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Coherence-based sufficient condition for support recovery using block generalized orthogonal matching pursuit

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ABSTRACT

Challenge is to find the support vectors of the unknown block sparse vector with compressed measurements in an underdetermined system where the number of unknowns is more than that of measurements. To recover unknown block sparse vector, restricted isometry property (RIP) is a sufficient condition need to be satisfied. Finding the restricted isometric constant is a non-polynomial hard problem for large values of n. In this paper coherence-based recovery guarantee has been proposed to recover the support vectors using block generalized orthogonal matching pursuit (BGOMP). It is proved that BGOMP can able to recover the support vectors with lesser number of iteration than block orthogonal matching pursuit (BOMP) by selecting multiple block support elements per iteration. Simulation results show detection performance of BGOMP is better than BOMP, block subspace pursuit (BSP) and block compressive sampling matching pursuit (BCoSaMP) for different block sparsity and block length. In most of the cases for different block sparsity and block length computation time for BGOMP is lesser than BCoSaMP, BSP and BOMP due to the multiple selection of elements in each iteration.

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1. INTRODUCTION

In an underdetermined system, number of unknown variables will be more than that of number of observations or measurements. Compressed sensing is a technique to recover the unknown sparse vector using compressed measurements is as shown in Figure 1. Signal is transformed in to some other plane and it will be sparse i.e., fewer non zero elements in the vector or signal in the particular basis is much less than the dimension of the vector. By exploiting the sparsity of the unknown support column vectors of the sensing matrix will be recovered with the help of compressed measurements. To recover the unknown sparse signal many recovery guarantees are in the literature. One of the popular methods to guarantee the recovery of sparse vector need to satisfy the restricted isometry property (RIP). Restricted isometric constant (RIC) $\delta \in 0.1$ is the smallest constant that satisfy the relation given by $(1-\delta_p)\|x\|_2^2 \leq \|Hx\|_2^2 \leq (1+\delta_p)\|x\|_2^2$ to recover a P-sparse unknown vector. The smallest constant δ satisfying RIP is referred as RIC. The majority of the suggested recovery guarantee relies on the use of RIC, which effectively recovers the sparse signal by leveraging RIP [1]–[7]. Typically, it is computationally difficult to determine the Restricted Isometric constant for a specific matrix H in order to meet the rigorous criteria [8]. The deep learning mechanism is applied to estimate the images precisely [9]. Prior research has previously suggested using mutual coherence as a criterion for determining recovery conditions in the orthogonal matching pursuit algorithm. This criterion specifically applies when the number of selected support elements in the method is equal to one Q=1 [10]–[11]. Coherence statistics are valuable for resolving various signal processing issues, such as support detection and vector quantization [12]–[18].

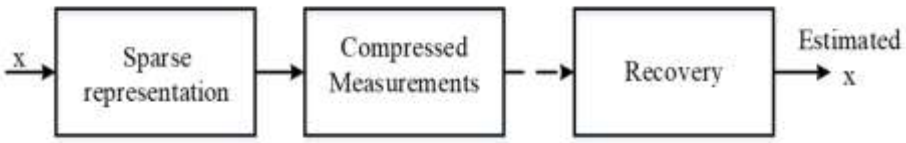

Figure 1. Compressed sensing system model

Proposed method establishes a sufficient criterion for ensuring the recovery performance of block generalized OMP, based on coherence. Several researchers have established a satisfactory criterion for orthogonal matching pursuit (OMP) [19]. OMP can precisely reconstruct the non-zero elements of an unknown P-sparse vector using a greater number of iterations compared to generalized orthogonal matching pursuit (GOMP). The proposed requirement in the literature is loosely constrained and relies on the coherence parameter.

