

Optimal Asymmetric S-shape Acceleration/Deceleration for Multi-axial Motion Systems

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Abstract

In this study, an optimization algorithm is proposed for asymmetric s-shape acceleration/deceleration to achieve better contour accuracy for biaxial systems. The optimization is based on the method of genetic algorithm incorporated with the constraints made by the motion system. Numerical simulations of an XY table driven by linear motors following a simple cornering path verify the effectiveness of the proposed algorithm.

Keywords: genetic algorithm, contouring error, interpolation, s-shape acceleration/deceleration

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1. introduction

Acceleration/deceleration (acc/dec) algorithm plays an important role in CNC machining. There are several methods, such as linear acc/dec, exponential acc/dec, trigonometric function acc/dec, and s-shape acc/dec, etc. [1], [2]. Among them, the linear acc/dec is the most commonly used one because of easy calculation. However, the sudden change of acceleration can cause the shocking vibration and low accuracy in the motion control, which is not suitable for high-speed and high-precision contouring. On the contrary, the s-shape acc/dec is more and more recognized because of its smooth velocity profile that can reduce the jerk limitation **Error! Reference source not found.** Shi *et al.* implement the effect of the s-shape acc/dec [1] and Hao *et al.* transform the linear acc/dec into the s-shape form [2]. Furthermore, Cao *et al.* integrate the look-ahead strategy with the s-shape acc/dec **Error! Reference source not found.** All of these recent studies point out the development of acc/dec control algorithm and the advantages of s-shape acc/dec.

In this article, the asymmetric s-shape acc/dec is introduced in section 2. It has a more flexible adjustment of the acc/dec intervals. There are four parameters in a acc/dec process with the constraint of the admissible set. We investigate the conditions of the practicability and analyze the bounds of these acc/dec parameters according to the path information and the limitation of this motion system. In section 3, the genetic algorithm **Error! Reference source not found.** is adopted to find the optimal set of acc/dec parameters in the same tracking performance of the motion system. The propagation is also discussed in detail. A numerical simulation of an XY table driven by linear motors is designed for validation of the proposed genetic algorithm by the cornering path, and a block change criterion "exact stop fine", is integrated to deal with the corner error. The setup and results are given in the section 4. Finally, conclusions are drawn in section 5.

2. Asymmetric S-shape Acc/Dec

This study proposes an optimization algorithm for asymmetric s-shape acc/dec using the method of genetic algorithm with the constraints made by the motion system. The algorithm is verified numerically in the simulations of an XY table driven by linear motors following a simple cornering path with the block change criterion "exact stop fine" to deal with the corner errors. The simulation results show that the contouring error and corner error can be reduced by meta-genetic cycle and the effect of asymmetric S-shape acc/dec is superior to the symmetric specification. We can obtain the optimal parameters of s-shape acc/dec.

There are two fields in the contouring problem which include interpolation and trajectory tracking control. The interpolation is to generate serial commands leading the controlled components to follow the desired path. The process of interpolate in a block (a row of code) contains acceleration, constant speed, and finally deceleration. This is the so-called acceleration/deceleration (acc/dec). There are several shapes of acc/dec, and it can be an option according to the different applications. There are several shapes of acc/dec, and the most universal shape of acc/dec is the trapezoidal velocity profile. According to the defined (acceptable) axis-acceleration, trapezoidal acc/dec accelerates to feed-rate and finally decelerates to zero-velocity at the end-point. The advantage of trapezoidal is time-optimized; the path will be finished in the shortest time. But, the efficiency also comes with the discontinuous of the acceleration profile that cause the infinity jerk which shock the system on the contrary. This impact will wear the system and reduce the accuracy in the contouring process. Another shape of acc/dec is called "s-shape acc/dec" which possesses a triangular acceleration profile as shown in Figure 1. In other words, the jerk is piecewise constants. It will lead to the smooth, s-shaped velocity profile shown in Figure 2. Compared with the commonly used trapezoidal acc/dec, it can generate more smooth commands, resulting in better accuracy. However, the s-shape acc/dec involves more parameters, making the tuning process more difficult. By application of acc/dec before interpolation, there are 4 parameters of s-shape acc/dec:

- T_{s1} : Time Interval from A to B
- T_{s2} : Time Interval from B to C
- T_{e1} : Time Interval from D to E
- T_{e2} : Time Interval from E to F

In a curve. If we consider the case of a corner path, there will be 8 parameters to optimize the contouring and corner errors at the same time (the curve before the corner and after). Most applications assume the s-shape acc/dec to be symmetric, i.e., the acceleration time interval (from A to C) equals deceleration one (from D to F). In this work, asymmetric shape is considered to keep the parameter tuning more flexible.

