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Study on VRPTW Based on Improved Particle Swarm Optimization

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Abstract

Vehicle routing problem with time windows (VRPTW) is a typical non-deterministic polynomial hard (NP-hard) optimization problem. In order to overcome PSO's slow astringe and premature convergence, an improved particle swarm optimization (IPSO) is put forward. In the algorithm, it uses the population entropy to makes a quantitative description about the diversity of the population, and adaptively adjusts the cellular structure according to the change of population entropy to have an effective balance between the local exploitation and the global exploration, thus enhance the performance of the algorithm. In the paper, the algorithm was applied to solve VRPTW, the mathematical model was established and the detailed implementation process of the algorithm was introduced. The simulation results show that the algorithm has better optimization capability than PSO.

Keywords*: vehicle routing problem with times window (VRPTW), particle swarm optimization (PSO), population entropy*

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1. Introduction

Vehicle routing problem was proposed by Dantzig Z and Ramser J in 1959 [1], it refers to the design of a set of minimum-cost vehicle routes, originating and terminating at a distribution center, for a fleet of vehicles that services a set of customers with certain demands. Each customer is serviced exactly once, furthermore, all customers must be assigned to vehicles which satisfy certain constraint conditions.

In our life, many problems in logistics vehicle can be attributed to the VRPTW. Although the problem has been widely applied, it is not still good solutions in the theory field. VRP has been proved to be NP problem, while VRPTW increase the time constraints on the basic vehicle routing problem and its implementation is more complex than VRP. When the scale of the problem is small, it is possible to get exact solution, but it is difficult to get the optimal solution for large dimension problems. In recent years, GA, ACA and other heuristic optimization algorithms have been applied to solve the problems [2-5], but these algorithms all have long search time and easily fall into local optimal. Therefore, how to construct optimized algorithm simple in formation and high in optimizing precision enjoys great significance in solving solve VRPTW.

Particle swarm optimization (PSO) is a set of new intelligent optimization algorithms which is simple calculation, fast convergence and easy to implement, etc [6]. The algorithm has been applied to VRP [7-8] and achieved very good results, but it exist such problems as being easy to stagnate and fall into local optimum. Therefore, in this paper, an adaptive particle swarm optimization based on population entropy is put forward by adaptively adjusting the cellular structure according to the change of population entropy and applied to solve VRPTW. The result of experiments indicates that the algorithm can efficiently solve the problem.

2. Description about VRPTW and Mathematical Model

2.1. Description about VRPTW

The VRPTW can be described as followed: There are a distribution center and N customers, distribution center has M vehicles of capacity Q, d_{ij} is transport distance from

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customer i to customer j, v is the speed of the vehicle, g_i $(i = 1,..., N)$ is demand goods for customer i and g_i is less than Q. Each vehicle route must start and finish at the same depot, now find driving routes from the center to each customer site with the time is not later than l_i $(i = 1,..., N)$ and meeting the following requirements:

1) Overload is forbidden.
2) Delivery must be finished as soon as possible.
3) Supplies required by each demanding site must be delivered.
4) the total cost must be at minimum.

Supplementary conditions:

1) Adequate vehicles are available in the distribution center, and the capacity of each has been given;
2) There is only one distribution center and its location has been given;
3) There is only one type vehicle in the center. Each customer site is served by only one vehicle. Every site must be covered by the driving route;
4) the gross demand of each demanding site along the route cannot surpass the capacity of the vehicle;
5) In order to simplify the case, only the time consumed on road will be considered, irrespective of the service time.

2.2. Mathematic Mode

Here is the mathematic mode built according to the description above:
(1)---objective function (2)-(9)---constraint conditions

$$f = \min \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=0}^{M} d_{ij} x_{ijk} \tag{1}$$

$$\sum_{k=1}^{M} y_{0k} \le M \tag{2}$$

$$\sum_{i=1}^{N} g_i y_{ik} \le Q \ , \ k \in \{1,..., M\} \tag{3}$$

$$\sum_{k=1}^{M} y_{ik} = 1 \ , \quad i \in \{1,..., N\} \tag{4}$$

$$\sum_{i=0}^{N} x_{ijk} = y_{jk} \ , \ j \in \{1,..., N\} \ , \ k \in \{1,..., M\} \tag{5}$$

$$\sum_{j=0}^{N} x_{ijk} = y_{ik} \ , \ i \in \{1,..., N\} \ , \ k \in \{1,..., M\} \tag{6}$$

$$t_{jik} = d_{ij} / v \ , \quad i, j \in \{1,..., N\} \tag{7}$$

$$t_{ik} = t_{jk} + t_{jik} x_{ijk} \ , \quad i, j \in \{1,..., N\} \ , \quad k \in \{1,..., M\} \tag{8}$$

$$a_i \le t_{ik} \le b_i \ , \ i, j \in \{1,..., N\} \ , \ k \in \{1,..., M\} \tag{9}$$