In recent years, recovery criteria based on the RIC have been proposed to guarantee the accurate recovery of a P-sparse signal using the GOMP algorithm. Moreover, the author [20] suggests that the adequate requirement has been enhanced to $\delta_{NK+1} < \frac{1}{\sqrt{K/N+1}}$. Although the requirements impose stricter constraints, determining the RIC of a matrix H is an NP-hard problem. A suggested sufficient condition, based on mutual coherence, enables accurate recovery of the support indices of a P-sparse signal using GOMP in the presence of noise [21]. The block generalized orthogonal matching pursuit (BGOMP) method allows for the selection of a maximum of Q elements in each iteration, where Q is less than or equal to (m-1)/P. In the general case, when the number of elements picked per iteration is equal to 1, it is same as block orthogonal matching pursuit (BOMP), which has a lower performance than BGOMP. The wireless sensor network (WSN) system and permit more efficient emergency reaction systems [22]–[24].

2. METHOD

Consider the following linear system model equation $b = H\,c + v$K-block sparse signal $c \in R^n$ can be given as $[\underbrace{c_1, \dots, c_{d_1}}_{C[1]}, \underbrace{c_{d_1+1}, \dots, c_{d_{g-1}}}_{C[2]}, \dots, c_{d_g}]^T$ and v is AWGN noise where $c[j](j \in \omega = \{1,2,\dots,L\})$ represents the j^{th} block of c and the block size is d_i for the i^{th} block. $b \in R^{M\times 1}$ is an observation vector contains M elements. $b \in R^{M\times N}$ (M<N) is an observation matrix where $H = [\underbrace{h_1, \dots, h_{d_1}}_{H[1]}, \underbrace{h_{d_1+1}, \dots, h_{d_{g-1}}}_{H[2]}, \dots, h_{d_g}]^T$. $c \in R^{N\times 1}$ is the block sparse signal that we need to recover from the compressed measurements. c is considered as a K-block sparse signal, which is a signal consisting of 'K' non-zero blocks in an N-dimensional space where K<<N. The above model indicates the assumption that every block $c_i \in R^{d_i \times 1}$ satisfies a gaussian distribution. Block sparse signal c will be recovered from underdetermined linear measurements using an expression $b = H\,c + v$ is as shown in Figure 2.

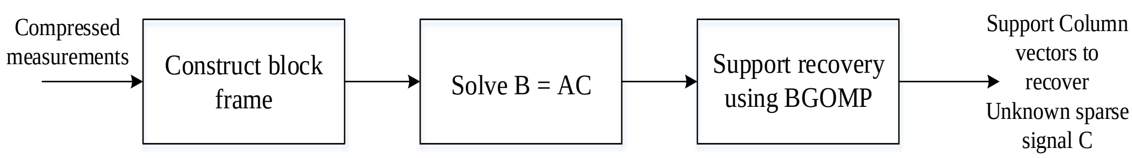

Figure 2. Block diagram for recovery of unknown block sparse signal c using BGOMP algorithm

2.1. Block restricted isometry property

Definition 1 Let H∈ $R^{m\times n}$ be a given matrix. Then H is said to have the block RIP over I={$d_1, d_2,\ d_L$} with constant $\delta_{K\,|\,I}$if for every c ∈ R^n that is block K -sparse signal over I. Let us assume that there is a matrix H∈ $R^{m\times}$. Then, if we know that for every c ∈ R^n that is a block K-sparse signal over I, H is said to have the block RIP over I={$d_1, d_2,\cdots, d_L$}with constant $\delta_{K\,|\,I}$, then we can say that H has the block RIP over I:

$$(1\text{-}\delta_{K\,|I})\|c\|_2^2 \leq \|Hc\|_2^2 \leq (1+\delta_{K\,|I})\|c\|_2^2 \tag{1}$$

It is worth noting that this study focuses on the detection of a block sparse signal with a same block size, i.e., I={d_1 , d_2,⋯, d_L}. Remember that a block K-sparse vector is also Kd sparse. Every block K -sparse signals, H must hold if it fulfills the RIP of order Kd. However, this may not be true for all Kd sparse signals if H fulfills the block RIP of order K. As a result, the block RIP (of order K) is a more relaxed criterion than the normal RIP (of order Kd). The probability that a random matrix will satisfy the block RIP is much higher than the probability that it will meet the standard RIP.

