The Non-equidistant Multivariable New Information MGM(1,n) Based on New Information Background Value Constructing

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Abstract

Applying the principle in which new information should be used fully and modeling method of grey system for the problem of lower precision as well as lower adaptability in non-equidistant multivariable MGM(1,n)model, a non-equidistant multivariable new information MGM(1,n) model was put forward which was taken the mth component as the initialization. As the background value is an important factor affecting the precision of grey system model, based on index characteristic of grey model, the characteristic of integral and new information principle, the new information background value in non-equidistant multivariable new information MGM(1,n) was researched and the discrete function with non-homogeneous exponential law was used to fit the accumulated sequence and the formula of new information background value was given. This new non-equidistant multivariable MGM(1,n) model can be used in equidistance & non-equidistance modeling and has the characteristic of high precision as well as high adaptability. Examples validate the practicability and reliability of the proposed model.

Keywords: Multivariable, background value, new information, non-equidistance sequence, non-equidistance MGM (1,n) model , least square method

1. Introduction

Grey model as an important part in grey system theory has been widely used in many fields since Professor J.L. Deng proposed the grey system. There are many grey models, foremost of which is GM (1,1), GM (1, n), MGM (1, n) [1-3], GOM (1,1) [4], and GRM (1,1) [5]. There often contain multiple variables which are intrinsically linked each other in social, economic and engineering systems. In spite of extending from GM (1,1) model in the case of n variables, MGM (1,n) model is not a simple combination of the GM (1,1) models, but also different from the GM (1, n) model establishing a single first-order differential equation with n variables. This model need to establish n differential equations with n variables to solute, and these parameters of MGM(1,n) can reflect the relationships of mutual influence and restriction among multiple variables [6]. Most of grey system models are based on equidistant sequence, but the original data obtained from the actual work are mostly non-equidistant sequence. So that establishing non-equidistant sequence model has a certain practical and theoretical significance. The optimizing model of MGM (1,n) was set up by taking the first component of the sequence $\mathbf{x}^{(1)}$ as the initial condition of the grey differential equation and modifying [2]. According to new information priority principle in the grey system, multivariable new information MGM(1,n) model taking the nth component of $\mathbf{x}^{(1)}$ as initial condition was established [7]. Taking the nth component of $\mathbf{x}^{(1)}$ as initial condition and optimizing the modified initial value and the coefficient of background value q where the form is $z_i^{(1)} = q x_i^{(1)}(k+1) + (1-q) x_i^{(1)}(k), \quad (q \in [0,1])$ the multivariable new information MGM(1,n) model was established [8]. These MGM(1,n) models are equidistance, the non-equidistance multivariable MGM(1,n) model with homogeneous exponent function fitting background value was established [9]. However, it is more widespread of non-homogeneous exponent function, so there are inherent defects in the modeling mechanism of this model. The non-equidistance multivariable MGM(1,n) model was established [10], where its background value is generated by mean value so as to bring about lower accuracy. The non-equidistance multivariable GM(1,n) model based on non-

homogeneous exponent function fitting background value was established [11], that improves the accuracy of the model. The building method for background value in MGM(1,n) was analyzed and a method of reconstructing background value was put forward which was based on vector continued fractions theory by using rational interpolation, trapezoidal rule in numerical integration and extrapolation formula [12]. This model can effectively improve simulation and prediction, but is a equidistance multivariable MGM(1,n) model. The constructing method for background value is a key factor affecting the prediction accuracy and the adaptability, so the optimization for background values is an important means of improving the model. In order to improve the accuracy of GM(1,1), some constructing methods for background value were proposed and some non-equidistance GM(1,1) model were established [13-17]. In this paper, the modeling method in [17] was absorbed. Based on index characteristic of grey model, the characteristic of integral and new information principle, the new information background value in non-equidistant multivariable new information MGM(1,n) was researched and the discrete function with non-homogeneous exponential law was used to fit the accumulated sequence and the formula of new information background value was given. The new non-equidistant multivariable MGM(1,n) model can be used in equidistance & non-equidistance modeling and extend the application range of the grey model. There is higher precision, better theoretical and practical value in this model.

2. Non-equidistant Multivariable New Information Grey Model MGM (1,n)

Definition 1: Supposed the sequence $\mathbf{X}_i^{(0)} = [x_i^{(0)}(t_1), x_i^{(0)}(t_2), \cdots, \ x_i^{(0)}(t_j), \cdots, x_i^{(0)}(t_m)]$, if $\Delta t_j = t_j - t_{j-1} \neq const$, where $i = 1, 2, \cdots, n, j = 2, \cdots, m$, n is the number of variables and m is the sequence number of each variable, $\mathbf{X}_i^{(0)}$ is called as non-equidistant sequence.

