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Robust Centralized Fusion Kalman Filters with Uncertain Noise Variances

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Abstract

This paper studies the problem of the designing the robust local and centralized fusion Kalman filters for multisensor system with uncertain noise variances. Using the minimax robust estimation principle, the centralized fusion robust time-varying Kalman filters are presented based on the worst-case conservative system with the conservative upper bound of noise variances. A Lyapunov approach is proposed for the robustness analysis and their robust accuracy relations are proved. It is proved that the robust accuracy of robust centralized fuser is higher than those of robust local Kalman filters. Specially, the corresponding steady-state robust local and centralized fusion Kalman filters are also proposed and the convergence in a realization between time-varying and steady-state Kalman filters is proved by the dynamic error system analysis (DESA) method and dynamic variance error system analysis (DVESA) method. A Monte-Carlo simulation example shows the robustness and accuracy relations.

Keywords: *multisensor information fusion, centralized fusion, uncertain noise variance, minimax robust Kalman filter*

1. Introduction

The aim of the multisensor information fusion is how to combine the local estimators or local measurements to obtain the fused estimators, whose accuracy is higher than that of each local estimator [1]. For the centralized fusion optimal Kalman filter, all the local measurement data are carried to the fusion centre to obtain a globally optimal fused state estimation [2].

The darwback of the Kalman filter is that it only suitable to handle the state estimation problmes for systems with exact model parameters and noise variances. However, in many application problems, there exist uncertainties of the model parameters and/or noise variances. Under these uncertainties the performance of the Kalman filter will degrade [3], and an inexact model may cause the filter to diverge. This has motivated the designing of the robust Kalman filters, which guarante to have a minimal upper bound of the actual filtering error variances for all admissible uncertainties.

In order to design the robust Kalman filters for the systems with the model parameters uncertainties, two important approaches are the Riccati equation approach [4-6] and the linear matrix inequality (LMI) approach [7-9]. The disadvantage of these two approaches is that only model parameters are uncertain while the noise variances are assumed to be exactly known. The robust Kalman filtering problems for systems with uncertain noise variances are seldom considered [10, 11], and the robust information fusion Kalman filter are also seldom researched [12, 13].

In this paper, using the minimax robust estimation principle, the local and centralized fusion robust time-varying and steady-state Kalman filters are presented based on the worst-case conservative system with the conservative upper bound of noise variances. The convergence in a realization between the time-varying and steady-state Kalman filters is rigorously proved by the dynamic error system analysis (DESA) method [14] and dynamic variance error system analysis (DVESA) method [15]. Furthermore, a Lyapunov equation approach is presented for the robustness analysis, which is different from the Riccati equation approach and the LMI approach. The concept of the robust accuracy is given and the robust accuracy relations are proved, it is proved that the robust accuracy of the centralized fuser is higher than that of the local robust Kalman filter.

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The remainder of this paper is organized as follows. Section 2 gives the problem formulation. The robust centralized fusion time-varying Kalman filters are presented in Section 3. The robust local and centralized fusion steady-state Kalman filters are presented in Section 4. The robust accuracy analysis is given in Section 5. The simulation example is given in Section 6. The conclusion is proposed in Section 7.

2. Problem Formulation

Consider the muiltisensor linear disceret time-varying system with uncertain noise varaince.

$$x(t+1)=\Phi(t)x(t)+\Gamma(t)w(t) \tag{1}$$

$$y_i(t)=H(t)x(t)+\eta(t)+\xi_i(t), i=1,\cdots,L \tag{2}$$

Where t represents the discrete time, $x(t)\in R^n$ is the state, $y_i(t)\in R^{m_i}$ is the measurement of the i th subsystem, $w(t)\in R^r$ is the input noise, $\eta(t)$ is the common disturbance noise, $\xi_i(t)\in R^{m_i}$ is the measurement noise of the i th subsystem, $\Phi(t)$, $\Gamma(t)$ and $H(t)$ are known time-varying matrices with appropriate dimensions. L is the number of sensors.

Assumption 1. $w(t)$, $\eta(t)$ and $\xi_i(t)$ are uncorrelated white noises with zero means and unknown uncertain actual variances $\bar{Q}(t)$, $\bar{R}_\eta(t)$ and $\bar{R}_{\xi_i}(t)$ at time t, respectively, $Q(t)$, $R_\eta(t)$ and $R_{\xi_i}(t)$ are known conservative upper bounds of $\bar{Q}(t)$, $\bar{R}_\eta(t)$ and $\bar{R}_{\xi_i}(t)$, satisfying:

$$\bar{Q}(t)\le Q(t),\bar{R}_\eta(t)\le R_\eta(t),\bar{R}_{\xi_i}(t)\le R_{\xi_i}(t), i=1,\cdots,L, \forall t \tag{3}$$

Assumption 2. The initial state $x(0)$ is independent of $w(t)$, $\eta(t)$ and $v_i(t)$ and has mean value μ and unknown uncertain actual variance $\bar{P}(0|0)$ which satisfies:

$$\bar{P}(0|0)\le P(0|0) \tag{4}$$

Where $P(0|0)$ is a known conservative upper bound of $\bar{P}(0|0)$.

Assumption 3. The system (1) and (2) is uniformly completely observable and completely controllable.