2.2. Proposed BGOMP algorithm

BGOMP algorithm consists of initialization, finding the index of highly correlated support vector index, appending in the list and updating the residual. It is unique in the way that it will not select the support vector twice. The various steps involved in proposed BGOMP algorithm 1 is given:

Algorithm 1. BGOMP Algorithm

Input: $H \in R^{m\times n}$, P, $Q \leq (m-d)/Pd$
Initialize the residual and the sub sampled measurements $b \in R^n$
For Each $t \leq P$ Initialize the for loop with a condition that t is less than or equal to P
Choose the Q block indices where the correlation of the active column indices is $i_t = \underset{n\in S_t}{argmax}\,|\widehat{H}_n[j]r_{t-1}|,\quad j \in \gamma = \{1,2,\ldots,L\}$;
Arrange the resulted i_t column indices in descending order and pick 'Q' largest value indices in the current iteration $i_t = \{i_1 \ldots i_Q\}$
Concatenate the newly identified column indices in the present iteration with the previous iteration entries $S_t = S_{t-1} \cap i_t$
Estimate the signal vector using the least squares $\hat{z}_t = \underset{z:supp(z)}{argmin}\|b - H_{s_t}z\|_2^2$ where H_{s_t} denotes s_t Column of H.
Update residual $r_t = b - H_{s_t}\hat{z}_{s_t}$ where $\hat{z}_{s_t}$ represents estimation of z with support indices s_t
Increment the loop count by one
Go to step 2
End for Return $\hat{S} = S_t$;
Output: $\hat{S}$
End

2.3. Description of proposed BGOMP algorithm

BGOMP comprises 4 steps that are initialization of input, identification, concatenation, and updating residual:

- In the initialization process from lines 1 to 3, sensing matrix H∈ $R^{m\times n}$, $b \in R^n$, P, $Q \leq (m-d)/Pd$ were initialized where His a sensing matrix whose number of columns is higher than the number of rows, $b$is compressed measurement column vector which has lesser number of elements compared to the unknown vector and the residual.
- Line 4 and 5 of the proposed algorithms have a sample space of all the column indices of H. Algorithm in step 4 will find out the Q-independent column indices by calculating the correlation between the columns of the sensing matrix and the residual vector of the previous iteration represented by r_{t-1}. Arrange the resulting column indices in descending order and pick 'Q' largest value indices in the current iteration $i_t = \{i_1 \ldots i_Q\}$.
- In line 6, concatenate the newly identified column and symmetric indices in the current iteration with the previous iteration entries $S_t = S_{t-1} \cap i_t$.

Line 7 to 9 is the residual update step; the support column indices in the current iteration will be transformed into the orthogonal space. In that way, the same column indices will not repetitively occur in the selection process. In that way, a unique set of support vector column indices will be found. With the help of it, the residual will be updated as specified in line 8. The loop count will be incremented, and the condition will be checked to execute the upcoming iteration. If the condition is false, it will come out of the loop and consolidate the active column indices to reconstruct the signal.

The key distinction between the BOMP method and the proposed approach lies in the fact that the BGOMP algorithm selects N block indices throughout each iteration. A larger value of N will lead to the selection of more block indices, hence reducing the computation time. In contrast, a smaller value of N leads to smaller block indices at each time, resulting in a longer computation time. In addition, the BGOMP algorithm, like the BOMP algorithm, requires prior knowledge of block sparsity [25]–[29]. It is worth noting that the residual r in the kth iteration of the BOMP is perpendicular to the column of $H_{\wedge^k}$, means that:

$$\langle H_{\wedge^k}, r^k \rangle = \langle H_{\wedge^k}, P_{\wedge^k}^{\perp} b \rangle = \langle H_{\wedge^k}^T, P_{\wedge^k}^{\perp} b \rangle = H_{\wedge^k}^T (P_{\wedge^k}^{\perp})^T b = 0 \quad (2)$$

As stated in the BOMP algorithm, the block indices that are newly selected do not overlap with the preceding ones. In other words, the cardinality of the intersection of the k-th block indices, denoted as $|\wedge^k|$is equal to Nk. However, it is possible that some of the indices chosen in each iteration are incorrect, resulting in $S \cap \wedge^k$ being less than or equal to Nk. Furthermore, the convergence within a maximum of K steps implies that, in each iteration, at least one accurate block index is selected, which can be expressed as $S \cap \wedge^k \geq k$. The BGOMP algorithm can be utilised in various applications, such as spectrum sensing [23]–[27].