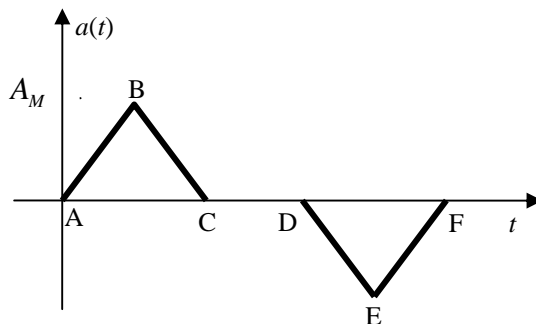


Figure 1. Acceleration Profile

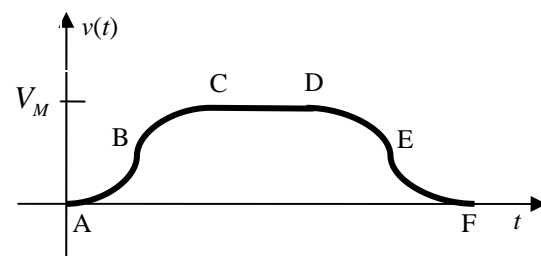


Figure 2. Velocity Profile

$$a(t) = \begin{cases} J_{s1}t, & 0 \leq t \leq T_{s1} \\ (J_{s1} + J_{s2})T_{s1} - T_{s2}t, & T_{s1} \leq t \leq T_{s1} + T_{s2} \end{cases} \quad (1)$$

Where J_{s1} is the jerk of the interval from A to B, and J_{s2} is the jerk of the interval from B to C. Due to the restriction of mathematical representation, we only derive the equations of the acceleration zone (from A to C). The deceleration zone is similar to the acceleration part. (Replace T_{s1} and T_{s2} with T_{e2} and T_{e1} , respectively). From Equation (1), we can get the velocity of acceleration interval.

$$v(t) = \begin{cases} \frac{1}{2} J_{s1} t^2, & 0 \leq t \leq T_{s1} \\ -\frac{1}{2} J_{s2} t^2 + (J_{s1} + J_{s2}) T_{s1} t - \frac{1}{2} (J_{s1} + J_{s2}) T_{s1}^2, & T_{s1} \leq t \leq T_{s1} + T_{s2} \end{cases}$$

And the total displacement from A to C.

$$L_s = \int_0^{T_{s1}+T_{s2}} v(t) dt \quad (2)$$

$$= \frac{1}{6} J_{s1} T_{s1}^3 + \frac{1}{2} J_{s1} T_{s1} T_{s2} (T_{s1} + T_{s2}) - \frac{1}{6} J_{s2} T_{s2}^3$$

For the four parameters (time intervals) to be adjusted in an acc/dec process, some constraint conditions must be imposed. Assume that the total length of path is L , maximum feed-rate is V_M , maximum acceleration is A_M , and maximum jerk is J_M . These constraints restrict the upper and lower bounds of parameters as follow:

- 1) The total movement of acceleration and deceleration time interval (from A to C and from D to F) should be less than L .
- 2) The velocity must arrive to V_M in the constant velocity time interval (C to D).
- 3) Acceleration and deceleration cannot be larger than A_M .
- 4) Jerk can't be larger than J_M .