Defining:

$$v_i(t)=\eta(t)+\xi_i(t), i=1,\cdots,L \tag{5}$$

Where $v_i(t)$ are white noises with zero means and the conservative and actual variances are given as:

$$R_{v_i}(t)=R_\eta(t)+R_{\xi_i}(t), \bar{R}_{v_i}(t)=\bar{R}_\eta(t)+\bar{R}_{\xi_i}(t), i=1,\cdots,L \tag{6}$$

$$R_{v_{ij}}(t)=R_\eta(t), \bar{R}_{v_{ij}}(t)=\bar{R}_\eta(t), i\ne j \tag{7}$$

From (3), we have:

$$\bar{R}_{v_i}(t)\le R_{v_i}(t), i=1,\cdots,L, \forall t \tag{8}$$

3. Robust Centralized Fusion Time-varying Kalman Filters

Introduce the centralized fusion measurement equation:

$$y_c(t) = H_c(t)x(t) + v_c(t) \tag{9}$$

With the definition:

$$y_c(t) = \left[y_1^{\mathrm{T}}(t),\cdots,y_L^{\mathrm{T}}(t)\right]^{\mathrm{T}}, H_c(t) = \left[H^{\mathrm{T}}(t),\cdots,H^{\mathrm{T}}(t)\right]^{\mathrm{T}}, v_c(t) = \left[v_1^{\mathrm{T}}(t),\cdots,v_L^{\mathrm{T}}(t)\right]^{\mathrm{T}} \tag{10}$$

And $v_c(t)$ has the conservative and actual variance matrice R_c and $\bar{R}_c$ as:

$$R_c = \begin{bmatrix} R_{v_i} & R_\eta & \cdots & R_\eta \\ R_\eta & \ddots & \ddots & \vdots \\ \vdots & \ddots & R_{v_i} & R_\eta \\ R_\eta & \cdots & R_\eta & R_{v_i} \end{bmatrix}, \quad \bar{R}_c = \begin{bmatrix} \bar{R}_{v_i} & \bar{R}_\eta & \cdots & \bar{R}_\eta \\ \bar{R}_\eta & \ddots & \ddots & \vdots \\ \vdots & \ddots & \bar{R}_{v_i} & \bar{R}_\eta \\ \bar{R}_\eta & \cdots & \bar{R}_\eta & \bar{R}_{v_i} \end{bmatrix} \tag{11}$$

Therefore from (3) and (8), accroding to the Lemma 1 and Lemma 2 in Appendix, we obtain:

$$\bar{R}_c(t) \le R_c(t) \tag{12}$$

Based on the worst-case conservative system (1) and (9) with Assumptions 1-3 and conservative upper bounds $Q(t)$ and $R_c(t)$, the globally optimal centralized fused time-varying robust Kalman filters are given as:

$$\hat{x}_c(t|t) = \Psi_c(t)\hat{x}_c(t-1|t-1) + K_c(t)y_c(t) \tag{13}$$

$$\Psi_c(t) = \left[I_n - K_c(t)H_c(t)\right]\Phi(t-1) \tag{14}$$

$$K_c(t) = P_c(t|t-1)H_c^{\mathrm{T}}(t)\left[H_c(t)P_c(t|t-1)H_c^{\mathrm{T}}(t) + R_c(t)\right]^{-1} \tag{15}$$

$$P_c(t+1|t) = \Phi(t)P_c(t|t)\Phi^{\mathrm{T}}(t) + \Gamma(t)Q(t)\Gamma^{\mathrm{T}}(t) \tag{16}$$

The fused conservative filtering error variance $P_c(t|t)$ is given as:

$$P_c(t|t) = \left[I_n - K_c(t)H_c(t)\right]P_c(t|t-1) \tag{17}$$

It can be rewritten as the Lyapunov equation:

$$\begin{aligned} P_c(t|t) &= \Psi_c(t)P_c(t-1|t-1)\Psi_c^{\mathrm{T}}(t) + \left[I_n - K_c(t)H_c(t)\right] \\ &\quad \times \Gamma(t-1)Q(t-1)\Gamma^{\mathrm{T}}(t-1)\left[I_n - K_c(t)H_c(t)\right]^{\mathrm{T}} + K_c(t)R_c(t)K_c^{\mathrm{T}}(t) \end{aligned} \tag{18}$$

With the initial values $\hat{x}_c(0|0) = \mu$, and $P_c(0|0) = P(0|0)$, where I_n is the $n \times n$ identity matrix.

The actual prediction and filtering errors are obtained as:

$$\tilde{x}_c(t+1|t) = x(t+1) - \hat{x}_c(t+1|t) = \Phi(t+1)\tilde{x}_c(t|t) + \Gamma(t)w(t) \tag{19}$$

$$\tilde{x}_c(t|t) = x(t) - \hat{x}_c(t|t) = \left[I_n - K_c(t)H_c(t)\right]\tilde{x}_c(t|t-1) - K_c(t)v_c(t) \tag{20}$$

Substituting (19) into (20) yields:

$$\tilde{x}_c(t|t)=\Psi_c(t)\tilde{x}_c(t-1|t-1)+\left[I_n-K_c(t)H_c(t)\right]\Gamma(t)w(t-1)-K_c(t)v_c(t) \tag{21}$$

The actual fused filtering error variance $\bar{P}_c(t|t)=\mathrm{E}\left[\tilde{x}_c(t|t)\tilde{x}_c^{\mathrm{T}}(t|t)\right]$, according to (21), we have:

$$\begin{aligned}\bar{P}_c(t|t)&=\Psi_c(t)\bar{P}_c(t-1|t-1)\Psi_c^{\mathrm{T}}(t)+\left[I_n-K_c(t)H_c(t)\right]\\&\times\Gamma(t-1)\bar{Q}(t-1)\Gamma^{\mathrm{T}}(t-1)\left[I_n-K_c(t)H_c(t)\right]^{\mathrm{T}}+K_c(t)\bar{R}_c(t)K_c^{\mathrm{T}}(t)\end{aligned} \tag{22}$$

With the initial value $\bar{P}_c(0|0)=\bar{P}(0|0)$.