2.4. Recovery guarantee condition

In this section, a constraint has been proposed to ensure the recovery of unknown signal based on the coherence for BGOMP, as shown in Figure 3.

Lemma 1: let Hsatisfy the block RIP of order P and Λ be set with $|\Lambda| \leq P$. Then for any$c \in R^m \|H_\Lambda^T c\| \leq (1 + \delta_P)\|c\|_2$.

Lemma 2: let Λ_1, Λ_2 be two sets of Ω with $|\Lambda_2 - \Lambda_1| \geq 1$ if a matrix A satisfies the block RIP of order $|\Lambda_1 \cup \Lambda_2|$, then for any vector.

Lemma 3: an arbitrary matrix A, the RIP constant δ_P bounded by $\delta_S \leq \mu(P-1)$ where P is the sparsity of the vector x and μ represents coherence of matrix A.

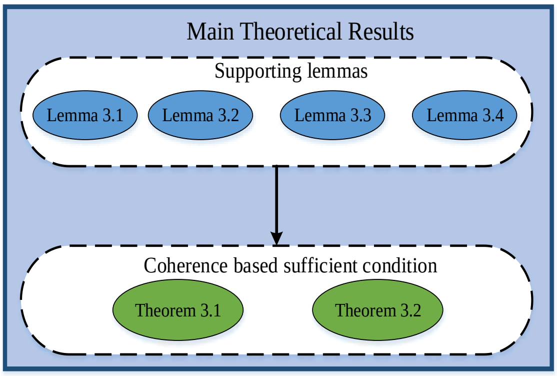

Figure 3. Block diagram of theoretical results based on the theorems and lemmas

Theorem1: let A satisfy the coherence iterations with $\mu \leq \frac{1}{1-\sqrt{\frac{|\omega|}{Q}+1}\,[(Q(P+1)+|\omega|-P)-1]\mu}$ for an integer Q with $1 \leq Q \leq (m-d)/Pd$. Then BGOMP identifies at least t_0 indices in ω if GOMP terminates after performing t_0 iterations with $1 \leq t_0 \leq P$ or recovers ω in P iterations if and only if $\min_{i \in P}\|c[i]\|_2 > \frac{2\varepsilon}{1-\sqrt{\frac{|\omega|}{Q}+1}\,[(Q(P+1)+|\omega|-P)-1]\mu}$.

Theorem 2: let A satisfy coherence with $\mu \leq \frac{1}{1-\sqrt{\frac{|\omega|}{Q}+1}\,(Q|\omega|)}$ for an integer Q with $1 \leq Q \leq (m-d)/Pd$. Then BGOMP identifies at least t_0 indices in ω if BGOMP terminates after performing t_0 iterations with $1 \leq t_0 \leq P$ or recovers ω in P iterations if and only if $\min_{i \in P}\|c[i]\|_2 > \frac{2\varepsilon}{1-\sqrt{\frac{|\omega|}{Q}+1}\,(Q|\omega|)\mu}$.

Lemma 4: let set $\Lambda \subseteq \omega$ satisfy $\Lambda = kP$ and $P \cap \Lambda = 1$ for some integers P, k and l with $0 \leq k \leq l \leq |P| - 1$ and $N(k+1) + |P| - k \leq m/d$. Let $W \subseteq P^c$ satisfy $|W| = Q$ and $W \cap \Lambda = \emptyset$. If H satisfies the block

RIP of order $Q(k+1)+|P|-l \leq m/d$, then $\left\|H_{P-\Lambda}^{T} P_{\Lambda}^{\perp} H_{P-\Lambda} c_{P-\Lambda}\right\|_{2,\infty} - \frac{1}{Q}\sum_{j\in W}\left\|H^{T}[j] H_{j}^{T} P_{\Lambda}^{\perp} H_{P-\Lambda} c_{P-\Lambda}\right\|_{2} \geq \frac{\left(1-\sqrt{\frac{|P|-l}{N}+1}\,\delta_{Q(k+1)+|P|-l}\right)\|c_{P-\Lambda}\|_2}{\sqrt{|P|-l}}$.