In the model, formulation (1) is the target function; formulation (2) limit that the total number of vehicles dispatched from depot does not exceed the total number of the distribution center has; formulation (3) ensure each vehicle load does not exceed its carrying capacity; formulation (4) show the task of customer can only be delivery by a car; formulation (5) and formulation (6) guarantee that each customer will be visited exactly once; formulation (7) , formulation (8)and formulation (9) are time constraint conditions of vehicle . x_{ijk} and y_{0k} are defined as follows in formula (10) and formula (11):

$$x_{k0} = \begin{cases} 1 & \text{emergency vehicle k is in use} \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$y_{0k} = \begin{cases} 1 & \text{vehicle k is in use} \\ 0 & otherwise \end{cases} \tag{11}$$

3. Improved Particle Swarm Optimization

3.1. Particle Swarm Optimization Algorithm

PSO is a newly-emerged bionic algorithm imitating birds to find food developed by Dr. Eberhart and Dr. Kennedy in 1995. In the PSO, each particle keeps track of its coordinates in the problem space which are associated with the best solution it has achieved so far, the value is called $pbest$. When a particle takes all the population as its topological neighbors, the best value is called $gbest$. The particle swarm optimization concept consists of, at each time step, changing the velocity of each particle toward its $pbest$ and $gbest$ locations. Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward $pbest$ and $gbest$ locations.

After finding the two best values, the particle updates its velocity and positions with following Equation (12) and (13).

$$v_{id}^{k+1} = wv_{id}^{k} + c_1 * r_1(pbest - x_{id}^{k}) + c_2 * r_2(gbest - x_{id}^{k}) \tag{12}$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1} \tag{13}$$

In the formula: x_{id}^{k} is the position of particle i, v_{id}^{k} is the velocity of particle i, $v_{id}^{k} \in [V_{\min}, V_{\max}]$, $V_{\min}$ and $V_{\max}$ defined by the user are two constants, w is inertia weighting factor, c1 and c2 are learning factors, , usually c1 = c2 = 2, r_1 and r_2 are two random number between (0, 1).

3.2. Population Entropy

Selecting proper A and B to meet particle fitness values are interval $[A, B]$, the interval $[A, B]$ is divided into N equal parts, the particle numbers of N subintervals are S_1, S_2 ,..., S_N , the population entropy are defined as shown in Equation (14).

$$E = -\sum_{i=1}^{N} \frac{S_i}{N} \lg \frac{S_i}{N} \tag{14}$$

3.3. Improved Particle Swarm Optimization

In the paper, improved particle swarm optimization is designed by using two-dimensional annular cellular structure, all particles are placed inside and each particle has 5 parts which is the particle itself and its four neighbors. When the population entropy decreases very quickly, the cellular structure is narrowed to strengthen the global search ability of algorithm; otherwise, the cellular structure is widened to strengthen the local searching ability of algorithm. Because each particle learns the best particle information of their neighbors, the location and velocity of new algorithm is updated with the Equation (15) and the Equation (16).

$$v_{id}(t+1)=wv_{id}(t)+c_1r_1(pbest_{id}(t)-x_{id}(t))+c_2r_2(gbest_{id}(t)-x_{id}(t)) \quad (15)$$

$$x_{id}(t+1)=x_{id}(t)+v_{id}(t+1) \quad (16)$$

In the paper, only two-dimensional cellular structures are respectively written as $r' \times c'$ and $r \times c$ which is adopted in order to facilitate the description, $r \times c$ is corresponding to wide cellular structure, while $r' \times c'$ is corresponding to narrow cellular structure and $r \times c = r' \times c'$,among of which, r and r' express the line number , c and c' express the column number, so the particle position of conversing cellular structure is defined with the Equation (17) [9].

$$(i, j) \rightarrow ([i \times c + j] div c', [i \times c + j] \bmod c') \quad (17)$$

4. The Solution to VRPTW Based on the Algorithm Proposed in the Paper

4.1. Coding Strategy of Particles

The position of particle corresponding with the answer to the question is the key idea of PSO. In the paper, a new particle encoding of three-dimensional vectors Z on the basis of reference [10] is constructed to VRPTW. In the vector Z , the first dimension Z_{ix} of particles is vehicle information; the second dimension Z_{iy} of particles is traveling distance of the vehicle.