Definition 2: Supposed the sequence $\mathbf{X}_i^{(1)} = \{x_i^{(1)}(t_1), x_i^{(1)}(t_2), \cdots, \ x_i^{(1)}(t_j), \cdots, x_{t_j}^{(1)}(t_m)\}$, if $x_i^{(1)}(t_1) = x_i^{(0)}(t_1)$ and $x_i^{(1)}(t_j) = x_i^{(1)}(t_{j-1}) + x_i^{(0)}(t_j) \cdot \Delta t_j$ where $j = 2, \cdots, m, \ i = 1, 2, \cdots, n,$ and $\Delta t_j = t_j - t_{j-1}$, $\mathbf{X}_i^{(1)}$ is one-time accumulated generation of non-equidistant sequence $\mathbf{X}_i^{(0)}$, and it is denoted by 1-AG0.

Supposed the original data matrix:

$$\mathbf{X}^{(0)} = \{\mathbf{X}_1^{(0)}, \mathbf{X}_2^{(0)}, \cdots, \mathbf{X}_n^{(0)}\}^{\mathbf{T}} = \begin{vmatrix} x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \cdots & x_1^{(0)}(t_m) \\ x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \cdots & x_2^{(0)}(t_m) \\ \cdots & \cdots & \cdots & \cdots \\ x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \cdots & x_n^{(0)}(t_m) \end{vmatrix} \tag{1}$$

Where, $\mathbf{X}^{(0)}(t_j) = [x_1^{(0)}(t_j), x_2^{(0)}(t_j), \cdots, x_n^{(0)}(t_j)]$ $(j = 1, 2, \cdots m)$ is the observation value of each variable at t_j, and the sequence $[x_i^{(0)}(t_1), x_i^{(0)}(t_2), \cdots, x_i^{(0)}(t_j), \cdots, x_i^{(0)}(t_m)]$ $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, m)$ is non-equidistant, that is, the distance $t_j - t_{j-1}$ is not constant.

In order to establish the model, firstly the original data is accumulated one time to generate a new matrix as:

$$\mathbf{X}^{(1)} = \{\mathbf{X}_1^{(1)}, \mathbf{X}_2^{(1)}, \cdots, \mathbf{X}_n^{(1)}\}^{\mathbf{T}} = \begin{bmatrix} x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m) \\ x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m) \\ \cdots & \cdots & \cdots & \cdots \\ x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m) \end{bmatrix} \tag{2}$$

Where, $x^{(1)}(t_j)(j = 1, 2, \cdots, m)$ meets the conditions in the definition 2, that is,

$$x_i^{(1)}(t_j) = \begin{cases} \sum_{j=1}^{k} x_i^{(0)}(t_j)(t_j - t_{j-1})(k = 2, \cdots, m) \\ x_i^{(0)}(t_1) \qquad\qquad (k = 1) \end{cases} \tag{3}$$

Non-equidistant multivariable MGM (1, n) model can be expressed as first-order differential equations with n variables:

$$\begin{cases} \dfrac{dx_1^{(1)}}{dt} = a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + \cdots + a_{1n}x_n^{(1)} + b_1 \\ \dfrac{dx_2^{(1)}}{dt} = a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + \cdots + a_{2n}x_n^{(1)} + b_2 \\ \qquad\qquad\qquad \cdots \\ \dfrac{dx_n^{(1)}}{dt} = a_{n1}x_1^{(1)} + a_{n2}x_2^{(1)} + \cdots + a_{nn}x_n^{(1)} + b_n \end{cases} \tag{4}$$

Assumed $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}$, Equation (4) can be expressed as:

$$\frac{d\mathbf{X}^{(1)}(t)}{dt} = \mathbf{A}\mathbf{X}^{(1)}(t) + \mathbf{B} \tag{5}$$

According to new information priority principle in the grey system, it is inadequate for utilizing new information when the first component of the sequence $\mathbf{x}_i^{(1)}(t_j)(j = 1,2,\cdots,m)$ is taken as initial condition of grey differential equation. Regarded the m th component as the initial conditions of the grey differential equation, the continuous time response of Equation (5) is as:

$$\mathbf{X}^{(1)}(t) = e^{At}\mathbf{X}^{(1)}(t_m) + \mathbf{A}^{-1}(e^{At} - \mathbf{I})\mathbf{B} \tag{6}$$

Where, $e^{At} = \mathbf{I} + \sum_{k=1}^{\infty} \dfrac{\mathbf{A}^k}{k!}t^k$, $\mathbf{I}$ is a unit matrix.