Proof: initially, we must demonstrate that the act of choosing Q elements that have at least one support index during iterations would be deemed as a successful detection. It is important to provide proof for the initial iteration. Let us assume:

$$V = \{j_1, j_2, \dots, j_N\} \tag{3}$$

Instead of proving $P_{t+1}\backslash P_t \cap \omega \neq \emptyset$, we will show:

$$|H_{j1}^{T} r_t| \geq \cdots \geq |H_{jN}^{T} r_t| \geq |H_{j\in\,\omega^c\backslash V}^{T} r_t| \tag{4}$$

$$\max_{j\in\,\omega} |H_i^T r_t| > |H_{jN}^T r_t| \tag{5}$$

By expression (6), $|H_{jQ}^{T} r_t \leq \frac{1}{Q}\max_{j\in W} |H_j^T r_t|$. Thus, to show $P_{t+1}\backslash P_t \cap \omega = \emptyset$, it suffices to show:

$$\max_{i\in W} |H_i^T r_t| > \frac{1}{Q}\max_{j\in W} |H_j^T r_t| \tag{6}$$

residual in the algorithm at the t^{th} iteration can be expressed as:

$$r_t = y - H_{S_t}\widehat{c_{S_t}} = (I - H_{S_t}(H_{S_t}^T H_{S_t})^{-1} H_{S_t}^T)y = P_{S_t}^{\perp}(H_\omega c_\omega + v) \tag{7}$$

$$= P_{S_t}^{\perp}(H_{\omega\cap S_t} c_{\omega\cap S_t} + H_{\omega\backslash S_t} c_{\omega\backslash S_t} + v)$$

$$= P_{S_t}^{\perp} H_{\omega\backslash S_t} c_{\omega\backslash S_t} + P_{S_t}^{\perp} v \tag{8}$$

Thus, by (9) and the triangular inequality.

$$\max_{i\in\omega} |H_i^T r_t| \geq \max_{i\in\omega\backslash S_t} \left(\left|H_i^T P_{S_t}^{\perp} H_{\omega\backslash S_t} c_{\omega\backslash S_t}\right| - |H_i^T P_{S_t}^{\perp} v|\right) \tag{9}$$

$$\frac{1}{N}\sum_{j\in W} |H_j^T r_t| \;\leq\; \frac{1}{N}\sum_{j\in W} |H_j^T P_{S_t}^{\perp} H_{\omega\backslash S_t} c_{\omega\backslash S_t}| + \max_{j\in W} |H_i^T P_{S_t}^{\perp} v| \tag{10}$$

To find the lower bound on$\max_{i}\omega|H_i^T r_t|$. A lower bound on $\max_{i\in\omega}|H_i^T r_t|$will be derived, requiring $Q \leq P$. Thus, to show (10), it suffices to show:

$$\beta_1 > \beta_2 \tag{11}$$

$$\beta_1 = \max_{i\in\omega\backslash S_t} |H_i^T P_{S_t}^{\perp} H_{\omega\backslash S_t} c_{\omega\backslash S_t}| - \frac{1}{N}\sum_{j\in W} |H_j^T P_{S_t}^{\perp} H_{\omega\backslash S_t} c_{\omega\backslash S_t}| \tag{12}$$

$$\beta_2 = \max_{i\in\omega\backslash S_t} |H_i^T P_{S_t}^{\perp} v| + \max_{j\in W} |H_j^T P_{S_t}^{\perp} v| \tag{13}$$

$$\max_{i\in\omega\backslash S_t} |H_i^T P_{S_t}^{\perp} v| = |H_{i_0}^T P_{S_t}^{\perp} v| \tag{14}$$

$$\max_{j\in W} |H_i^T P_{S_t}^{\perp} v| + \max_{j\in W} |H_{j0}^T P_{S_t}^{\perp} v| \tag{15}$$