Here L , V_M , A_M , and J_M are given constants. According to the Figure 1 and 2), we have:

$$A_s = J_{s1} T_{s1} = J_{s2} T_{s2}, \quad \frac{1}{2} A_s (T_{s1} + T_{s2}) = V_M \quad (3)$$

Hence,

$$A_s = \frac{2V_M}{T_{s1} + T_{s2}} \leq A_M \Rightarrow T_{s1} + T_{s2} \geq \frac{2V_M}{A_M} \quad (4)$$

By constraint 3). Combining Equation (3) with Equation (4), we obtain the jerk equation:

$$J_{s1} = \frac{2V_M}{T_{s1}(T_{s1} + T_{s2})}, \quad J_{s2} = \frac{2V_M}{T_{s2}(T_{s1} + T_{s2})} \quad (5)$$

Furthermore,

$$T_{s1}(T_{s1} + T_{s2}) \geq \frac{2V_M}{J_M}, \quad T_{s2}(T_{s1} + T_{s2}) \geq \frac{2V_M}{J_M} \quad (6)$$

Is available by constraint 4). Addition of the two equation in Equation (5), we have:

$$J_{s1} + J_{s2} = \frac{2V_M}{T_{s1} T_{s2}} \leq 2J_M \Rightarrow T_{s1} T_{s2} \geq \frac{V_M}{J_M} \quad (7)$$

Finally, we obtain the total displacement of acceleration interval.

$$L_s = \frac{1}{3} V_M (T_{s1} + 2T_{s2}) \quad (8)$$

By substituting Equation (5) into Equation (2).

In the same way, the restrictions of parameters in deceleration interval are also available.

$$T_{e1} + T_{e2} \geq \frac{2V_M}{A_M} \quad (9)$$

$$T_{e1}T_{e2} \geq \frac{V_M}{J_M} \quad (10)$$

$$L_e = \frac{1}{3}V_M(2T_{e1} + T_{e2}) \quad (11)$$

By constraint 1), $L_s + L_e \leq L$.

$$T_{s1} + 2T_{s2} + 2T_{e1} + T_{e2} \leq \frac{3L}{V_M} \quad (12)$$

It also means:

$$T_{s1} + 2T_{s2} \leq \frac{3L}{V_M} \quad (13)$$

As a result, acceleration parameters (T_{s1}, T_{s2}) must satisfy the equations (4), (7), and (13). In other words, the intersection of Equation (4), (7), and (13) should be the non-empty set in the two-dimensional space composed of (T_{s1}, T_{s2}) . Then, we can apply the s-shape acc/dec in this case and find the optimal solution.

By the simple calculation of algebraic geometry, we can obtain the necessary conditions of the s-shape acc/dec are:

$$8V_M^3 < 9J_M L^2 \text{ and } \frac{8V_M^2}{A_M} < 9L + \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \quad (14)$$

Equation (14) shows that the admissible set won't exist if the feed-rate, V_M , is too large or the limit of L , A_M , and J_M are too small. These results are reasonable. Under the condition of Equation (14), there are exactly two intersection points crossed by the boundary of the Equation (7) and Equation (13). These two points are:

$$Q_1 : \left(\frac{3L}{2V_M} - \frac{1}{2V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}}, \frac{3L}{4V_M} + \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \right)$$

$$Q_2 : \left(\frac{3L}{2V_M} + \frac{1}{2V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}}, \frac{3L}{4V_M} - \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \right)$$

If the boundary of Equation (4) pass through Q_1 , this results:

$$\frac{2V_M}{A_M} = \frac{9L}{4V_M} - \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}}$$

Or if the boundary of Equation (4) pass through Q_2 , this results:

$$\frac{2V_M}{A_M} = \frac{9L}{4V_M} + \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}}$$

We have the similar results in deceleration part (T_{e1}, T_{e2}) .

Synthesizing these analyses of acc/dec parameters above, there are three possible cases of the admissible set:

(i) Case 1: $\frac{2V_M}{A_M} \leq \sqrt{\frac{4V_M}{J_M}}$. As shown in Figure 3, the admissible set is surrounded by the boundaries of Equation (7) and Equation (13). Then, the upper and lower bound of the acc/dec parameters are limited by the points, (Q_1, Q_2) . That is:

$$\frac{3L}{2V_M} - \frac{1}{2V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \leq T_{s1} \text{ (or } T_{e2}) \leq \frac{3L}{2V_M} + \frac{1}{2V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \quad (15)$$

$$\frac{3L}{4V_M} - \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \leq T_{s2} \text{ (or } T_{e1}) \leq \frac{3L}{4V_M} + \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \quad (16)$$

(ii) Case 2: $\sqrt{\frac{4V_M}{J_M}} \leq \frac{2V_M}{A_M} \leq \frac{9L}{4V_M} - \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}}$. As shown in Figure 4, the admissible set is the same region as (i).

