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Robust Weighted Measurement Fusion Kalman Predictors with Uncertain Noise Variances

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Abstract

For the multisensor system with uncertain noise variances, using the minimax robust estimation principle, the local and weighted measurement fusion robust time-varying Kalman predictors are presented based on the worst-case conservative system with the conservative upper bound of noise variances. The actual prediction error variances are guaranteed to have a minimal upper bound for all admissible uncertainties of noise variances. A Lyapunov approach is proposed for the robustness analysis and their robust accuracy relations are proved. It is proved that the robust accuracy of weighted measurement robust fuser is higher than that of each local robust Kalman predictor. Specially, the corresponding steady-state robust local and weighted measurement fusion Kalman predictors are also proposed and the convergence in a realization between time-varying and steady-state Kalman predictors is proved by the dynamic error system analysis (DESA) method. A Monte-Carlo simulation example shows the effectiveness of the robustness and accuracy relations.

Keywords: *multisensor information fusion, weighted measurement fusion, uncertain noise variance, minimax robust Kalman predictor*

1. Introduction

Multisensor information fusion is widely applied to many filelds including defence, target tracking, GPS position and so on [1, 2]. Its aim is to combine the local estimators or local measurements to obtain the fused estimators of the system state, whose accuracy is higher than that of each local estimator. Kalman filtering approach is the basic tool of the information fusion with the assumption that the model parameters and noise variances are exactly known. When there exist uncertainties, the performance of the Kalman filter can be very poor [3], and an inexact model may cause the divergent filter. In order to handle this problem, various studies on designing of the robust Kalman filters have been reported [4-6]. The robust Kalman filters guarantee to have a minimal upper bound of the actual filtering error variances for all admissible uncertainties.

For the systems with the model parameters uncertainties, there are two important approaches for designing the robust Kalman filters such that the Riccati equation approach [4], [7-8] and the linear matrix inequality (LMI) approach [5-6], [9]. The disadvantage of these two approaches is that only model parameters are uncertain while the noise variances are assumed to be exactly known. The robust Kalman filtering problems for systems with uncertain noise variances are seldom considered [10, 11], and the robust information fusion Kalman filter are also seldom researched [12, 13].

For information fusion based on the Kalman filtering, there exist two methodologies [14, 15], the state and measurement fusion methods, the former method can give a fused state estimator by combing or weighting the local state estimators, while the later fusion method is to weight all the local measurement to obtain a fused measurement equation, and then to obtain global optimal state estimator based on a single Kalman filter.

In this paper, using the minimax robust estimation principle, the local and weighted measurement fusion robust time-varying and steady-state Kalman predictors are presented based on the worst-case conservative system with the conservative upper bound of noise variances. The convergence in a realization between the time-varying and steady-state Kalman predictors is rigorously proved by the dynamic error system analysis (DESA) method [16, 17].

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Furthermore, a Lyapunov equation approach is presented for the robustness analysis, which is different from the Riccati equation approach and the LMI approach. The concept of the robust accuracy is given and the robust accuracy relations are proved, it is proved that the robust accuracy of the robust weighed measurement fusion Kalman predictor is higher than that of each local robust Kalman predictor.

The remainder of this paper is organized as follows. Section 2 gives problem formulation. The robust weighted measurement fusion time-varying Kalman predictors are presented in Section 3. The robust local and fused steady-state Kalman predictors are presented in Section 4. The robust accuracy analysis is given in Section 5. The simulation example is given in Section 6. The conclusion is proposed in Section 7.

2. Problem Formulation

Consider muiltisensor linear disceret time-varying system with uncertain noise varaince and identical measurement matrix.

$$x(t+1)=\Phi(t)x(t)+\Gamma(t)w(t) \tag{1}$$

$$y_i(t)=H(t)x(t)+\eta(t)+\xi_i(t), i=1,\cdots,L \tag{2}$$

Where t represents the discrete time, $x(t)\in R^n$ is the state, $y_i(t)\in R^m$ is the measurement of the i th subsystem, $w(t)\in R^r$ is the input noise, $\eta(t)\in R^m$ is the common disturbance noise, $\xi_i(t)\in R^m$ is the measurement noise of the i th subsystem, $\Phi(t)$, $\Gamma(t)$ and $H(t)$ are known time-varying matrices with appropriate dimensions. L is the number of sensors.

Assumption 1. $w(t)$, $\eta(t)$ and $\xi_i(t)$ are uncorrelated white noises with zero means and unknown uncertain actual variances $\bar{Q}(t)$, $\bar{R}_\eta(t)$ and $\bar{R}_{\xi_i}(t)$ at time t, respectively, $Q(t)$, $R_\eta(t)$ and $R_{\xi_i}(t)$ are known conservative upper bounds of $\bar{Q}(t)$, $\bar{R}_\eta(t)$ and $\bar{R}_{\xi_i}(t)$, satisfying:

$$\bar{Q}(t)\le Q(t), \bar{R}_\eta(t)\le R_\eta(t), \bar{R}_{\xi_i}(t)\le R_{\xi_i}(t), i=1,\cdots,L, \forall t \tag{3}$$

Assumption 2. The initial state $x(0)$ is independent of $w(t)$, $\eta(t)$ and $v_i(t)$ and has mean value μ and unknown uncertain actual variance $\bar{P}(0|0)$ which satisfies:

$$\bar{P}(0|0)\le P(0|0) \tag{4}$$

Where $P(0|0)$ is a known conservative upper bound of $\bar{P}(0|0)$.

Assumption 3. The system (1) and (2) is uniformly completely observable and completely controllable.