Theorem 1. For multisensor uncertain system (1) and (9) with Assumptions 1-3, the actual centralized fusion time-varying Kalman filters with the conservative upper bound $Q(t)$, $R_c(t)$ and $P_c(0|0)$ are robust in the sense that for all admissible actual variances $\bar{Q}(t),\bar{R}_c(t)$ and $\bar{P}_c(0|0)$ satisfying (3), (4) and (12), for arbitrary time t, we have:

$$\bar{P}_c(t|t)\le P_c(t|t) \tag{23}$$

And $P_c(t|t)$ is the minimal upper bound of $\bar{P}_c(t|t)$ for all admissible uncertainties of noise variances. We call the actual fused Kalman filters as the robust centralized fusion Kalman filters.

Proof. Defining $\Delta P_c(t|t)=P_c(t|t)-\bar{P}_c(t|t)$, subtracting (22) from (18) yields the Lyapunov equation.

$$\Delta P_c(t|t)=\Psi_c(t)\Delta P_c(t-1|t-1)\Psi_c^{\mathrm{T}}(t)+U_c(t) \tag{24}$$

$$\begin{aligned}U_c(t)&=\left[I_n-K_c(t)H_c(t)\right]\Gamma(t-1)\left(Q(t-1)-\bar{Q}(t-1)\right)\Gamma^{\mathrm{T}}(t-1)\left[I_n-K_c(t)H_c(t)\right]^{\mathrm{T}}\\&+K_c(t)\left(R_c(t)-\bar{R}_c(t)\right)K_c^{\mathrm{T}}(t)\end{aligned} \tag{25}$$

Applying (3), (12) and (25) yields that $U_c(t)\ge 0$, and from (4) we have:

$$\Delta P_c(0|0)=P_c(0|0)-\bar{P}_c(0|0)=P(0|0)-\bar{P}(0|0)\ge 0 \tag{26}$$

Hence from (24), we have $\Delta P_c(1|1)\ge 0$. Applying the mathematical induction method yields $\Delta P_c(t|t)\ge 0$, for all time t, i.e. the inequality (23) holds. Taking $\bar{Q}(t)=Q(t),\bar{R}_c(t)=R_c(t)$ and $\bar{P}(0|0)=P(0|0)$, then comparing (18) with (22), we have $\bar{P}_c(t|t)=P_c(t|t)$. For arbitrary other upper bound $P_c^*(t|t)$, we have $P_c(t|t)=\bar{P}_c(t|t)\le P_c^*(t|t)$ which yields that $P_c(t|t)$ is the minimal upper bound of $\bar{P}_c(t|t)$. The proof is completed.

Corollary 1. For uncertain multisensor system (1) and (2) with Assumptions 1-3 and conservative upper bounds $Q(t)$ and $R_{v_i}(t)$, similar to the robust centralized fusion time-varying Kalman filters, the robust local time-varying Kalman filters are given by:

$$\hat{x}_i(t|t)=\Psi_i(t)\hat{x}_i(t-1|t-1)+K_i(t)y_i(t)\ ,\ i=1,\cdots,L \tag{27}$$

$$\Psi_i(t)=\left[I_n-K_i(t)H(t)\right]\Phi(t-1),\ K_i(t)=P_i(t|t-1)H^{\mathrm{T}}(t)R_{\varepsilon_i}^{-1}(t) \tag{28}$$

$$R_{\varepsilon_i}(t)=H(t)P_i(t|t-1)H^{\mathrm{T}}(t)+R_{v_i}(t) \tag{29}$$

$$P_i(t+1|t)=\Phi(t)P_i(t|t)\Phi^{\mathrm{T}}(t)+\Gamma(t)Q(t)\Gamma^{\mathrm{T}}(t) \tag{30}$$

$$P_i(t|t)=\left[I_n-K_i(t)H(t)\right]P_i(t|t-1) \tag{31}$$

The conservative local filtering error variance $P_i(t|t)$ can be rewritten as the Lyapunov equation [2].

$$\begin{aligned}P_i(t|t)=&\Psi_i(t)P_i(t-1|t-1)\Psi_i^{\mathrm{T}}(t)+\left[I_n-K_i(t)H(t)\right]\Gamma(t-1)Q(t-1)\Gamma^{\mathrm{T}}(t-1)\\&\times\left[I_n-K_i(t)H(t)\right]^{\mathrm{T}}+K_i(t)R_{v_i}(t)K_i^{\mathrm{T}}(t)\end{aligned} \tag{32}$$

With the initial values $P_i(0|0)=P(0|0)$. And the actual filtering error variances are given by the Lyapunov equations.

$$\begin{aligned}\bar{P}_i(t|t)=&\Psi_i(t)\bar{P}_i(t-1|t-1)\Psi_i^{\mathrm{T}}(t)+\left[I_n-K_i(t)H(t)\right]\Gamma(t-1)\bar{Q}(t-1)\Gamma^{\mathrm{T}}(t-1)\\&\times\left[I_n-K_i(t)H(t)\right]^{\mathrm{T}}+K_i(t)\bar{R}_{v_i}(t)K_i^{\mathrm{T}}(t)\end{aligned} \tag{33}$$