$$\beta_2 = \left\|H_{i_0\cup j_0}^T P_{S_t}^{\perp} v\right\|$$

$$\leq \sqrt{2}\left\|H_{i_0\cup j_0}^T P_{S_t}^{\perp} v\right\|_2$$

$$\leq \sqrt{2(1+\delta_{Q(P+1)+|\omega|-P})}\|v\|_2 \tag{16}$$

Lower bound on β_1 will be derived $0 \leq t \leq |\omega \cap S_t| = l \leq |\omega| - 1$;

$$\beta_1 \geq \frac{1-\sqrt{\frac{|\omega|}{Q}+1}\,\delta_{Q(P+1)+|\omega|-l)}\left\|x_{\omega\setminus s_t}\right\|_2}{\sqrt{|\omega|-l}} \tag{17}$$

$$\left\|x_{\omega\setminus s_t}\right\|_2 \geq \sqrt{\frac{|\omega-l|}{P(1+\delta_{Q(P+1)+|\omega|-P})}}\sqrt{\text{MAVR.SNR}}\|v\|_2 \tag{18}$$

$$\left\|x_{\omega\setminus s_t}\right\|_2 \geq \sqrt{|\omega|-l}\min_{i\in\omega}|x_i| \tag{19}$$

$$= \sqrt{|\omega|-l}\,(\sqrt{\text{MAVR}}\|x\|_2/\sqrt{K})$$

$$\geq \sqrt{\frac{|\omega-l|}{P(1+\delta_{Q(P+1)+|\omega|-P})}}\sqrt{\text{MAVR}}\|Hc\|_2$$

$$\geq \sqrt{\frac{|\omega-l|}{P(1+\delta_{Q(P+1)+|\omega|-P})}}\sqrt{\text{MAVR.SNR}}\|v\|_2$$

$$\|Hc\| = \|H_\omega c_\omega\|_2 \tag{20}$$

$$\leq \sqrt{(1+\delta_{Q(P+1)+|\omega|-P})}$$

$$\beta_1 \geq \frac{1-\sqrt{\frac{|\omega|}{Q}+1}\,\delta_{Q(P+1)+|\omega|-P})\sqrt{MAVR.\,SNR}\|v\|_2}{\sqrt{P(1+\delta_{Q(P+1)+|\omega|-P})}} \tag{21}$$

$$\frac{(1-\sqrt{\frac{|\omega|}{Q}+1}\,\delta_{Q(P+1)+|\omega|-P})\sqrt{MAVR.\,SNR}\|v\|_2}{\sqrt{P(1+\delta_{Q(P+1)+|\omega|-P})}} > \sqrt{2(1+\delta_{Q(P+1)+|\omega|-P})}\|v\|_2 \tag{22}$$

$$\min_{i\in S}\|c[i]\|_2 > \frac{2\varepsilon}{1-\sqrt{\frac{|\omega|}{Q}+1}\delta_{Q(P+1)+|\omega|-P}} \tag{23}$$

By applying the Lemma 3 in the expression (23), will be given as:

$$\min_{i\in P}\|c[i]\|_2 > \frac{2\varepsilon}{1-\sqrt{\frac{|\omega|}{Q}+1}\,(Q|\omega|)\mu} \tag{24}$$

Hence, the proof. From the (23), it has been proved that BGOMP can recover t_0 block indices in t_0 iterations, and $\mu \leq \frac{1}{1-\sqrt{\frac{|\omega|}{Q}+1}\,(Q|\omega|)}$ is the coherence-based sufficient condition. Table 1 explains the comparison of the proposed results with the state-of-the-art methods.