(iii) Case 3: $\frac{9L}{4V_M} - \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \leq \frac{2V_M}{A_M} \leq \frac{9L}{4V_M} + \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}}$. As shown in Figure 5, there is an intersection point.

$$Q_3 : \left(\frac{4V_M}{A_M} - \frac{3L}{V_M}, \frac{3L}{V_M} - \frac{2V_M}{A_M} \right)$$

Crossed by the boundary of eq(4) and eq(13). This situation causes the upper and lower bound become:

$$\frac{4V_M}{A_M} - \frac{3L}{V_M} \leq T_{s1} \text{ (or } T_{e2}) \leq \frac{3L}{2V_M} + \frac{1}{2V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \quad (17)$$

$$\frac{3L}{4V_M} - \frac{1}{4V_M} \sqrt{9L^2 - \frac{8V_M^3}{J_M}} \leq T_{s2} \text{ (or } T_{e1}) \leq \frac{3L}{V_M} - \frac{2V_M}{A_M} \quad (18)$$

By synthesizing the discussions, we have the conclusions below:

1. Given the data, L , V_M , A_M and J_M , we can check whether the eq(14) is satisfied or not. If not, the setting of the path or feed-rate should be modified.

2. In order to meet the constraint 1 to 4, these acc/dec parameters should satisfy the Equation (4), (7), (8), (12) and (13).

3. According to the given information like L , V_M , A_M and J_M , roughly analysis the upper and lower bound of the acc/dec parameters in admissible set by the intersection set of the Equation (4), (7), and (13). However, it's just the beginning if we want to randomly choose a set of parameters satisfying the upper and lower bound. We still need to check if the set of data, $(T_{s1}, T_{s2}, T_{e1}, T_{e2})$, locate in the admissible set or not.

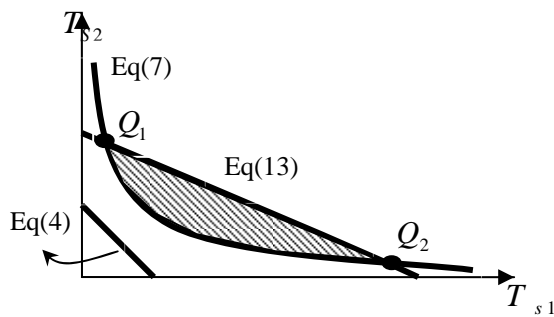


Figure 3. Case 1 of the Admissible Set

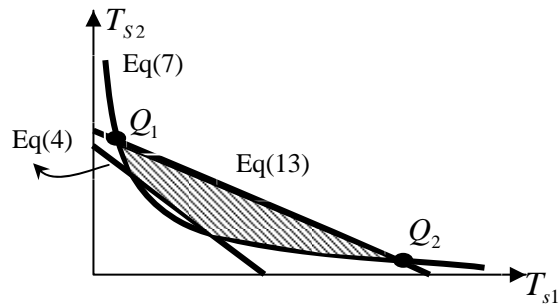


Figure 4. Case 2 of the Admissible Set

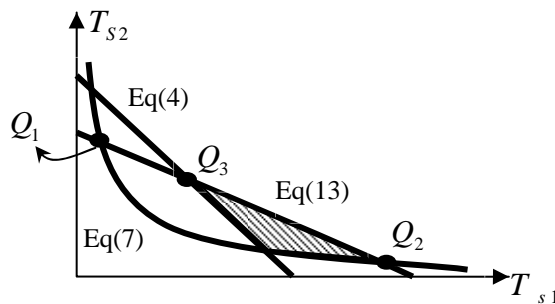


Figure 5. Case 3 of the Admissible Set

3. Optimization By Genetic Algorithm

This study proposes an optimization algorithm for asymmetric s-shape acc/dec using the method of genetic algorithm with the constraints made by the motion system. The algorithm is verified numerically in the simulations of an XY table driven by linear motors following a simple cornering path with the block change criterion “exact stop fine” to deal with the corner errors. The simulation results show that the contouring error and corner error can be reduced by meta-genetic cycle and the effect of asymmetric s-shape acc/dec is superior to the symmetric specification. We can obtain the optimal parameters of s-shape acc/dec.