Similarly, the local time-varying Kalman filters are also robust, i.e.,

$$\bar{P}_i(t|t)\le P_i(t|t),\ i=1,\cdots,L \tag{34}$$

4. Robust Local and Centralized Fusion Steady-state Kalman Filters

Theorem 2. For multisensor uncertain time-invariant system (1) and (9) with Assumption 1 and 3, where $\Phi(t)=\Phi$, $\Gamma(t)=\Gamma$, $H(t)=H$, $Q(t)=Q, R_\eta(t)=R_\eta, R_{\xi_i}(t)=R_{\xi_i}$,and $\bar{Q}(t)=\bar{Q}$, $\bar{R}_\eta(t)=\bar{R}_\eta$, $\bar{R}_{\xi_i}(t)=\bar{R}_{\xi_i}$ are all the constant matrices, then the actual centralized fusion steady-state Kalman filters are given by:

$$\hat{x}_c^s(t|t)=\Psi_c\hat{x}_c^s(t-1|t-1)+K_cy_c(t) \tag{35}$$

$$\Psi_c=\left[I_n-K_cH_c\right]\Phi,\ K_c=\Sigma_cH_c^{\mathrm{T}}\left[H_c\Sigma_cH_c^{\mathrm{T}}+R_c\right]^{-1} \tag{36}$$

$$\Sigma_c=\Phi\Sigma_c\Phi^{\mathrm{T}}+\Gamma Q\Gamma^{\mathrm{T}}\quad,\quad P_c=\left[I_n-K_cH_c\right]\Sigma_c \tag{37}$$

The prediction error variance Σ_c satisfies the steady-state Riccati equation:

$$\Sigma_c=\Phi\left[\Sigma_c-\Sigma_cH_c^{\mathrm{T}}\left(H_c\Sigma_cH_c^{\mathrm{T}}+R_c\right)^{-1}H_c\Sigma_c\right]\Phi^{\mathrm{T}}+\Gamma Q\Gamma^{\mathrm{T}} \tag{38}$$

Where the superscript s denotes "steady-state", the fused conservative filtering error variance P_c is given as:

$$P_c=\Psi_cP_c\Psi_c^{\mathrm{T}}+\left[I_n-K_cH_c\right]\Gamma Q\Gamma^{\mathrm{T}}\left[I_n-K_cH_c\right]^{\mathrm{T}}+K_cR_cK_c^{\mathrm{T}} \tag{39}$$

The fused actual filtering error variance $\bar{P}_c$ is given as:

$$\bar{P}_c = \Psi_c \bar{P}_c \Psi_c^{\mathrm{T}} + [I_n - K_c H_c]\Gamma\bar{Q}\Gamma^{\mathrm{T}}[I_n - K_c H_c]^{\mathrm{T}} + K_c \bar{R}_c K_c^{\mathrm{T}} \quad (40)$$

The actual centralized fusion steady-state Kalman filters (35) are robust in the sense that for all admissible uncertainties of noise variances $\bar{Q}$ and $\bar{R}_{v_i}$ satisfying (3) and (8), we have:

$$\bar{P}_c \le P_c \quad (41)$$

And P_c is the minimal upper bound of $\bar{P}_c$.

Proof. As $t \to \infty$, taking the limit operations for (13)-(18), (22) and (23), we obtain (35)-(41). Taking $\bar{Q} = Q, \bar{R}_c = R_c$, from (39) and (40), we have $\bar{P}_c = P_c$. If P_c^* is arbitrary other upper bound of $\bar{P}_c$ for all admissible $\bar{Q}$ and $\bar{R}_c$ satisfying $\bar{Q} \le Q, \bar{R}_c \le R_c$, then we have $P_c = \bar{P}_c \le P_c^*$, which yields that P_c is minimal upper bound of $\bar{P}_c$. The proof is completed.

Similarly, the actual local steady-state Kalman filters are given by:

$$\hat{x}_i^s(t|t) = \Psi_i \hat{x}_i^s(t-1|t-1) + K_i y_i(t), i = 1, \cdots, L \quad (42)$$

$$\Psi_i = [I_n - K_i H]\Phi, K_i = \Sigma_i H^{\mathrm{T}}(H\Sigma_i H^{\mathrm{T}} + R_{v_i})^{-1}, P_i = [I_n - K_i H]\Sigma_i \quad (43)$$

The prediction error variance Σ_i satisfies the steady-state Riccati equation.

$$\Sigma_i = \Phi\left[\Sigma_i - \Sigma_i H^{\mathrm{T}}(H\Sigma_i H^{\mathrm{T}} + R_{v_i})^{-1} H\Sigma_i\right]\Phi^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} \quad (44)$$

The conservative and actual local filtering error variances satisfy the steady-state Lyapunov equations.

$$P_i = \Psi_i P_i \Psi_i^{\mathrm{T}} + [I_n - K_i H]\Gamma Q \Gamma^{\mathrm{T}}[I_n - K_i H]^{\mathrm{T}} + K_i R_{v_i} K_i^{\mathrm{T}} \quad (45)$$

$$\bar{P}_i = \Psi_i \bar{P}_i \Psi_i^{\mathrm{T}} + [I_n - K_i H]\Gamma \bar{Q} \Gamma^{\mathrm{T}}[I_n - K_i H]^{\mathrm{T}} + K_i \bar{R}_{v_i} K_i^{\mathrm{T}} \quad (46)$$

The actual local steady-state Kalman filters (42) are robust, i.e.,

$$\bar{P}_i \le P_i,\ i = 1, \cdots, L \quad (47)$$

And P_i is the minimal upper bound of $\bar{P}_i$.