Defining:

$$v_i(t)=\eta(t)+\xi_i(t), i=1,\cdots,L \tag{5}$$

Where $v_i(t)$ are white noises with zero means, the conservative and actual variances and cross-covariances are given as:

$$R_{v_i}(t)=R_\eta(t)+R_{\xi_i}(t), \bar{R}_{v_i}(t)=\bar{R}_\eta(t)+\bar{R}_{\xi_i}(t), i=1,\cdots,L \tag{6}$$

$$R_{v_{ij}}(t)=R_\eta(t), \bar{R}_{v_{ij}}(t)=\bar{R}_\eta(t), i\ne j \tag{7}$$

From (3), we have:

$$\bar{R}_{v_i}(t) \le R_{v_i}(t)\,,\ i=1,\cdots,L\,,\ \forall t \tag{8}$$

3. Robust Weighted Measurement Fusion Time-varying Kalman Predictors

Introduce the centralized fusion measurement equation.

$$y_c(t) = H_c(t)x(t) + v_c(t) = eH(t)x(t) + v_c(t) \tag{9}$$

With the definitions:

$$y_c(t) = \left[y_1^{\mathrm{T}}(t),\cdots,y_L^{\mathrm{T}}(t)\right]^{\mathrm{T}},\ H_c(t) = \left[H^{\mathrm{T}}(t),\cdots,H^{\mathrm{T}}(t)\right]^{\mathrm{T}},\ v_c(t) = \left[v_1^{\mathrm{T}}(t),\cdots,v_L^{\mathrm{T}}(t)\right]^{\mathrm{T}}$$
$$e^{\mathrm{T}} = \left[I_m,\cdots,I_m\right] \tag{10}$$

And the fused noise $v_c(t)$ respectively has the conservative and actual variances as:

$$R_c = \begin{bmatrix} R_{v_i} & R_\eta & \cdots & R_\eta \\ R_\eta & \ddots & \ddots & \vdots \\ \vdots & \ddots & R_{v_i} & R_\eta \\ R_\eta & \cdots & R_\eta & R_{v_i} \end{bmatrix},\ \bar{R}_c = \begin{bmatrix} \bar{R}_{v_i} & \bar{R}_\eta & \cdots & \bar{R}_\eta \\ \bar{R}_\eta & \ddots & \ddots & \vdots \\ \vdots & \ddots & \bar{R}_{v_i} & \bar{R}_\eta \\ \bar{R}_\eta & \cdots & \bar{R}_\eta & \bar{R}_{v_i} \end{bmatrix} \tag{11}$$

Applying the weighted least square method [18], from (9), we have the conservative weighted fusion measurement equation.

$$y_M(t) = H(t)x(t) + v_M(t) \tag{12}$$

Where $y_M(t)$ is the conservative weighted fusion measurement and $v_M(t)$ is the conservative fused measurement white noise with conservative variance $R_M(t)$, such that:

$$y_M(t) = (e^{\mathrm{T}}R_c^{-1}(t)e)^{-1}e^{\mathrm{T}}R_c^{-1}(t)y_c(t) \tag{13}$$

$$v_M(t) = (e^{\mathrm{T}}R_c^{-1}(t)e)^{-1}e^{\mathrm{T}}R_c^{-1}(t)v_c(t) \tag{14}$$

$$R_M(t) = (e^{\mathrm{T}}R_c^{-1}(t)e)^{-1} \tag{15}$$

Based on the worst-case conservative system (1) and (12) with Assumptions 1-3 and conservative upper bounds $Q(t)$ and $R_{v_i}(t)$, the conservative optimal weighted measurement fused time-varying Kalman predictors $\hat{x}_M(t+N\,|\,t)$, $N \ge 1$ are given as.

When $N=1$, the one-step predictor is given as:

$$\hat{x}_M(t+1\,|\,t) = \Psi_M(t)\hat{x}_M(t\,|\,t-1) + K_M(t)y_M(t) \tag{16}$$

$$\Psi_M(t) = \Phi(t) - K_M(t)H(t) \tag{17}$$

$$K_M(t) = \Phi(t)P_M(t\,|\,t-1)H^{\mathrm{T}}(t)\left[H(t)P_M(t\,|\,t-1)H^{\mathrm{T}}(t) + R_M(t)\right]^{-1} \tag{18}$$

$$P_M(t+1\,|\,t) = \Psi_M(t)P_M(t\,|\,t-1)\Psi_M^{\mathrm{T}}(t) + \Gamma(t)Q(t)\Gamma^{\mathrm{T}}(t) + K_M(t)R_M(t)K_M^{\mathrm{T}}(t) \tag{19}$$

With $\hat{x}_M(1|0)=\mu$, $P_M(1|0)=P(0|0)$, and $P_M(t+1|t)$ satisfies the Riccati equation.

$$\begin{aligned}P_M(t+1|t)=\Phi(t)\Big[&P_M(t|t-1)-P_M(t|t-1)H^{\mathrm{T}}(t)\big(H(t)P_M(t|t-1)H^{\mathrm{T}}(t)+R_M(t)\big)^{-1}\\&\times P_M(t|t-1)\Big]\Phi^{\mathrm{T}}(t)+\Gamma(t)Q(t)\Gamma^{\mathrm{T}}(t)\end{aligned}\tag{20}$$

When $N\geq 2$, the multi-step predictor is given by:

$$\hat{x}_M(t+N|t)=\Phi(t+N,t+1)\hat{x}_M(t+1|t),\ N\geq 2\tag{21}$$

With the definition $\Phi(t,i)=\Phi(t-1)\Phi(t-2)\cdots\Phi(i),\Phi(t,t)=I_n$.

The conservative N-step prediction error variance $P_M(t+N|t)$ is given by:

$$\begin{aligned}P_M(t+N|t)=&\Phi(t+N,t+1)P_M(t+1|t)\Phi^{\mathrm{T}}(t+N,t+1)\\&+\sum_{j=2}^{N}\Phi(t+N,t+j)\Gamma(t+j-1)Q(t+j-1)\Gamma^{\mathrm{T}}(t+j-1)\Phi^{\mathrm{T}}(t+N,t+j)\end{aligned}\tag{22}$$

Substituting the actual measurement $y_i(t)$ into the conservative weighted measurement fusion Kalamn predictors (16) and (21), we obtain the actual one-step and N-step time-varying Kalman predictors.

The actual prediction errors are given as:

$$\tilde{x}_M(t+1|t)=x(t+1)-\hat{x}_M(t+1|t)=\Phi(t+1)\tilde{x}_M(t|t)+\Gamma(t)w(t)\tag{23}$$

$$\tilde{x}_M(t|t)=x(t)-\hat{x}_M(t|t)=\left[I_n-K_M(t)H(t)\right]\tilde{x}_M(t|t-1)-K_M(t)v_M(t)\tag{24}$$

Substituting (24) into (23) yields:

$$\tilde{x}_M(t+1|t)=\Psi_M(t)\tilde{x}_M(t|t-1)+\Gamma(t)w(t)-K_M(t)v_M(t)\tag{25}$$

The actual weighted measurement fused one-step prediction error variance satisfies the Lyapunov equation.

$$\bar{P}_M(t+1|t)=\Psi_M(t)\bar{P}_M(t|t-1)\Psi_M^{\mathrm{T}}(t)+\Gamma(t)\bar{Q}(t)\Gamma^{\mathrm{T}}(t)+K_M(t)\bar{R}_M(t)K_M^{\mathrm{T}}(t)\tag{26}$$

