bmatrix} e_3(t) - y_2(t)e_1(t) - x_1(t)e_2(t) \\ e_1(t)\big(x_1(t) + y_1(t)\big) \\ 0 \\ \vartheta\big(y_2(t)e_1(t) + x_1(t)e_2(t)\big) \end{bmatrix} \tag{17}$$

Figure 6 illustrates the transient behavior of the errors and control input signals by the control effort (17). Figure 6(a) shows the synchronization error trajectories behaviour by the control effort (17), illustrating that synchronization takes a long time. Slow convergence rates can lead to unpredictable behavior in closed-loop systems, often caused by oscillation or instability. Additionally, these systems are more vulnerable to external disturbances, which can result in a loss of synchronization. Figure 6(b) depicts the control input (17) transient behaviour that demonstrates initial signal oscillations and takes approximately 18 seconds to reach the steady-state. Figures 5(b) and (c) and Figures 6(a) and (b) illustrate that the controller (7) realizes the HF systems (2-3) synchronization in less time, and the control signals reach zero steady-state quickly with reduced active oscillations than the controller (17).

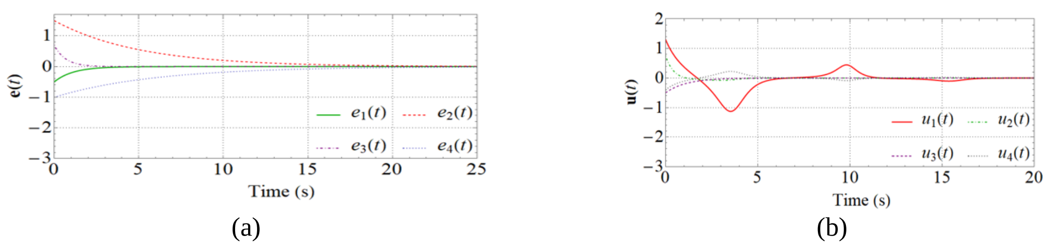

Figure 6. Transient behaviour of the error trajectories and control input signals; (a) synchronization errors and (b) control input signals

Figure 7 depicts the comparison of the energy functions and gradient of the Lyapunov function candidates. Figures 7(a) and (b) depicts the comparative simulation results of the energy functions $\varphi(t) =$

$\varphi_1(t) = \sum_{i=1}^{3} e_i(t)$ obtained by the TENLC approach (7) and control strategy (17) for 2 seconds and 12 seconds, respectively. As shown in Figure 7(a), the proposed TENLC technique (7) takes less than 2 seconds to drive the system synchronization error energy to zero, whereas the controller attains the same after 11 seconds, as shown in Figure 7(b). These comparative computer simulations clearly demonstrate the efficiency advantages of the presented TENCL technique (7) over the controller in (17). This advancement addresses the challenges posed by financial markets' complex and fluctuating nature, where traditional control methods often struggle to maintain stable synchronization. The behaviour of energy dissipation rates by the controllers (7) and (17) is compared for 2 seconds and 4.5 seconds, respectively, as depicted in Figures 7(c) and (d). The time gradient of the energy function by (7) is denoted by $\dot{V}(t)$ and $\dot{V}_1(t)$ by (17). Figure 7(c) shows that $\dot{V}(t) = 0$ after 1.8 seconds, and the closed-loop remains at zero level forever. $\dot{V}_1(t) = 0$ depicts that controller (12) takes 4 seconds to make the energy level zero, as illustrated in Figure 7(d). This comparative analysis demonstrates that controller (7) utilizes less energy for faster convergence of the synchronization error to zero with reduced fluctuations than (17). Precise synchronization of diverse financial instruments enables more effective portfolio management strategies, optimizing risk and return profiles.

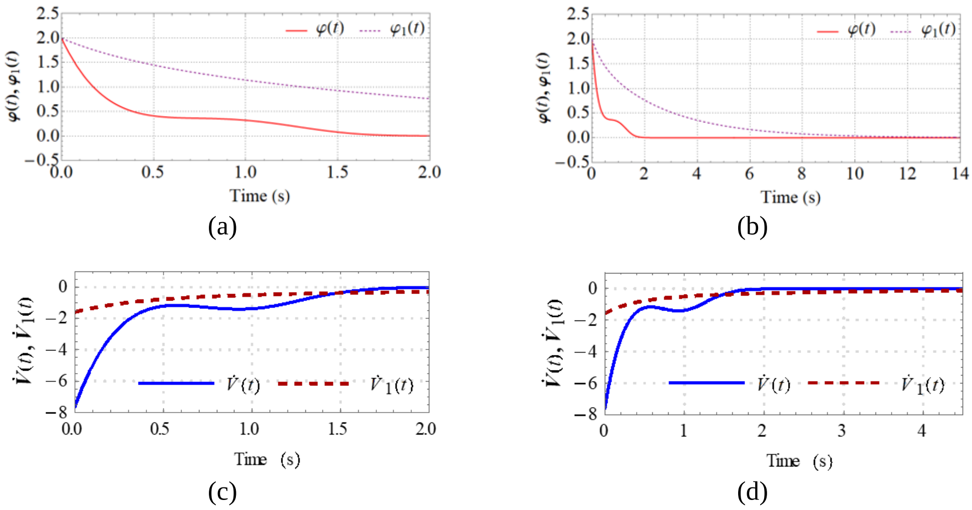

Figure 7. Comparison of the energy functions and gradient of the Lyapunov function candidates: (a) $\varphi(t)$, (b) $\varphi_1(t)$, (c) $\dot{V}(t)$, and (d) $\dot{V}_1(t)$

5. CONCLUSIONS

This study introduces an efficient and time-effective robust nonlinear control algorithm, employing a drive-response system model for achieving synchronization in two hyperchaotic finance systems. The algorithm retains the closed-loop's nonlinear terms and achieves global stable synchronization with reduced fluctuations and faster convergence rates. The Lyapunov stability analysis ensures that the closed-loop is globally stable. The proposed nonlinear control scheme is designed for quick convergence of state errors to zero and reduces input and output signal fluctuations. The control scheme's rigorous theoretical analysis is based on the Lyapunov stability principles and validated by mathematical proofs and computer simulations. Additionally, extensive comparative simulations have demonstrated the superiority of this synchronization approach compared to existing methods for the same chaotic system. The proposed approach shows high performance compared to existing methods in terms of convergence speed and synchronization behavior. The research team will focus on exploring fixed-time synchronization of fractional-order uncertain hyperchaotic financial systems in the future.

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