In order to identify $\mathbf{A}$ and $\mathbf{B}$, Equation (4) is made the integration in $[t_{j-1}, t_j]$ and we can obtain:

$$x_i^{(0)}(t_j)\Delta t_j = \sum_{l=1}^{n} a_{il}\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt + b_i\Delta t_j (i = 1,2,\cdots,n; j = 2,3,\cdots,m) \tag{7}$$

Noting $z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$, the common formula for background value that is actually based on the trapezoidal area $z_i^{(1)}(t_j)\Delta t_j$ is appropriate when the time interval is small, that is, the change of sequence data is slow. But when this change is sudden, the background value using the common formula often brings out the larger error, and the model parameters obtained by the common formula for background value in the new information model is the same as the one in the ordinary model. It is also unreasonable, so it is more suitable for Equation (4) that parameter matrix $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ estimated by the background value in $[t_{j-1}, t_j]$

are obtained by $z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$ substituting for $x_i^{(1)}(t_j)$. Based on quasi-exponentially law of the grey model and the modeling principles and methods in [7, 9], we set that $x_i^{(1)}(t) = G_i e^{a_i(t-t_1)} + C_i$, where a_i, G_i, C_i are the undetermined coefficients and $x_i^{(1)}(t_j) = G_i e^{a_i(t-t_1)} + C_i$ is meet. It can be determined by the grey modeling when the data are known. $x_i^{(1)}(t_j)$ is regressively generated to obtain:

$$x_i^{(0)}(t_j) = \frac{x_i^{(1)}(t_j) - x_i^{(1)}(t_{j-1})}{\Delta t_j} = \frac{G_i(1-e^{a_i \Delta t_j})}{\Delta t_j} e^{-a_i(t_j-t_m)} = g_i e^{-a_i(t_j-t_m)} \tag{8}$$

Where,

$$g_i = \frac{G_i(1-e^{a \Delta t_j})}{\Delta t_j} = \frac{G_i(1-(1+(a_i \Delta t_j)+\frac{(a_i \Delta t_j)^2}{2!}+\cdots)}{\Delta t_j}$$

When a_i and Δt_j are small, the first two items are taken after expanding $e^{a_i \Delta t_j}$, they can be obtained as follows:

$$g_i = \frac{G_i(1-e^{a_i \Delta t_j})}{\Delta t_j} = \frac{G_i(-a_i \Delta t_j)}{\Delta t_j} = -G_i a_i$$

$$\frac{x_i^{(0)}(t_j)}{x_i^{(0)}(t_{j-1})} = \frac{e^{-a_i(t_j-t_m)}}{e^{-a_i(t_{j-1}-t_m)}} = e^{-a_i \Delta t_j}$$

Then,

$$a_i = \frac{\ln x_i^{(0)}(t_j) - \ln x_i^{(0)}(t_{j-1})}{\Delta t_j} (j=2,3,\cdots;m) \tag{9}$$

That Equation (9) substituting into Equation (8), we can obtain:

$$\begin{cases} g_i = \frac{x_i^{(0)}(t_j)}{e^{-a_i(t_j-t_m)}} = \frac{x_i^{(0)}(t_j)}{[x_i^{(0)}(t_j)/x_i^{(0)(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}} \\ \\ G_i = \frac{x_i^{(0)}(t_j)\Delta t_j[x_i^{(0)}(t_j)/x_{i1}^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}{1 - \frac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}} \end{cases} \tag{10}$$

Accounting to the initial conditions as $x_i^{(1)}(t_1) = G_i e^{a_i(t_1-t_1)} + C_i = G_i + C_i$, it can be obtained:

$$C_i = x_i^{(0)}(t_m) - G_i = x_i^{(0)}(t_m) - \frac{x_i^{(0)}(t_j)\Delta t_j[x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m-t_j}{\Delta t_j}}}{1 - \frac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_{j-1})}} \tag{11}$$