With the initial value $\bar{P}_M(1|0)=\bar{P}(0|0)$, and from (4), we have:

$$\bar{P}_M(1|0)\leq P_M(1|0)\tag{27}$$

Where $\bar{R}_M(t)$ is the actual variance of $v_M(t)$, and from (14) and (15) we have:

$$\bar{R}_M(t)=(e^{\mathrm{T}}R_c^{-1}(t)e)^{-1}e^{\mathrm{T}}R_c^{-1}(t)\bar{R}_c(t)R_c^{-1}(t)e(e^{\mathrm{T}}R_c^{-1}(t)e)^{-1}\tag{28}$$

$$\bar{R}_M(t)\leq R_M(t)\tag{29}$$

Iterating (1), we have the non-recursive formula:

$$x(t+N)=\Phi(t+N,t+1)x(t+1)+\sum_{i=t+2}^{t+N}\Phi(t+N,i)\Gamma(i-1)w(i-1),\ N\geq 2 \quad (30)$$

The actual prediction errors are given as:

$$\begin{aligned}\tilde{x}_M(t+N\mid t)&=x(t+N)-\hat{x}_M(t+N\mid t)\\&=\Phi(t+N,t+1)\tilde{x}_M(t+1\mid t)+\sum_{i=t+2}^{t+N}\Phi(t+N,i)\Gamma(i-1)w(i-1),\ N\geq 2\end{aligned} \quad (31)$$

So we have the actual N step weighted measurement fused prediction error variances.

$$\begin{aligned}\bar{P}_M(t+N\mid t)&=\Phi(t+N,t+1)\bar{P}_M(t+1\mid t)\Phi^{\mathrm{T}}(t+N,t+1)\\&+\sum_{j=2}^{N}\Phi(t+N,t+j)\Gamma(t+j-1)\bar{Q}(t+j-1)\Gamma^{\mathrm{T}}(t+j-1)\Phi^{\mathrm{T}}(t+N,t+j)\end{aligned} \quad (32)$$

Theorem 1. For multisensor uncertain system (1) and (12) with Assumptions 1-3, the actual weighted measurement fusion time-varying Kalman predictors are robust in the sense that for all admissible actual variances $\bar{Q}(t),\bar{R}_{v_i}(t)$ and $\bar{P}_M(1\mid 0)$ satisfying (3) and (4), for arbitrary time t, we have:

$$\bar{P}_M(t+N\mid t)\leq P_M(t+N\mid t),\quad N\geq 1 \quad (33)$$

And $P_M(t+N\mid t)$ is the minimal upper bound of $\bar{P}_M(t+N\mid t)$ for all admissible uncertainties of noise variances. We call the actual fused Kalman predictors as the robust weighted measurement fusion Kalman predictors.

Proof. When $N=1$, defining $\Delta P_M(t+1\mid t)=P_M(t+1\mid t)-\bar{P}_M(t+1\mid t)$, subtracting (26) from (19) yields the Lyapunov equation.

$$\Delta P_M(t+1\mid t)=\Psi_M(t)\Delta P_M(t\mid t-1)\Psi_M^{\mathrm{T}}(t)+\Delta_M(t) \quad (34)$$

$$\Delta_M(t)=\Gamma(t)\left(Q(t)-\bar{Q}(t)\right)\Gamma^{\mathrm{T}}(t)+K_M(t)\left(R_M(t)-\bar{R}_M(t)\right)K_M^{\mathrm{T}}(t) \quad (35)$$

Applying (3), (29) and (35) yields that $\Delta_M(t)\geq 0$, and from (4) we have:

$$\Delta P_M(1\mid 0)=P_M(1\mid 0)-\bar{P}_M(1\mid 0)=P(0\mid 0)-\bar{P}(0\mid 0)\geq 0 \quad (36)$$

Hence from (34), we have $\Delta P_M(2\mid 1)\geq 0$. Applying the mathematical induction method yields $\Delta P_M(t+1\mid t)\geq 0$, for all time t, i.e. the inequality (33) holds for $N=1$. When $N\geq 2$, Defining $\Delta P_M(t+N\mid t)=P_M(t+N\mid t)-\bar{P}_M(t+N\mid t)$, subtracting (32) from (22) yields:

$$\begin{aligned}\Delta P_M(t+N\mid t)&=\Phi(t+N,t+1)\left(P_M(t+1\mid t)-\bar{P}_M(t+1\mid t)\right)\Phi^{\mathrm{T}}(t+N,t+1)\\&+\sum_{j=2}^{N}\Phi(t+N,t+j)\Gamma(t+j-1)\left(Q(t+j-1)-\bar{Q}(t+j-1)\right)\Gamma^{\mathrm{T}}(t+j-1)\Phi^{\mathrm{T}}(t+N,t+j)\end{aligned} \quad (37)$$

Applying the robustness of the one-step predictor (33) and (3), we get $\Delta P_M(t+N\mid t)\geq 0$, therefore (33) holds for $N\geq 2$. Taking $\bar{Q}(t)=Q(t),\bar{R}_{v_i}(t)=R_{v_i}(t)$ and $\bar{P}_M(1\mid 0)=P_M(1\mid 0)$, then comparing (19) with (26) and (22) with (37), we have $\bar{P}_M(t+N\mid t)=P_M(t+N\mid t)$, $N\geq 1$. For arbitrary other upper bound $P_M^*\left(t+N\mid t\right)$, we have $P_M\left(t+N\mid t\right)=\bar{P}_M\left(t+N\mid t\right)\le P_M^*\left(t+N\mid t\right)$ which yields that $P_M\left(t+N\mid t\right)$ is the minimal upper bound of $\bar{P}_M\left(t+N\mid t\right)$. The proof is completed.