4.2. Decoding Strategy of Particles

To get to the vehicle order of the travel path, Z_{iy} must be adjusted. Adjustment function can be got according to the size sequence of the vector Z_{iy} , that is, finding Z_{iy} of the vehicle for customer i first, and then sorted from small to large according to the size of Z_{iy} , thus the driving path order of vehicle i is obtained. For example, if there are 10 customers and 4 vehicles. If the position vector Z_{iy} of a particle is shown as Table 1, then vector Z_{iy} of the position after adjustment is shown as Table 2. So the corresponding vehicle routings are as follows (0 represents the distribution center):

vehicle 1，0→2→6→0；
vehicle 2，0→1→4→3→10→0；
vehicle 3，0→7→9→5→0；
vehicle 3，0→8→0。

Table 1. The Vector of Particle i before Decoding

customer	1	2	3	4	5	6	7	8	9	10
Z_{ix}	2	1	2	2	3	1	3	4	3	2
Z_{iy}	1.3	2.6	3.3	2.4	4.6	3.4	2.8	0.9	3.6	5.7

Table 2. The Vector of Particle i after Decoding

customer	1	2	3	4	5	6	7	8	9	10
Z_{ix}	2	1	2	2	3	1	3	4	3	2
Z_{iy}	1	1	3	2	3	2	1	1	2	4

4.3. Process Description of Algorithm

Step 1: initialize: Set the population size N , inertia weight coefficient w , accelerating factor c_1 and c_2 , maximum number of iterations $N_{\max}$, constant ε . The current iteration number t is set to 1 , random generation initial position $x_1, x_2, ...x_N$ and initial velocity $v_1, v_2, ..., v_N$ of N particles; the N particles are placed in the cellular of $r \times c$ according to the order of initialization.

Step 2: Calculate the fitness value f_i of the i particle, and seek the best particle information $gbest_i$ $(i = 1,2,..., N)$ of their neighbors.

Step 3: For all the particles, if $f_i > pbest_i$, then $pbest_i = f_i$, $x_{pbest_i} = x_i$, update $gbest_i$ according to the new particle fitness value.

Step 4: Calculate population entropy E_t of t iteration, update each particle's position and speed with the formula (15) and formula (16).

Step 5: Separately calculate $\Delta E_t = E_t - E_{t-1}$ and $\Delta E_{t-1} = E_{t-1} - E_{t-2}$, if satisfying $\Delta E_t > (2-\varepsilon) \times \Delta E_{t-1}$, the structure of cellular is converted to $r' \times c'$, if satisfying $\Delta E_t < (1+\varepsilon) \times \Delta E_{t-1}$, the structure of cellular is converted to $r \times c$.

Step 6: Judge whether the current iteration number t is greater than the iteration number $N_{\max}$, if not satisfied, $t = t + 1$ return Step2, otherwise, the optimal solutions is output.

5. Experimental Examples

5.1. Example 1

To verify the viability of algorithm, randomly generated VRPTW through the computer within the range of 100×100(unit: km), the location (unit: km) and demand (unit: t) and time windows (unit: minute) of customers is shown as Table 3. Distribution center has five vehicles, each vehicle's speed is 65 km/hour and each vehicle's maximum capacity is 25.

Table 3. Customer Information

customers	X coordinates	Y coordinates	demand	times windows
0	50	50		
1	39	4	7	(0,100)
2	19	94	1	(0,90)
3	97	42	2	(0,120)
4	31	72	8	(0,60)
5	37	26	2	(0,150)
6	63	55	3	(0,240)
7	95	82	2	(0,180)
8	28	59	3	(0,30)
9	54	28	5	(0,180)
10	7	22	2	(0,60)
11	16	2	4	(0,90)
12	80	3	1	(0,60)
13	39	98	5	(0,120)
14	66	29	6	(0,30)
15	86	60	1	(0,150)
16	4	76	4	(0,60)
17	96	8	5	(0,90)
18	21	38	3	(0,60)
19	74	73	4	(0,210)
20	50	78	3	(0,150)

The equation is calculated respectively by PSO and IPSO. Through calculation, the optimal scheduling schemes of vehicle are shown as Table 4, the objective function changes as number of iterations is shown as Figure 1.