The origin of Genetic algorithm comes from the Darwin’s “Natural Selection” and “Survival of the Fittest”. In 1960, John Holland applied these concepts, including propagation, mutation, and selection repetitively, to search the optimal solution by mathematical calculation. Recently, it has been broadly applied to search all kinds of optimal problems.

The procedure of the genetic algorithm is listed as follow:

- Step 1) Random initialization of population
- Step 2) The fittest of chromosome in fitness function
- Step 3) Natural selection
- Step 4) Crossover
- Step 5) Mutation

Repeating the step 2) to 5), the superior offspring may appear and it’s possible to get the best one. In section 2, the upper and lower bound of the acc/dec parameters have been approximately estimated. We want to search the optimal solution in the admissible set of the space constructed by T_{s1} , T_{s2} , T_{e1} , and T_{e2} . Because a set of parameters $(T_{s1}, T_{s2}, T_{e1}, T_{e2})$ which satisfy the bound are not necessary satisfying the admissible set, the genetic algorithm which we apply to find the optimal solution of s-shape acc/dec should be modified to adapt the constraints. We will go into detail as follows.

Step 1) Random initialization: According to the data, L, V_M, A_M and J_M , randomly generate 4 chromosomes $(T_{s1}, T_{s2}, T_{e1}, T_{e2})$ of the initial populations in the interval with the lower and upper bound analyzed in section 2, until we have 8 sets of acc/dec parameters, which

belong to the admissible set (omit the non-coincident one). Then, use the binary encoding for these initial populations in 10 bits (gene) with 0000000000 corresponding to the lower bound and 1111111111 corresponding to the upper bound.

Step 2) Survival of the Fittest: Decode these chromosomes of population and simulate the case of s-shape acc/dec. Calculate the contouring errors and corner error from the simulation results. Then, compare the fitness of each population by:

$$Fitness = 10000 * (0.8 * contouring\ error + 0.2 * corner\ error)$$

Where 10000 is a multiplier to increase the sensitivity of the fitness function.

Step 3) Natural selection: We reserve 4 superior populations (selection rate=50%) to be the survival base on the fitness, and weight these four population to generate the offspring possibly by the weighting function.

$$P_n = \frac{N_{keep} - n + 1}{\sum_{n=1}^{N_{keep}} n}$$

Where N_{keep} is the population number which we reserve, and P_n is the probability of the n -th individual chosen to propagate.

Step 4) Crossover: We randomly generate two numbers from the interval [0, 1] twice. According to the weighting probability, we have two pairs to propagate 4 filial generations. The method of propagation is single point crossover, and the cross point is random. So, we have totally 8 samples (4 populations and 4 filial generations) after propagation.

Step 5) Mutation: Except the optimal generation, the mutation possibly occurs to all gene of each individual. The numbers of mutation is controlled by the mutation rate, $\mu = 5\%$ (usually). Then,

$$Mutation\ numbers = \mu \times (N_{pop} - 1) \times N_{bits}$$

Where $N_{bits} = N_{chromosome} \times N_{gene}$ is the total bits of the individual. We randomly mutate the gene (1 becomes to 0 or contrarily) directly and repeat to the mutation numbers. Finally, we decode the new mutative generation and check if the set of acc/dec parameters locate in the admissible set or not. (If not, we throw it out.) In this way, we execute step 2) to step 5) repetitively to find the optimal solution.

4. Simulation Verification

Considering a complete motion control system, it includes command code, acc/dec, interpolation, and plant with servo loop control, as shown in Figure 6. The plant is the XY table driven by linear motors which refer to the manual of Siemens. We choose the motor type "1FN3" and cite the excogitative parameters. After the servo loop of the PPI control, the bandwidth of the position is about 100Hz.