Corollary 1. For uncertain multisensor system (1) and (2) with Assumptions 1-3 and conservative upper bounds $Q(t)$ and $R_{v_i}(t)$, similar to the robust weighted measurement fusion time-varying Kalman predictors, the robust local time-varying Kalman one-step and multi-step predictors are given by:

$$\hat{x}_i\left(t+1\mid t\right)=\Psi_i\left(t\right)\hat{x}_i\left(t\mid t-1\right)+K_i\left(t\right)y_i\left(t\right),\ i=1,\cdots,L \tag{38}$$

$$\Psi_i\left(t\right)=\Phi\left(t\right)-K_i\left(t\right)H\left(t\right)\ ,\ K_i\left(t\right)=\Phi\left(t\right)P_i\left(t\mid t-1\right)H^{\mathrm{T}}\left(t\right)Q_{\varepsilon i}^{-1}\left(t\right) \tag{39}$$

$$Q_{\varepsilon i}\left(t\right)=H\left(t\right)P_i\left(t\mid t-1\right)H^{\mathrm{T}}\left(t\right)+R_i\left(t\right) \tag{40}$$

With the initial value $\hat{x}_i\left(1\mid 0\right)=\mu, P_i\left(1\mid 0\right)=P\left(0\mid 0\right)$, and we have the Riccati equation.

$$\begin{aligned}P_i\left(t+1\mid t\right)=\Phi\left(t\right)\Big[&P_i\left(t\mid t-1\right)-P_i\left(t\mid t-1\right)H^{\mathrm{T}}\left(t\right)\left(H\left(t\right)P_M\left(t\mid t-1\right)H^{\mathrm{T}}\left(t\right)+R_M\left(t\right)\right)^{-1}\\&\times H\left(t\right)P_i\left(t\mid t-1\right)\Phi^{\mathrm{T}}\left(t\right)\Big]+\Gamma\left(t\right)Q\left(t\right)\Gamma^{\mathrm{T}}\left(t\right)\end{aligned} \tag{41}$$

The conservative and the actual one-step prediction error variances satisfy the Lyapunov equations.

$$P_i\left(t+1\mid t\right)=\Psi_i\left(t\right)P_i\left(t\mid t-1\right)\Psi_i^{\mathrm{T}}\left(t\right)+\Gamma\left(t\right)Q\left(t\right)\Gamma^{\mathrm{T}}\left(t\right)+K_i\left(t\right)R_i\left(t\right)K_i^{\mathrm{T}}\left(t\right) \tag{42}$$

$$\bar{P}_i\left(t+1\mid t\right)=\Psi_i\left(t\right)\bar{P}_i\left(t\mid t-1\right)\Psi_i^{\mathrm{T}}\left(t\right)+\Gamma\left(t\right)\bar{Q}\left(t\right)\Gamma^{\mathrm{T}}\left(t\right)+K_i\left(t\right)\bar{R}_i\left(t\right)K_i^{\mathrm{T}}\left(t\right) \tag{43}$$

With the initial values $P_i\left(1\mid 0\right)=P\left(0\mid 0\right)$, $\bar{P}_i\left(1\mid 0\right)=\bar{P}\left(0\mid 0\right)$.

The conservative local optimal time-varying Kalman multi-step predictors are given by:

$$\hat{x}_i\left(t+N\mid t\right)=\Phi\left(t+N,t+1\right)\hat{x}_i\left(t+1\mid t\right),\ i=1,\cdots,L\ ,\ N\ge 2 \tag{44}$$

The conservative optimal N step prediction error variances $P_i\left(t+N\mid t\right)$ are given by:

$$\begin{aligned}P_i\left(t+N\mid t\right)&=\Phi\left(t+N,t+1\right)P_i\left(t+1\mid t\right)\Phi^{\mathrm{T}}\left(t+N,t+1\right)\\&+\sum_{s=2}^{N}\Phi\left(t+N,t+s\right)\Gamma\left(t+s-1\right)Q\left(t+s-1\right)\Gamma^{\mathrm{T}}\left(t+s-1\right)\Phi^{\mathrm{T}}\left(t+N,t+s\right)\end{aligned} \tag{45}$$

The actual N step prediction error variances are give by:

$$\begin{aligned}\bar{P}_i\left(t+N\mid t\right)&=\Phi\left(t+N,t+1\right)\bar{P}_i\left(t+1\mid t\right)\Phi^{\mathrm{T}}\left(t+N,t+1\right)\\&+\sum_{s=2}^{N}\Phi\left(t+N,t+s\right)\Gamma\left(t+s-1\right)\bar{Q}\left(t+s-1\right)\Gamma^{\mathrm{T}}\left(t+s-1\right)\Phi^{\mathrm{T}}\left(t+N,t+s\right)\end{aligned} \tag{46}$$

Similarly, the local time-varying Kalman one-step and multi-step predictors are also robust, i.e.,

$$\bar{P}_i\left(t+1\mid t\right)\le P_i\left(t+1\mid t\right),\ i=1,\cdots,L \tag{47}$$

$$\bar{P}_i\left(t+N\mid t\right)\le P_i\left(t+N\mid t\right),\ N\ge 2\ ,\ i=1,\cdots,L \tag{48}$$

And $P_i(t+N|t)$ is the minimal upper bound of $\bar{P}_i(t+N|t)$, $N \geq 1$.

4. Robust Local and Fused Steady-state Kalman Predictors

Theorem 2. For multisensor uncertain time-invariant system (1) and (12) with Assumptions 1-3, where $\Phi(t)=\Phi$, $\Gamma(t)=\Gamma$, $H(t)=H$, $Q(t)=Q, R_\eta(t)=R_\eta, R_{\xi_i}(t)=R_{\xi_i}$, and $\bar{Q}(t)=\bar{Q}$, $\bar{R}_\eta(t)=\bar{R}_\eta$, $\bar{R}_{\xi_i}(t)=\bar{R}_{\xi_i}$ are all constant matrices. Assume that the measurements $y_i(t)$, $i=1,\cdots,L$ are bounded, then the actual weighted measurement fusion steady-state Kalman predictors are given by:

$$\hat{x}_M^s(t+1|t)=\Psi_M \hat{x}_M^s(t|t-1)+K_M y_M(t),\ N=1 \tag{49}$$

$$\hat{x}_M^s(t+N|t)=\Phi^{N-1}\hat{x}_M^s(t+1|t),\ N\geq 2 \tag{50}$$

$$\Psi_M=[I_n-K_M H]\Phi,\ K_M=\Phi\Sigma_M H^{\mathrm{T}}\left[H\Sigma_M H^{\mathrm{T}}+R_M\right]^{-1} \tag{51}$$

$$\Sigma_M=\Psi_M\Sigma_M\Psi_M^{\mathrm{T}}+\Gamma Q\Gamma^{\mathrm{T}}+K_M R_M K_M^{\mathrm{T}} \tag{52}$$

$$\bar{\Sigma}_M=\Psi_M\bar{\Sigma}_M\Psi_M^{\mathrm{T}}+\Gamma\bar{Q}\Gamma^{\mathrm{T}}+K_M\bar{R}_M K_M^{\mathrm{T}} \tag{53}$$