Table 4. The Optimal Scheduling Scheme of Example

Vehicle	driving route	driving distance	driving time
Vehicle 1	0→8→4→16→2→13→20→0	159.1	146.8
Vehicle 2	0→18→10→11→1→5→9→0	159.2	147.0
Vehicle 3	0→14→12→17→3→15→7→19→6→0	209.4	193.3
Total		527.7	487.1

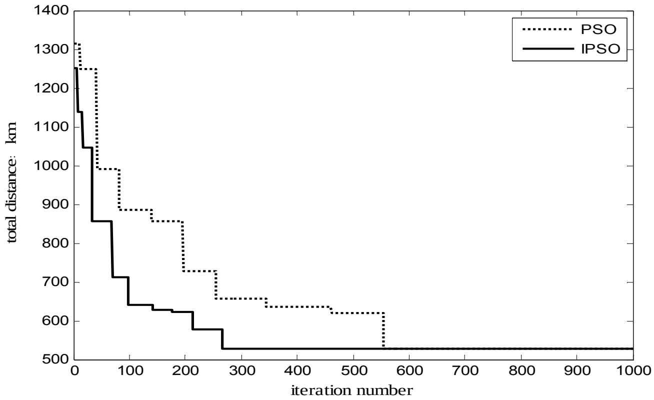

Figure 1. Performance Comparison of PSO and IPSO

5.2. Example 2

An area has 25 demand sites needing resource delivered from distribution center. There are enough vehicles. The loading capacity is 30 ton for each vehicle. The average speed is 25km/hour. The coordinate of distribution center is 0 point (6km, 4km), position coordinates of demand sites (unit: km), the demand q_i (unit: ton)and time windows (unit: minute) are displayed in Table 5 (0 represents for the center, 1-25 represents for the demand sites).

The equation is calculated respectively by PSO and IPSO. Through calculation, the optimal scheduling schemes of vehicle are shown as Table 6, the objective function changes as number of iterations is shown as Figure 2.

Table 5. Customer Information

customers	X coordinates	Y coordinates	demand	times windows
0	6	4		
1	5	8	2	(0,20)
2	9	7	9	(0,20)
3	5	3	6	(30,60)
4	7	7	8	(0,10)
5	0	2	2	(10,30)
6	10	8	4	(10,30)
7	2	9	3	(10,30)
8	4	1	3	(10,30)
9	4	5	3	(30,60)
10	9	7	2	(10,30)
11	7	1	8	(20,50)
12	4	4	4	(0,10)
13	3	6	1	(30,60)
14	9	10	5	(10,20)
15	1	1	3	(10,40)
16	10	1	5	(10,30)
17	0	6	9	(20,50)
18	2	3	8	(0,20)
19	8	5	6	(30,60)
20	3	0	1	(20, 50)
21	7	9	5	(0,20)
22	8	0	6	(10,30)
23	4	10	7	(0,20)
24	5	1	2	(20,50)
25	5	7	4	(0,10)

Table 6. The Optimal Scheduling Scheme of Example

Vehicle	driving route	driving distance	driving time
Vehicle 1	0→4→21→14→6→10→19→0	15.53	37.27
Vehicle 2	0→25→1→23→7→17→13→9→0	18.9	45.36
Vehicle 3	0→12→18→5→15→20→8→24→3→0	15.95	38.28
Vehicle 4	0→2→16→22→11→0	17.13	41.11
Total		67.51	162.02

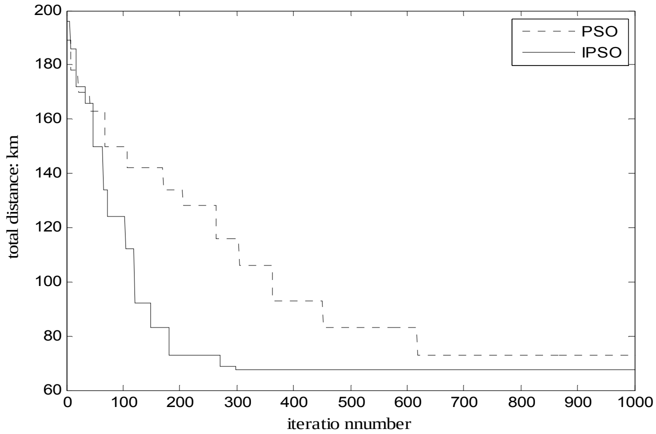

Figure 2. Performance Comparison of PSO and IPSO

6. Conclusion

From the simulation result of the two experiments, it could quickly and accurately find the optimal solution to VRPTW using IPSO, which provides a new idea for VRPTW.

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