Theorem 3. Under the conditions of Theorem 2, and assume that the measurements $y_i(t), i = 1, \cdots, L$ are bounded, then the robust time-varying and steady-state Kalman filters $\hat{x}_i(t|t)$ and $\hat{x}_i^s(t|t)$, $\hat{x}_c(t|t)$ and $\hat{x}_c^s(t|t)$ given by (27) and (42), (13) and (35) have each other the convergence in a realization, such that:

$$\left[\hat{x}_i(t|t) - \hat{x}_i^s(t|t)\right] \to 0, \text{ as } t \to \infty, \text{ i.a.r} \quad (48)$$

$$\left[\hat{x}_c(t|t) - \hat{x}_c^s(t|t)\right] \to 0, \text{ as } t \to \infty, \text{ i.a.r} \quad (49)$$

Where the notation “i.a.r” denotes the convergence in a realization [15], and we have the convergence of variances.

$$P_i(t|t) \to P_i,\ \bar{P}_i(t|t) \to \bar{P}_i, \text{ as } t \to \infty, i = 1, \cdots, L \quad (50)$$

$$P_c(t|t) \to P_c,\ \bar{P}_c(t|t) \to \bar{P}_c, \text{ as } t \to \infty \tag{51}$$

Proof. According to the complete observability and complete controllability of each subsystem, the time-varying local Kalman filters (27) have the convergence that [16]:

$$P_i(t|t-1) \to \Sigma_i, \text{ as } t \to \infty,\ i = 1,\cdots,L \tag{52}$$

From (28) and (31), we have:

$$\Psi_i(t) \to \Psi_i,\ K_i(t) \to K_i,\ P_i(t|t) \to P_i, \text{ as } t \to \infty,\ i = 1,\cdots,L \tag{53}$$

Setting $\Psi_i(t) = \Psi_i + \Delta\Psi_i(t)$, $K_i(t) = K_i + \Delta K_i(t)$ in (27), applying (53) yields $\Delta\Psi_i(t) \to 0$ $\Delta K_i(t) \to 0$, as $t \to \infty$. Subtracting (42) from (27), and defining $\delta_i(t) = \hat{x}_i(t|t) - \hat{x}_i^s(t|t)$, we have:

$$\delta_i(t) = \Psi_i \delta_i(t-1) + u_i(t) \tag{54}$$

With $u_i(t) = \Delta\Psi_i(t)\hat{x}_i(t-1|t-1) + \Delta K_i(t) y_i(t)$. Noting that $\Psi_i(t)$ is uniformly asymptotically stable [17], and $\Delta K_i(t) y_i(t)$ is bounded, applying Lemma 4 to (27) yields the boundedness of $\hat{x}_i(t|t)$. Hence we have $u_i(t) \to 0$. Applying Lemma 4 to (54), noting that Ψ_i is a stable matrix, so it is also uniformly asymptotically stable, hence $\delta_i(t) \to 0$, i.e. the convergence (48) holds. The convergence of (49) can be proved similarly.

From (33) and (46), defining $\Delta_i(t) = \bar{P}_i(t|t) - \bar{P}_i$ yield the Lyapunov equation.

$$\Delta_i(t) = \Psi_i \Delta_i(t-1) \Psi_i^{\mathrm{T}} + U_i(t) \tag{55}$$

$$\begin{aligned} U_i(t) = {} & [I_n - K_i(t)H]\Gamma\bar{Q}\Gamma^{\mathrm{T}}[I_n - K_i(t)H]^{\mathrm{T}} + K_i(t)\bar{R}_{v_i}K_i^{\mathrm{T}}(t) \\ & - [I_n - K_iH]\Gamma\bar{Q}\Gamma^{\mathrm{T}}[I_n - K_iH] - K_i\bar{R}_{v_i}K_i^{\mathrm{T}} + \Psi_i\bar{P}_i(t-1|t-1)\Delta\Psi_i^{\mathrm{T}}(t) \\ & + \Delta\Psi_i(t)\bar{P}_i(t-1|t-1)\Psi_i + \Delta\Psi_i(t)\Delta\Psi_i^{\mathrm{T}}(t) \end{aligned} \tag{56}$$

From (33), noting that $\Psi_i(t)$ is uniformly asymptotically stable, applying $K_i(t) \to K_i$, $\Delta\Psi_i(t) \to 0$ and Lemma 3 yields $\bar{P}_i(t-1|t-1)$ is bounded. From (56) yields that $U_i(t) \to 0$. Applying Lemma 3 to (55) yields $\Delta_i(t) \to 0$, as $t \to \infty$, i.e., $\bar{P}_i(t|t) \to \bar{P}_i$ holds. Similarly, we can prove (51) holds. The proof is completed.

5. The Accuracy Analysis

Definition 1. The trace $\mathrm{tr}\,P(t|t)$ of the upper bound $P(t|t)$ of the actual filtering error variances $\bar{P}(t|t)$ for all admissible uncertainties is called the robust accuracy or global accuracy of a robust Kalman filter, and $\mathrm{tr}\,\bar{P}(t|t)$ is called as its actual accuracy.

From this definition, the smaller $\mathrm{tr}\,P(t|t)$ or $\mathrm{tr}\,\bar{P}(t|t)$ means the higher robust accuracy or actual accuracy. The robust accuracy gives the lowest bound of all possible actual accuracies yielded from the uncertainties of noise variances.