Where the superscript s denotes “steady-state”, the initial value $\hat{x}_M^s(0|0)$ can arbitrarily be selected, $y_i(t)$ are the actual measurements, and:

$$y_M(t)=(e^{\mathrm{T}}R_c^{-1}e)^{-1}e^{\mathrm{T}}R_c^{-1}y_c(t) \tag{54}$$

$$R_M=(e^{\mathrm{T}}R_c^{-1}e)^{-1},\ \ \bar{R}_M=(e^{\mathrm{T}}R_c^{-1}e)^{-1}e^{\mathrm{T}}R_c^{-1}\bar{R}_c R_c^{-1}e(e^{\mathrm{T}}R_c^{-1}e)^{-1} \tag{55}$$

The conservative and actual steady-state prediction error variances satisfy the Lyapunov equations.

$$P_M(N)=\Phi^{N-1}\Sigma_M\left(\Phi^{N-1}\right)^{\mathrm{T}}+\sum_{s=0}^{N-2}\Phi^s\Gamma Q\Gamma^{\mathrm{T}}\left(\Phi^s\right)^{\mathrm{T}},\ N\geq 2 \tag{56}$$

$$\bar{P}_M(N)=\Phi^{N-1}\bar{\Sigma}_M\left(\Phi^{N-1}\right)^{\mathrm{T}}+\sum_{s=0}^{N-2}\Phi^s\Gamma\bar{Q}\Gamma^{\mathrm{T}}\left(\Phi^s\right)^{\mathrm{T}},\ N\geq 2 \tag{57}$$

The actual steady-state Kalman predictors are robust, in the sense that:

$$\bar{\Sigma}_M\leq\Sigma_M,\ \ \bar{P}_M(N)\leq P_M(N) \tag{58}$$

And Σ_M and $P_M(N)$ are the minimal upper bounds of $\bar{\Sigma}_M$ and $\bar{P}_M(N)$, respectively.

Proof. As $t\to\infty$, taking the limit operations for (14)-(33) with Φ,Γ,H,Q and R_i are constant matrices yields (49)-(58). Taking $\bar{Q}=Q$ and $\bar{R}_{v_i}=R_{v_i}$, from (55), we have $\bar{R}_M=R_M$, so that from (52) and (53) yields $\bar{\Sigma}_M=\Sigma_M$, hence from (56) and (57), we have $\bar{P}_M(N)=P_M(N)$. If Σ_M^* or $P_M^*(N)$ is the arbitrary other bound of $\bar{\Sigma}_M$ or $\bar{P}_M(N)$, we have $\Sigma_M=\bar{\Sigma}_M\leq\Sigma_M^*$ or $P_M(N)=\bar{P}_M(N)\leq P_M^*(N)$, which yields that Σ_M and $P_M(N)$ are minimal. The proof is completed.

Similarly, the actual local steady-state Kalman predictors are given by:

$$\hat{x}_i^s(t+1|t)=\Psi_i\hat{x}_i^s(t|t-1)+K_iy_i(t),\ i=1,\cdots,L \quad (59)$$

$$\Psi_i=[I_n-K_iH]\Phi,\ K_i=\Phi\Sigma_iH^{\mathrm{T}}\left(H\Sigma_iH^{\mathrm{T}}+R_{v_i}\right)^{-1} \quad (60)$$

The conservative and actual prediction error variances satisfy Lyapunov equation.

$$\Sigma_i=\Psi_i\Sigma_i\Psi_i^{\mathrm{T}}+\Gamma Q\Gamma^{\mathrm{T}}+K_iR_iK_i^{\mathrm{T}},\ i=1,\cdots,L \quad (61)$$

$$\bar{\Sigma}_i=\Psi_i\bar{\Sigma}_i\Psi_i^{\mathrm{T}}+\Gamma\bar{Q}\Gamma^{\mathrm{T}}+K_i\bar{R}_iK_i^{\mathrm{T}},\ i=1,\cdots,L \quad (62)$$

The actual steady-state fused Kalman multi-step predictor is given as:

$$\hat{x}_i^s(t+N|t)=\Phi^{N-1}\hat{x}_i^s(t+1|t),\ i=1,\cdots,L,\ N>1 \quad (63)$$

The conservative and actual local steady-state N step prediction error variances are given as:

$$P_i(N)=\Phi^{N-1}\Sigma_i\left(\Phi^{N-1}\right)^{\mathrm{T}}+\sum_{s=0}^{N-2}\Phi^s\Gamma Q\Gamma^{\mathrm{T}}\left(\Phi^s\right)^{\mathrm{T}},\ N\ge 2,\ i=1,\cdots,L \quad (64)$$

$$\bar{P}_i(N)=\Phi^{N-1}\bar{\Sigma}_i\left(\Phi^{N-1}\right)^{\mathrm{T}}+\sum_{s=0}^{N-2}\Phi^s\Gamma\bar{Q}\Gamma^{\mathrm{T}}\left(\Phi^s\right)^{\mathrm{T}},\ N\ge 2\ i=1,\cdots,L \quad (65)$$

The actual local steady-state Kalman predictors (59) and (63) are robust in the sense that for all admissible uncertainties of $\bar{Q}$ and $\bar{R}_{v_i}$ satisfying $\bar{Q}\le Q,\bar{R}_{v_i}\le R_{v_i}$, we have:

$$\bar{\Sigma}_i\le\Sigma_i,\quad \bar{P}_i(N)\le P_i(N),\ i=1,\cdots,L \quad (66)$$

And Σ_i and $P_i(N)$ are the minimal upper bounds of $\bar{\Sigma}_i$ and $\bar{P}_i(N)$, respectively. Hence they are called the robust steady-state Kalman predictors.

Lemma 1. [16, 17] Consider a dynamic error system.

$$\delta(t)=F(t)\delta(t-1)+u(t) \quad (67)$$

Where $t\ge 0$, $\delta(t)\in R^n$, $u(t)\in R^n$, and $F(t)$ is uniformly asymptotically stable, i.e., there exist constants $0<\rho<1$ and $c>0$ such that:

$$\|F(t,i)\|\le c\rho^{t-i},\forall t\ge i\ge 0 \quad (68)$$

Where the notation $\|\ \|$ denotes the norm of matrix, $F(t,i)=F(t)F(t-2)\cdots F(i+1)$, $F(i,i)=I_n$. If $u(t)$ is bounded, then $\delta(t)$ is bounded. If $u(t)\to 0$, then $\delta(t)\to 0$, as $t\to\infty$.