Table 1. Comparison of proposed results with the state-of-the-art methods

Parameter	BOMP	BCoSaMP [15]	Proposed
Method	RIP based BOMP	RIP based BCoSAMP	Coherence based BGOMP
Performance guarantee	$\delta_{d(K+1)} < \frac{1}{\sqrt{K+1}}$	$\delta_{4dK} < 0.5$	$\mu \leq \frac{1}{1-\sqrt{\frac{\lvert\omega\rvert}{Q}+1}\,(Q\lvert\omega\rvert)}$
Maximum number of iterations	Sparsity P	Sparsity P	Sparsity P
Number of selection blocks per iteration	1	2P	$Q \leq (m-d)/Pd$

Table 1 shows coherence based sufficient condition is within the polynomial time and it can select multiple active block index per iteration. BOMP, compressive sampling matching pursuit (CoSaMP) recovery guarantees are based on RIC. It is NP hard to find RIC to satisfy RIP that has been overcome by finding a sufficient condition based on coherence for BGOMP algorithm. BGOMP can able to recover at least ‘P’ number of support vectors in ‘P’ iterations implies that it is has more probable to recover all the support vectors in lesser iterations than the BOMP and CoSaMP. Though CoSaMP chooses '2P’ number elements in every iteration, there is a chance of choosing the same column indices chosen in the previous iterations.

3. RESULTS AND DISCUSSION

Simulation of an underdetermined system to detect the support vector index will be discussed: construct a m by n sensing matrix with elements derived from a Gaussian distribution with N (0, 1) as the range, and normalize each column to unity [30]. The observation vector can be computed using the equation b=A c, where m=50 and n=128 remain constant during the experiment. Block sparse signal c is generated, and the position of the non-zero elements is randomly chosen. Figures 4 and 5 shows the detection performance of greedy algorithm for different block length levels d=4, 8 for d=4 BGOMP outperforms the other greedy algorithms. BGOMP can able to exactly recover the support column indices even when it is equal to d=4. It is evident from the Figures 4 and 5, BGOMP can able to recover all the occupied bands for d=4 and K≤28. When the block length is increased from d=4 to d=8 algorithms like BCoSAMP, BOMP and BSP cannot able to recover successfully for K>14.

Figure 6 shows the recovery performance of BGOMP algorithm for varying number of elements picked per iteration. As the number of selection element increases the detection perform got improved slightly. Next consider the computation time of BGOMP with other algorithms like BOMP, BSP, and BCoSaMP. Simulation has been carried out using Intel® Core™ i5-8265 2.3GHz with 8GB memory in Windows 10.

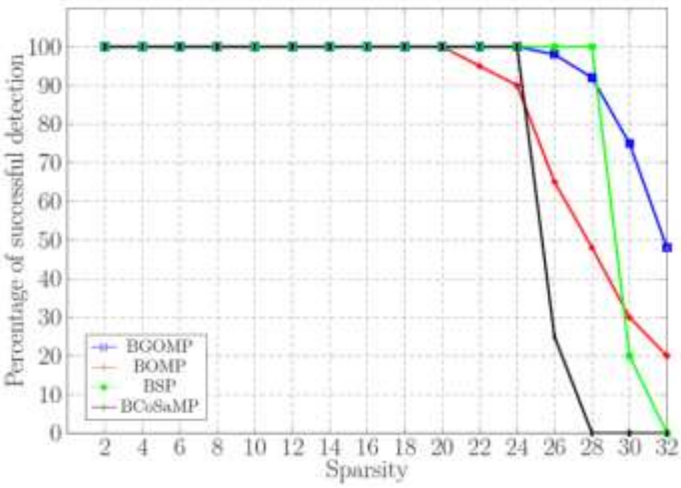

Figure 4. Detection of support vectors of the unknown sparse signal where block length is 4 using different greedy algorithms

BGOMP can able to select multiple active support column indices per iteration. When the block sparsity increases i.e., number of non-zero elements increase BGOMP computation time will be lesser than BOMP and BCoSaMP because BOMP can choose only one active element per iteration leads to higher computation time. In the case of BCoSaMP it can choose ‘2P’ elements in each iteration it will choose the same active index repetitively in the upcoming the iteration leads to a greater number of iterations to recover the support column indices. When the block sparsity is greater than 24, 64 BGOMP can able to recover successfully in lesser computation time than the BOMP, block subspace pursuit (BSP) and BCoSaMP. Table 2 shows the average computation time of other algorithms. BGOMP is quicker than BOMP, BCoSaMP and BSP for different block length and block sparsity.