Theorem 4. For multisensor uncertain system (1) and (2) with Assumptions 1-3, the accuracy comparison of the local and fused robust Kalman filters is given by:

$$\bar{P}_i(t|t) \le P_i(t|t),\ i=1,\cdots,L \tag{57}$$

$$\bar{P}_c(t|t) \le P_c(t|t) \le P_i(t|t),\ i=1,\cdots,L \tag{58}$$

$$\mathrm{tr}\,\bar{P}_i(t|t) \le \mathrm{tr}\,P_i(t|t),\ \mathrm{tr}\,\bar{P}_c(t|t) \le \mathrm{tr}\,P_c(t|t) \le \mathrm{tr}\,P_i(t|t),\ i=1,\cdots,L \tag{59}$$

$$\bar{P}_i \le P_i, \bar{P}_c \le P_c \le P_i, i=1,\cdots,L, \tag{60}$$

$$\mathrm{tr}\,\bar{P}_i \le \mathrm{tr}\,P_i,\ i=1,\cdots,L,\ \mathrm{tr}\,\bar{P}_c \le \mathrm{tr}\,P_c \le \mathrm{tr}\,P_i \tag{61}$$

Proof. According to the robustness (23) and (34), we have (57) and the first inequality of (58). The second inequality of (58) has been proven in [18]. Taking the trace operations for (57) and (58) yields the inequalities (59). As $t \to \infty$, taking the limit operations for (57), (58) and (59) yields (60) and (61). The proof is completed.

From the inequalities (59), we can see that all admissible actual traces $\mathrm{tr}\,\bar{P}_i(t|t)$ and $\mathrm{tr}\,\bar{P}_c(t|t)$ are globally controlled by the upper bound $\mathrm{tr}\,P_i(t|t)$ and $\mathrm{tr}\,P_c(t|t)$, respectively, and the robust accuracy of the centralized robust fuser is higher than that of each local robust Kalman filter.

6. Similation Example

Consider a three-sensor time-invariant tracking system with uncertain noise variances.

$$x(t+1) = \Phi x(t) + \Gamma w(t),\ y_i(t) = Hx(t) + \eta(t) + \xi_i(t), i=1,2,3 \tag{62}$$

$$\Phi = \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.5T_0^2 \\ T_0 \end{bmatrix}, H = I_2 \tag{63}$$

Where $T_0 = 0.25$ is the sampled period, $x(t) = [x_1(t), x_2(t)]^{\mathrm{T}}$ is the state, $x_1(t)$ and $x_2(t)$ are the position and velocity of target at time tT_0. $w(t)$, $\eta(t)$ and $\xi_i(t)$ are independent Gaussion white noises with zero mean and unknown uncertain actual variances $\bar{Q}$, $\bar{R}_\eta$ and $\bar{R}_{\xi_i}$ respectively. In the simulation, we take $Q=1$, $\bar{Q}=0.8$, $R_\eta = \mathrm{diag}(1.5, 2.5)$, $\bar{R}_\eta = \mathrm{diag}(1,2)$, $R_{\xi_1} = \mathrm{diag}(3.6, 2.5)$, $\bar{R}_{\xi_1} = \mathrm{diag}(3, 1.8)$, $R_{\xi_2} = \mathrm{diag}(8, 0.36)$, $\bar{R}_{\xi_2} = \mathrm{diag}(6, 0.25)$, $R_{\xi_3} = \mathrm{diag}(0.5, 2.8)$, $\bar{R}_{\xi_3} = \mathrm{diag}(0.38, 2)$, the initial values $x(0) = [0 \quad 0]^{\mathrm{T}}$, $\mu = 0$, $P(0|0) = \mathrm{diag}(1.1, 1.2), \bar{P}(0|0) = I_2$.

The comparisons of the filtering error variance matrices and their traces of the robust steady-state local and centralized fusion Kalman filters are shown in Table 1 and Table 2. These matrices and their traces verify the accuracy relations (60)-(61).

The traces of the conservative and actual robust filtering error variances are compared in Figure 1. We see that the traces of the local and fused robust time-varying Kalman filters quickly converge to these of the corresponding steady-state Kalman filters, which show the robust accuracy relations (59) and (61) hold.

Table 1. The Conservative and Actual Accuracy Comparison of P_i and $\bar{P}_i$ $i=1,2,3,c$

P_1	P_2	P_3	P_c

$\begin{bmatrix} 0.8247 & 0.3416 \\ 0.3416 & 0.3750 \end{bmatrix}$	$\begin{bmatrix} 1.0554 & 0.3278 \\ 0.3278 & 0.3405 \end{bmatrix}$	$\begin{bmatrix} 0.4360 & 0.2383 \\ 0.2383 & 0.3233 \end{bmatrix}$	$\begin{bmatrix} 0.3771 & 0.1956 \\ 0.1956 & 0.2805 \end{bmatrix}$
$\bar{P}_1$	$\bar{P}_2$	$\bar{P}_3$	$\bar{P}_c$
$\begin{bmatrix} 0.6442 & 0.2669 \\ 0.2669 & 0.2956 \end{bmatrix}$	$\begin{bmatrix} 0.7994 & 0.2545 \\ 0.2545 & 0.2689 \end{bmatrix}$	$\begin{bmatrix} 0.3119 & 0.1770 \\ 0.1770 & 0.2495 \end{bmatrix}$	$\begin{bmatrix} 0.2726 & 0.1478 \\ 0.1478 & 0.2191 \end{bmatrix}$