Theorem 3. Under the conditions of Theorem 2, the robust time-varying and steady-state Kalman local and fused one-step and multi-step predictors have each other the convergence in a realization, such that:

$$\left[\hat{x}_i(t+1|t)-\hat{x}_i^s(t+1|t)\right]\to 0,\ \text{as } t\to\infty,\ \text{i.a.r} \quad (69)$$

$$\left[\hat{x}_i(t+N|t)-\hat{x}_i^s(t+N|t)\right]\to 0,\ \text{as } t\to\infty,\ \text{i.a.r},\ N\ge 2 \quad (70)$$

$$\left[\hat{x}_M\left(t+1|t\right)-\hat{x}_M^s\left(t+1|t\right)\right]\to 0\text{, as } t\to\infty\text{, i.a.r} \quad (71)$$

$$\left[\hat{x}_M\left(t+N|t\right)-\hat{x}_M^s\left(t+N|t\right)\right]\to 0\text{, as } t\to\infty\text{, i.a.r, } N\ge 2 \quad (72)$$

Where the notation "i.a.r" denotes the convergence in a realization [17], and we have the convergence of variances.

$$P_i\left(t+1|t\right)\to\Sigma_i,\ P_M\left(t+1|t\right)\to\Sigma_M\text{, as } t\to\infty,\ i=1,\cdots,L \quad (73)$$

$$P_i\left(t+N|t\right)\to P_i\left(N\right),\ P_M\left(t+N|t\right)\to P_M\left(N\right)\text{, as } t\to\infty,\ i=1,\cdots,L \quad (74)$$

Proof. According to Assumption 3, we have [18]:

$$P_i\left(t+1|t\right)\to\Sigma_i\text{, as } t\to\infty,\ i=1,\cdots,L \quad (75)$$

Then from (19) and (39), we have:

$$\Psi_i\left(t\right)\to\Psi_i,\ K_i\left(t\right)\to K_i,\ P_M\left(t+1|t\right)\to\Sigma_M\text{ as } t\to\infty,\ i=1,\cdots,L \quad (76)$$

Similarly, we can prove (74) holds, Setting $\Psi_i\left(t\right)=\Psi_i+\Delta\Psi_i\left(t\right), K_i\left(t\right)=K_i+\Delta K_i\left(t\right)$ in (38), applying (76) yields $\Delta\Psi_i\left(t\right)\to 0\ \Delta K_i\left(t\right)\to 0$, as $t\to\infty$. Subtracting (59) from (38), and defining $\delta_i\left(t\right)=\hat{x}_i\left(t+N|t\right)-\hat{x}_i^s\left(t+N|t\right)$, we have:

$$\delta_i\left(t\right)=\Psi_i\delta_i\left(t-1\right)+u_i\left(t\right) \quad (77)$$

With $u_i\left(t\right)=\Delta\Psi_i\left(t\right)\hat{x}_i\left(t|t-1\right)+\Delta K_i\left(t\right)y_i\left(t\right)$. Noting that $\Psi_i\left(t\right)$ is uniformly asymptotically stable [19], and $\Delta K_i\left(t\right)y_i\left(t\right)$ is bounded, applying Lemma 1 to (38) yields the boundedness of $\hat{x}_i\left(t+1|t\right)$. Hence we have $u_i\left(t\right)\to 0$. Applying Lemma 1 to (77), noting that Ψ_i is a stable matrix, so it is also uniformly asymptotically stable, hence $\delta_i\left(t\right)\to 0$, i.e. the convergence (69) holds. The convergence of (70)-(72) can be proved similarly. The proof is completed.

5. The Accuracy Analysis

Definition 1. The trace $\operatorname{tr}P\left(t+N|t\right)$ of the upper bound $P\left(t+N|t\right)$ of the actual prediction error variances $\bar{P}\left(t+N|t\right)$ for all admissible uncertainties is called the robust accuracy or global accuracy of a robust Kalman predictor, and $\operatorname{tr}\bar{P}\left(t+N|t\right)$ is called as its actual accuracy. The smaller $\operatorname{tr}P\left(t+N|t\right)$ or $\operatorname{tr}\bar{P}\left(t+N|t\right)$ means the higher robust accuracy or actual accuracy. The robust accuracy gives the lowest bound of all possible actual accuracies yielded from the uncertainties of noise variances.

Theorem 4. For multisensor uncertain system (1) and (2) with Assumptions 1-3, the accuracy comparison of the local and fused robust Kalman predictors is given by:

$$\bar{P}_i\left(t+N|t\right)\le P_i\left(t+N|t\right),\ i=1,\cdots,L\ ,\ N\ge 1 \quad (78)$$

$$\bar{P}_M\left(t+N|t\right)\le P_M\left(t+N|t\right)\le P_i\left(t+N|t\right),\ i=1,\cdots,L,\ N\ge 1 \quad (79)$$

$$\operatorname{tr}\bar{P}_i\left(t+N|t\right)\le\operatorname{tr}P_i\left(t+N|t\right),\ i=1,\cdots,L\ ,\ N\ge 1 \quad (80)$$

$$\text{tr}\,\bar{P}_M\left(t+N\,|\,t\right)\le \text{tr}\,P_M\left(t+N\,|\,t\right)\le \text{tr}\,P_i\left(t+N\,|\,t\right),\ i=1,\cdots,L\ ,\ N\ge 1 \quad (81)$$

$$\bar{P}_i\left(N\right)\le P_i\left(N\right),\bar{P}_M\left(N\right)\le P_M\left(N\right)\le P_i\left(N\right),\ i=1,\cdots,L\ ,\ N\ge 1 \quad (82)$$

With the definitions $P_i\left(1\right)=\Sigma_i,\bar{P}_i\left(1\right)=\bar{\Sigma}_i,P_M\left(1\right)=\Sigma_M,\bar{P}_M\left(1\right)=\bar{\Sigma}_M$.

$$\text{tr}\,\bar{P}_i\left(N\right)\le \text{tr}\,P_i\left(N\right),\ \text{tr}\,\bar{P}_M\left(N\right)\le \text{tr}\,P_M\left(N\right)\le \text{tr}\,P_i\left(N\right),\ i=1,\cdots,L,\ N\ge 1 \quad (83)$$

Proof. According to the robustness (33), (47) and (48), we have (78) and the first inequality of (79). Since the conservative weighted measurement fuser is equivalent to the conservative centralized fuser [20], the second inequality of (79) has been proven in [21]. Taking the trace operations for (78) and (79) yields the inequalities (80) and (81). As $t\to\infty$, taking the limit operations for (78)-(81) yields (82) and (83). The proof is completed.