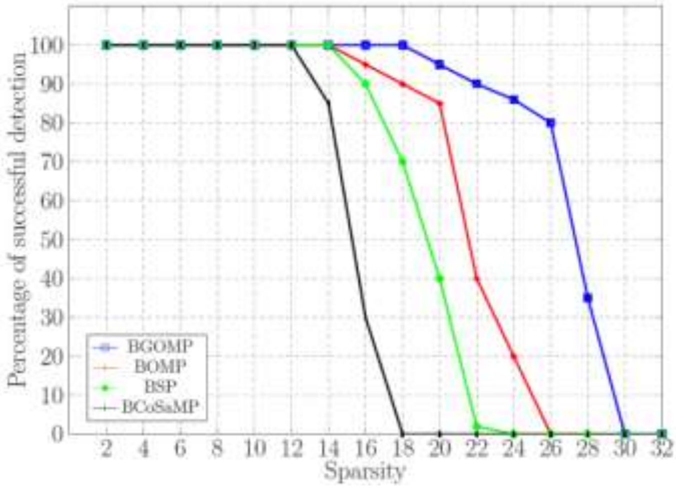

Figure 5. Detection of support vectors of unknown sparse signal where block length is 8 using different greedy algorithms

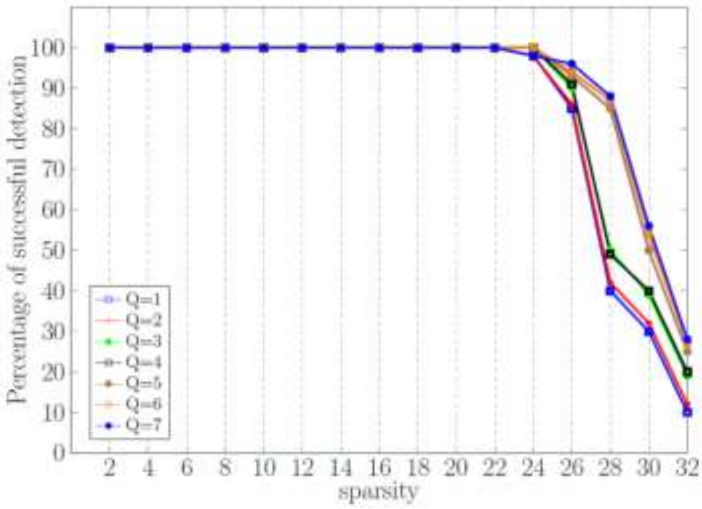

Figure 6. Detection of support vectors by varying the number of elements selected per iteration Q for different sparsity values using BGOMP

Table 2. Computation time versus block sparsity for state-of-the-art algorithms

Block length(d)	Block sparsity (K)	BGOMP(s) proposed	BOMP(s) [20]	BSP(s) [20]	BCOSaMP(s) [20]
2	2	0.0008	0.0009	0.0009	0.0007
2	24	0.0021	0.0043	0.0028	0.0024
2	64	0.0221	0.0236	0.0034	0.0448
4	2	0.0004	0.0005	0.0005	0.0003
4	12	0.0015	0.0023	0.0019	0.0016
4	32	0.0111	0.0122	0.0036	0.4455
16	2	0.0004	0.0005	0.0006	0.0005
16	3	0.0006	0.0008	0.0009	0.0007
16	8	0.0032	0.0033	0.0041	0.0183

4. CONCLUSION

Coherence based sufficient condition to recover the unknown sparse vector using block generalized orthogonal matching pursuit has been derived. The coherence based condition for BGOMP is a novel condition to recover the unknown block sparse vector from an under determined system. Computation time for block sparsity>2 for BGOMP is lesser than BOMP in all the cases. BGOMP can able to exactly recover the active sub band indices even when block length is equal to d=4. It is evident that BGOMP outperforms all the other greedy algorithms for different block length and block sparsity. BGOMP outperforms BOMP in all the cases of block sparsity ranges from 2, 12, 32, 64 because BGOMP can select multiple blocks per iteration to process leads to lesser number of iteration than BOMP. Coherence based sufficient condition for BGOMP has improved the detection performance for different block sparsity and block length. BGOMP computation time outperforms other state of the art algorithms like BOMP, BSP and BCoSaMP.

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