Table 2. The Conservative and Actual Accuracy Comparison of $\mathrm{tr}\,P_i$, $\mathrm{tr}\,\bar{P}_i$, $i = 1,2,3,c$

$\mathrm{tr}\,P_1$, $\mathrm{tr}\,\bar{P}_1$	$\mathrm{tr}\,P_2$, $\mathrm{tr}\,\bar{P}_2$	$\mathrm{tr}\,P_3$, $\mathrm{tr}\,\bar{P}_3$	$\mathrm{tr}\,P_c$, $\mathrm{tr}\,\bar{P}_c$
1.1998,0.9398	1.3959,1.0683	0.7593,0.5613	0.6576,0.4917

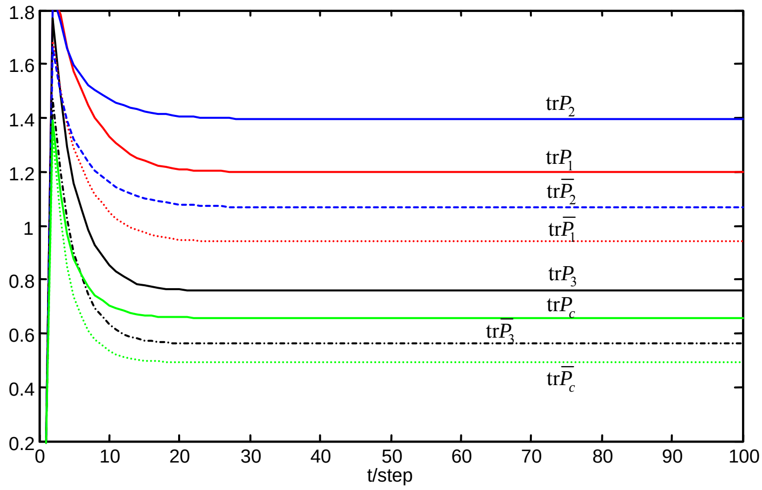

Figure 1. The Traces of the Conservative and Actual Local and Fused Kalman Filters

In order to verify the above theoretical accuracy relations, taking $\rho = 200$ Monte Carlo simulation runs, the mean square error (MSE) values at time t of local or fused robust Kalman filters are defined as:

$$\mathrm{MSE}_\theta(t) = \frac{1}{\rho}\sum_{j=1}^{\rho}\left(x^{(j)}(t) - \hat{x}_\theta^{(j)}(t|t)\right)^{\mathrm{T}}\left(x^{(j)}(t) - \hat{x}_\theta^{(j)}(t|t)\right), \theta = 1,2,3,c \tag{64}$$

Where $x^{(j)}(t)$ or $\hat{x}_\theta^{(j)}(t|t)$ denotes the j th realization of $x(t)$ or $\hat{x}_\theta(t|t)$.

According to the ergodicity [19], we have:

$$\mathrm{MSE}_\theta(t) \to \mathrm{tr}\bar{P}_\theta \text{, as } t \to \infty, \rho \to \infty, (\theta = 1,2,3,c) \tag{65}$$

The MSE curves of the local and fused time-varying robust Kalman filters are shown in Figure 2, which verify the accuracy relations (59) and (61), and verify the ergodicity (65).

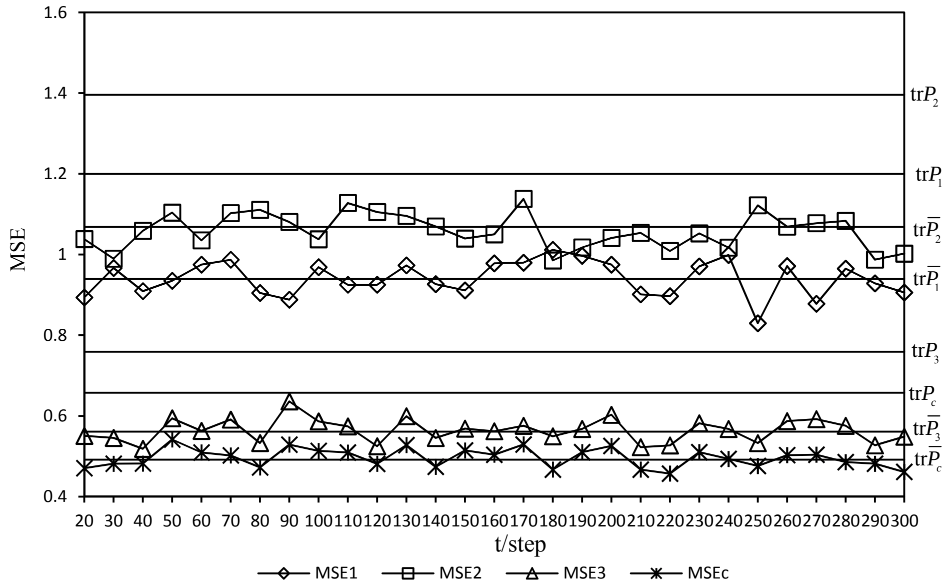

Figure 2. The Comparison of $\mathrm{MSE}_\theta(t)$ and $\mathrm{tr}P_\theta$, $\theta = 1,2,3,c$

7. Conclusion

For multisensor system with uncertain noise variances, using the minimax robust estimation principle, the lcoal and centralized fusion robust Kalman time-varying Kalman filters are presented. Based on the Lyapunov equation approach, their robustness are proved and their robust accuracy relations are also proved. It is proved that the robust accruacies of the centralized fusion Kalman filters are higher than those of the local robust Kalman filters. The convergence problem of the robust local and centralized fusion time-varying and steady-state Kalman filters is proved by the dynamic error system analysis (DESA) method and the dynamic varaince error system analysis (DVESA) method. This extension of this paper to systems with uncertain noise variances and model parameters is under study.