6. Similation Example

Consider a three-sensor time-invariant tracking system with uncertain noise variances.

$$x\left(t+1\right)=\Phi x\left(t\right)+\Gamma w\left(t\right),\ y_i\left(t\right)=Hx\left(t\right)+\eta\left(t\right)+\xi_i\left(t\right),i=1,2,3 \quad (84)$$

$$\Phi=\begin{bmatrix}1 & T_0\\ 0 & 1\end{bmatrix},\Gamma=\begin{bmatrix}0.5T_0^2\\ T_0\end{bmatrix},H=I_2 \quad (85)$$

Where $T_0=0.35$ is the sampled period, $x\left(t\right)=\left[x_1\left(t\right),x_2\left(t\right)\right]^{\mathrm{T}}$ is the state, $x_1\left(t\right)$ and $x_2\left(t\right)$ are the position and velocity of target at time tT_0. $w\left(t\right)$, $\eta\left(t\right)$ and $\xi_i\left(t\right)$ are independent Gaussion white noises with zero mean and unknown uncertain actual variances $\bar{Q}$, $\bar{R}_\eta$ and $\bar{R}_{\xi_i}$ respectively. In the simulation, we take $Q=1$, $\bar{Q}=0.8$, $R_\eta=\text{diag}(1.5,2.5)$, $\bar{R}_\eta=\text{diag}(1,2)$, $R_{\xi_1}=\text{diag}(3.6,2.5)$, $\bar{R}_{\xi_1}=\text{diag}(3,1.8)$, $R_{\xi_2}=\text{diag}(8,0.36)$, $\bar{R}_{\xi_2}=\text{diag}(6,0.25)$, $R_{\xi_3}=\text{diag}(0.5,2.8)$, $\bar{R}_{\xi_3}=\text{diag}(0.38,2)$, $N=1$, $N=2$. The initial values $x\left(0\right)=\begin{bmatrix}0 & 0\end{bmatrix}^{\mathrm{T}}$, $\mu=0$, $P\left(0\,|\,0\right)=\text{diag}(1.1,1.2),\bar{P}\left(0\,|\,0\right)=I_2$.

The comparisons of the prediction error variance matrices and their traces of the robust steady-state local and weighted measurement fusion Kalman predictors are shown in Table 1-Table 3. These matrices and their traces verify the accuracy relations (82) and (83).

The traces of the conservative and actual robust one-step and two-step prediction error variances are compared in Figure 1 and Figure 2. We see that the traces of the local and fused robust time-varying Kalman one-step and two-step predictors quickly converge to these of the corresponding steady-state Kalman predictors, which show the robust accuracy relations (80), (81) and (83) hold.

Table 1. The Conservative and Actual Accuracy Comparison of One-step Prediction Error Variances Matrices Σ_i and $\bar{\Sigma}_i$, $i=1,2,3,M$

Σ_1	Σ_2	Σ_3	Σ_M
$\begin{bmatrix}1.4931 & 0.6538\\ 0.6538 & 0.6314\end{bmatrix}$	$\begin{bmatrix}1.7995 & 0.6200\\ 0.6200 & 0.5833\end{bmatrix}$	$\begin{bmatrix}0.8558 & 0.4877\\ 0.4877 & 0.5592\end{bmatrix}$	$\begin{bmatrix}0.7315 & 0.4098\\ 0.4098 & 0.4995\end{bmatrix}$
$\bar{\Sigma}_1$	$\bar{\Sigma}_2$	$\bar{\Sigma}_3$	$\bar{\Sigma}_M$
$\begin{bmatrix}1.1667 & 0.5123\\ 0.5123 & 0.4989\end{bmatrix}$	$\begin{bmatrix}1.3698 & 0.4836\\ 0.4836 & 0.4617\end{bmatrix}$	$\begin{bmatrix}0.6202 & 0.3672\\ 0.3672 & 0.4346\end{bmatrix}$	$\begin{bmatrix}0.5365 & 0.3134\\ 0.3134 & 0.3922\end{bmatrix}$

Table 2. The Conservative and Actual Accuracy Comparison of Two-step Prediction Error Variances Matrices P_i and $\bar{P}_i$, $i=1,2,3,M$

$P_1(2)$	$P_2(2)$	$P_3(2)$	$P_M(2)$
$\begin{bmatrix} 2.0319 & 0.8963 \\ 0.8963 & 0.7539 \end{bmatrix}$	$\begin{bmatrix} 2.3087 & 0.8455 \\ 0.8455 & 0.7058 \end{bmatrix}$	$\begin{bmatrix} 1.2694 & 0.7049 \\ 0.7049 & 0.6817 \end{bmatrix}$	$\begin{bmatrix} 1.0833 & 0.6060 \\ 0.6060 & 0.6220 \end{bmatrix}$
$\bar{P}_1(2)$	$\bar{P}_2(2)$	$\bar{P}_3(2)$	$\bar{P}_M(2)$
$\begin{bmatrix} 1.5894 & 0.7040 \\ 0.7040 & 0.5969 \end{bmatrix}$	$\begin{bmatrix} 1.7679 & 0.6624 \\ 0.6624 & 0.5597 \end{bmatrix}$	$\begin{bmatrix} 0.9335 & 0.5365 \\ 0.5365 & 0.5326 \end{bmatrix}$	$\begin{bmatrix} 0.8069 & 0.4678 \\ 0.4678 & 0.4902 \end{bmatrix}$

Table 3. The Robust and Actual Accuracy Comparison of $\text{tr}\,\Sigma_i$, $\text{tr}\,\bar{\Sigma}_i$, and $\text{tr}\,P_i$, $\text{tr}\,\bar{P}_i$ $i=1,2,3,M$

$\text{tr}\,\Sigma_1$, $\text{tr}\,\bar{\Sigma}_1$	$\text{tr}\,\Sigma_2$, $\text{tr}\,\bar{\Sigma}_2$	$\text{tr}\,\Sigma_3$, $\text{tr}\,\bar{\Sigma}_3$	$\text{tr}\,\Sigma_M$, $\text{tr}\,\bar{\Sigma}_M$
2.1245,1.6656	2.3828,1.8315	1.415,1.0548	1.231,0.9287
$\text{tr}\,P_1(2)$, $\text{tr}\,\bar{P}_1(2)$	$\text{tr}\,P_2(2)$, $\text{tr}\,\bar{P}_2(2)$	$\text{tr}\,P_3(2)$, $\text{tr}\,\bar{P}_3(2)$	$\text{tr}\,P_M(2)$, $\text{tr}\,\bar{P}_M(2)$
2.7858,2.1863	3.0144,2.3276	1.9511,1.4661	1.7053,1.2971