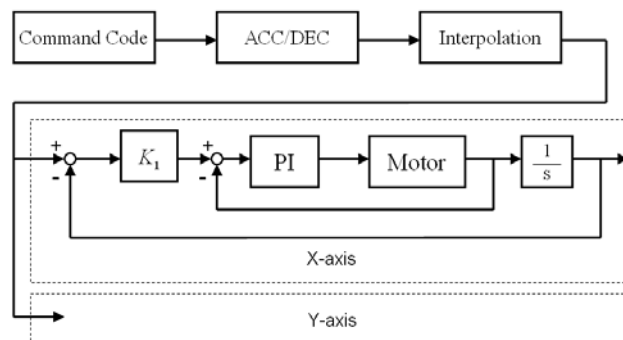


Figure 6. The Servo Loop of the XY Table

We are planning to contour a turning line by this XY table. It starts at the original point $(0, 0)$, and track a straight line with the length is L meter to the corner point along the direction of 45° upper right. Then, turn to the upper left side and track the other straight line to the endpoint. We adopt the criterion of exact stop fine to deal with the problem of corner accuracy. When both of the tolerances of X and Y-axis are less than 10^{-5} meter, the action of block change is executed to interpolate the second straight line. These two lines are specified to acceleration and deceleration by the S-shape acc/dec specification (analyzed in section 2), and the parameters of acc/dec time, T_{s1}, T_{s2}, T_{e1} , and T_{e2} are calculated optimally by the genetic algorithm discussed in section 3.

In this simulation, we set the length $L=0.1$ meter, and the feed-rate $V_{\max} = 6000\text{mm}/\text{min}$ ($0.1\text{ meter}/\text{sec}$). The limitation of acceleration, A_{\max} , is $1\text{m}/\text{s}^2$ and the jerk limitation, J_{\max} , is $10\text{m}/\text{s}^3$. There are 4 parameters, T_{s1}, T_{s2}, T_{e1} , and T_{e2} of the first straight line, and another 4 parameters, $T_{s12}, T_{s22}, T_{e12}$, and T_{e22} of the second one. So, there are totally 8 chromosomes in this case. The upper and lower bounds are defined by the conditions introduced in the section 3.

The section of the simulation about genetic algorithm follow the optimization method discussed in section 3 as a model. After the propagations of two hundreds generations passed through, the optimal parameters of the S-shape acc/dec are obtained. These solutions are $T_{s1} = 0.3191$, $T_{s2} = 0.1397$, $T_{e1} = 0.0228$, $T_{e2} = 1.0217$, $T_{s12} = 2.9328$, $T_{s22} = 0.4348$, $T_{e12} = 0.7037$, and $T_{e22} = 0.5424$, and the corresponding contouring error is $4.0348 \times 10^{-7}\text{m}$ and corner error is $3.9319 \times 10^{-7}\text{m}$. Figure 7 is the evolution of the fitness function, and Figure 8 shows the contouring performance.

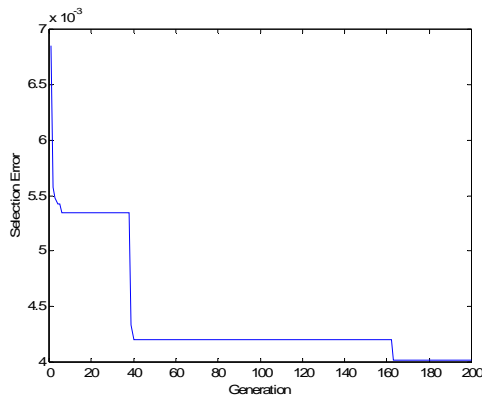


Figure 7. The Evolution of the Fitness Function

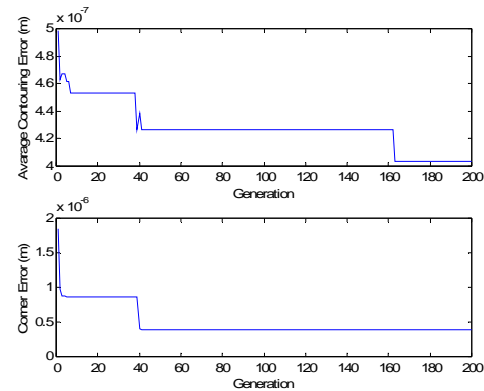


Figure 8. Optimal Performance: (a) the evolution of the average contouring error; (b) the evolution of the corner

5. Conclusion

This study proposes an optimization algorithm for asymmetric s-shape acc/dec using the method of genetic algorithm with the constraints made by the motion system. The algorithm is verified numerically in the simulations of an XY table driven by linear motors following a simple cornering path with the block change criterion “exact stop fine” to deal with the corner errors. The simulation results show that the contouring error and corner error can be reduced by meta-genetic cycle and the effect of asymmetric s-shape acc/dec is superior to the symmetric specification. We can obtain the optimal parameters of s-shape acc/dec.

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