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Appendix

Lemma 1. Let Λ be the $r\times r$ positive semi-definite matrix, i.e. $\Lambda \ge 0$, then the following $rL\times rL$ matrix Λ_δ is also positive semi-definite, i.e.,

$$\Lambda_\delta = \begin{bmatrix} \Lambda & \cdots & \Lambda \\ \vdots & \ddots & \vdots \\ \Lambda & \cdots & \Lambda \end{bmatrix}_{rL\times rL} \ge 0 \qquad \text{(A.1)}$$

Proof. Consider the characteristic polynomial of Λ_δ.

$$|\Lambda_\delta - \lambda I_{rL}| = \begin{vmatrix} \Lambda-\lambda I_r & \Lambda & \cdots & \Lambda \\ \Lambda & \Lambda-\lambda I_r & & \Lambda \\ \vdots & \ddots & \ddots & \vdots \\ \Lambda & \cdots & \Lambda & \Lambda-\lambda I_r \end{vmatrix} \qquad \text{(A.2)}$$

Adding all the other columns to the first column yields:

$$|\Lambda_\delta - \lambda I_{rL}| = \begin{vmatrix} L\Lambda-\lambda I_r & \Lambda & \cdots & \Lambda \\ L\Lambda-\lambda I_r & \Lambda-\lambda I_r & & \Lambda \\ \vdots & \Lambda & \ddots & \vdots \\ L\Lambda-\lambda I_r & \cdots & \Lambda & \Lambda-\lambda I_r \end{vmatrix} \qquad \text{(A.3)}$$

Subtracting the first row from each row starting off with the second row to the Lth row yields:

$$|\Lambda_\delta - \lambda I_{rL}| = \begin{vmatrix} L\Lambda-\lambda I_r & \Lambda & \cdots & \Lambda \\ 0 & -\lambda I_r & & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -\lambda I_r \end{vmatrix} = |L\Lambda-\lambda I_r|\,|-\lambda I_r|^{L-1} \qquad \text{(A.4)}$$

Which yields the characteristic equation:

$$|\Lambda_\delta - \lambda I_{rL}| = |L\Lambda-\lambda I_r|\,|-\lambda I_r|^{L-1} = 0 \qquad \text{(A.5)}$$

Its all eigenvalues are determined by:

$$\left|L\Lambda - \lambda I_r\right| = 0,\ \left|-\lambda I_r\right| = 0 \quad \text{(A.6)}$$

Since $\Lambda \geq 0$, then $L\Lambda \geq 0$, so that $L\Lambda$ has all the eigenvalues $\lambda_i \geq 0$, $i = 1, \cdots, r$ which are also the eigenvalues of Λ_δ. The other eigenvalues of Λ_δ are determined by $\left|-\lambda I_r\right|^{L-1} = 0$, i.e. $(-\lambda)^{r(L-1)} = 0$, which yields all the other eigenvalues of Λ_δ are $\lambda_i = 0$, $i = r+1, \cdots, rL$. Therefore all eigenvalues λ_i of Λ_δ are non-negative, i.e., $\Lambda_\delta \geq 0$. The proof is completed.

Lemma 2. Let R_i be the $m_i \times m_i$ positive semi-definite matrix, i.e. $R_i \geq 0$, the following $m \times m$ block-diagonal matrix R_δ is also positive semi-definite, i.e.,

$$R_\delta = \text{diag}\left(R_1, \cdots R_L\right) \geq 0 \quad \text{(A.7)}$$

With $m = m_1 + \cdots + m_L$.

Lemma 3. [14] Consider the time-varying Lyapunov equation.

$$P(t) = F_1(t)P(t-1)F_2^{\mathrm{T}}(t) + U(t) \quad \text{(A.8)}$$

Where $t \geq 0$, the output $P(t)$ and the input $U(t)$ are the $n \times n$ matrices, and the $n \times n$ matrices $F_1(t)$ and $F_2(t)$ are uniformly asymptotically stable, i.e., there exist constants $0 < \rho_j < 1$ and $c_j > 0$ such that:

$$\left\|F_j(t,i)\right\| \leq c_j \rho_j^{t-i}, \forall t \geq i \geq 0, j = 1, 2 \quad \text{(A.9)}$$

Where the notation $\| \ \|$ denotes the norm of matrix, $F_j(t,i) = F_j(t)F_j(t-2)\cdots F_j(i+1)$, $F_j(i,i) = I_n$. If $U(t)$ is bounded, then $P(t)$ is bounded. If $U(t) \to 0$, then $P(t) \to 0$, as $t \to \infty$. Notice that $U(t)$ is called to be bounded, if $\left\|U(t)\right\| \leq c$ (constant), for arbitrary $t \geq 0$.

Lemma 4. [15] Consider a dynamic error system.

$$\delta(t) = F(t)\delta(t-1) + u(t) \quad \text{(A.10)}$$

Where $t \geq 0$, $\delta(t) \in R^n$, $u(t) \in R^n$, and $F(t)$ is uniformly asymptotically stable. If $u(t)$ is bounded, then $\delta(t)$ is bounded. If $u(t) \to 0$, then $\delta(t) \to 0$, as $t \to \infty$.