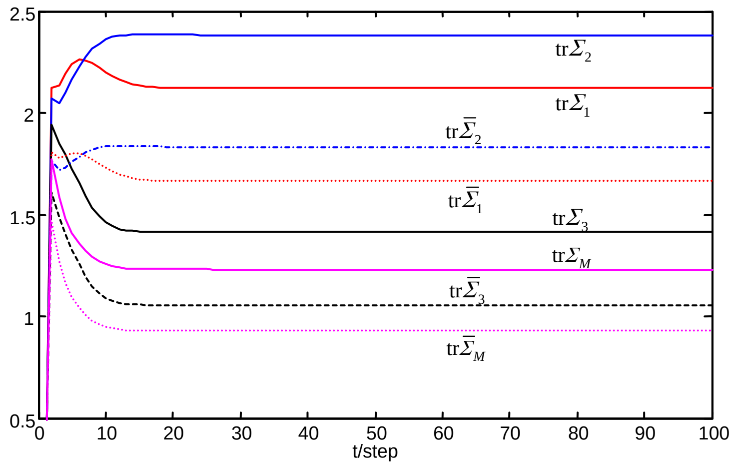

Figure 1. The Traces of the Conservative and Actual Local and Fused Kalman One-step Predictors

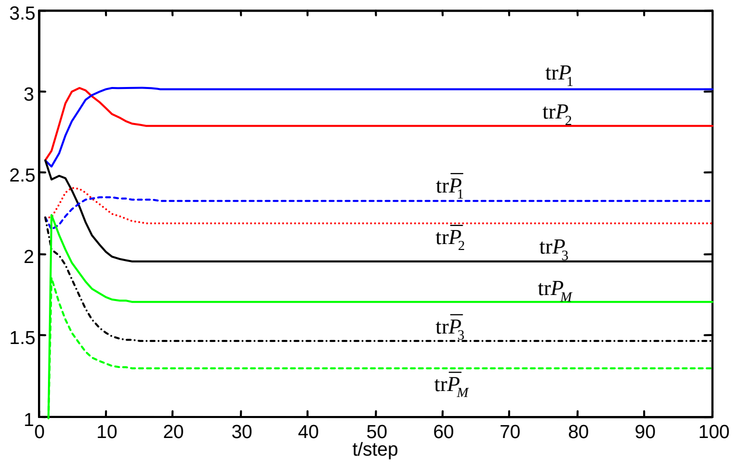

Figure 2. The Traces of the Conservative and Actual Local and Fused Kalman Two-step Predictors

In order to verify the above theoretical accuracy relations, taking $\rho = 200$ Monte Carlo simulation runs, According to the ergodicity [22], we have:

$$\mathrm{MSE}_\theta(t) \to \mathrm{tr}\bar{P}_\theta \text{, as } t \to \infty, \rho \to \infty, (\theta = 1,2,3,M) \quad (86)$$

The MSE curves of the local and fused time-varying robust Kalman predictors are shown in Figure 3, which verify the accuracy relations (80), (81) and (83), and verify the ergodicity (86).

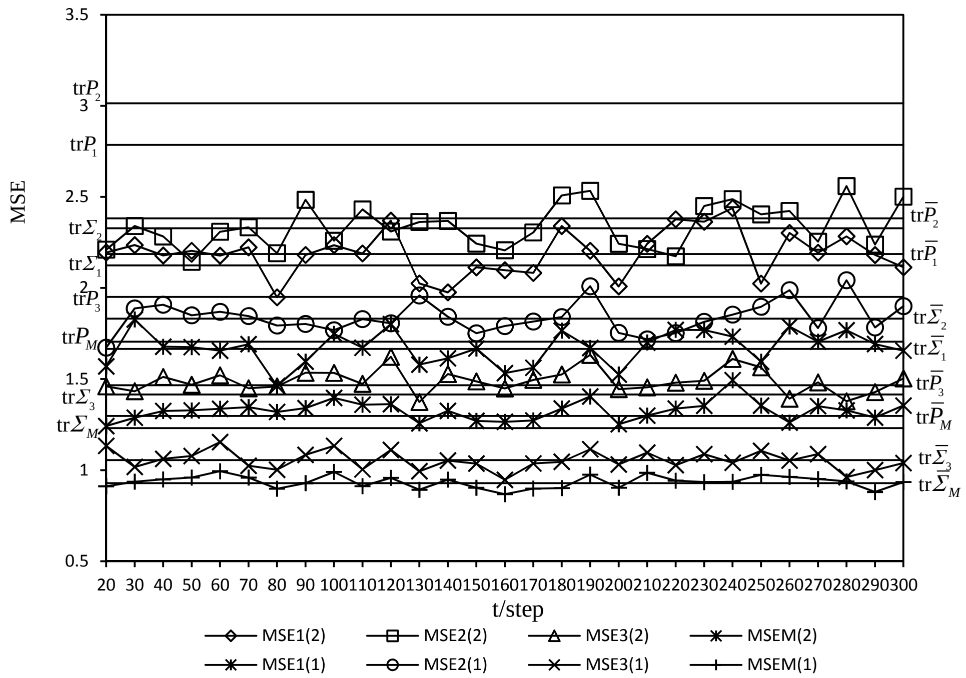

Figure 3. The Comparison of $\mathrm{MSE}_\theta(t)$ and $\mathrm{tr}P_\theta$, $\theta = 1,2,3,M$

7. Conclusion

For multisensor system with uncertain noise variances, using the minimax robust estimation principle, the lcoal and weighted measurement fusion robust Kalman time-varying predictors are presented. Based on the Lyapunov equation approach, their robustness are proved, and their robust accuracy relations are also proved. It is proved that the robust accruacies of the weighted measurement fusion Kalman predictors are higher than that of each local robust Kalman predictor. The convergence problem of the robust local and weighted measurement fusion time-varying and steady-state Kalman predictors is proved by the dynamic error system analysis (DESA) method. This extension of this paper to systems with uncertain noise variances and model parameters is under study.

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