That Equation (9) and Equation (11) substituting for the formula for background value $\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$ can be obtained:

$$z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}dt = -\frac{\Delta t_j x_i^{(0)}(t_j)}{a_i} + C_i \Delta t_j$$

$$= \frac{(\Delta t_j)^2 x_i^{(0)}(t_j)}{\ln x_i^{(0)}(t_j) - \ln x_i^{(0)}(t_j)} + x_i^{(0)}(t_m)\Delta t_j - \frac{x_i^{(0)}(t_m)(\Delta t_j)^2 [x_i^{(0)}(t_j)/x_i^{(0)}(t_{j-1})]^{\frac{t_m - t_j}{\Delta t_j}}}{1 - \frac{x_i^{(0)}(t_{j-1})}{x_i^{(0)}(t_j)}}$$

(12)

Noting $\mathbf{a}_i = (a_{i1}, a_{i2}, \cdots, a_{in}, b_i)^{\mathbf{T}}(i = 1,2,\cdots, n)$, the identified value $\hat{\mathbf{a}}_i$ of a_i can be obtained by using the least square method:

$$\hat{\mathbf{a}}_i = [\hat{a}_{i1}, \hat{a}_{i2}, \cdots, \hat{a}_{in}, \hat{b}_i]^{\mathbf{T}} = (\mathbf{L}^T \mathbf{L})^{-1}\mathbf{L}^T \mathbf{Y}_i, i = 1,2,\cdots, n$$

(13)

Where,

$$\mathbf{L} = \begin{bmatrix} z_1^{(1)}(t_2) & z_2^{(1)}(t_2) & \cdots & z_n^{(1)}(t_2) & \Delta t_2 \\ z_1^{(1)}(t_3) & z_2^{(1)}(t_3) & \cdots & z_n^{(1)}(t_3) & \Delta t_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ z_1^{(1)}(t_m) & z_2^{(1)}(t_m) & \cdots & z_n^{(1)}(t_m) & \Delta t_m \end{bmatrix}$$

(14)

$$\mathbf{Y}_i = [x_i^{(0)}(t_2)\Delta t_2, x_i^{(0)}(t_3)\Delta t_3, \cdots, x_i^{(0)}(t_m)\Delta t_m]^{\mathbf{T}}$$

(15)

Then the identified values of A and B can be obtained:

$$\hat{\mathbf{A}} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \cdots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \cdots & \hat{a}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \hat{a}_{n1} & \hat{a}_{n2} & \cdots & \hat{a}_{nn} \end{bmatrix}, \quad \hat{\mathbf{B}} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \cdots \\ \hat{b}_n \end{bmatrix}$$

(16)

The calculated value in new information MGM(1,n) model is:

$$\hat{\mathbf{X}}_i^{(1)}(t_j) = e^{\hat{\mathbf{A}}(t_j - t_m)}\mathbf{X}^{(1)}(t_m) + \hat{\mathbf{A}}^{-1}(e^{\hat{\mathbf{A}}(t_j - t_m)} - \mathbf{I})\hat{\mathbf{B}}, j = 1,2,\cdots, m$$

(17)

After restoring the fitting value of the original data is:

$$\hat{\mathbf{X}}_i^{(0)}(t_j) = \begin{cases} \lim_{\Delta t \to 0} \dfrac{\mathbf{X}_i^{(1)}(t_1) - \mathbf{X}_i^{(1)}(t - \Delta t)}{\Delta t}, & j = 1 \\ (\hat{\mathbf{X}}_i^{(1)}(t_j) - \hat{\mathbf{X}}_i^{(1)}(t_{j-1}))/(t_j - t_{j-1}), & j = 2,3,\cdots, m \end{cases}$$

(18)

The absolute error of the ith variable: $\hat{x}_i^{(0)}(t_j) - x_i^{(0)}(t_j)$.

The relative error of the i th variable: $e_i(t_j) = \dfrac{\hat{x}_i^{(0)}(t_j) - x_i^{(0)}(t_j)}{x_i^{(0)}(t_j)} * 100$.

The mean of the relative error of the i th variable: $\dfrac{1}{m}\sum_{j=1}^{m}|e_i(t_j)|$.

The average error of the whole data: $f = \dfrac{1}{nm} \sum\limits_{i=1}^{n} (\sum\limits_{j=1}^{m} |e_i(t_j)|)$

It can be seen that non-equidistant new information MGM(1,n) model is degraded into GM(1,n) when $n = 1$, and this MGM(1,n) model is a combination of n GM(1,n) models when $B = 0$. This MGM(1,n) can be used not only for modeling and predicting but also for data fitting and processing. The value of n accounting to the specific circumstances can get the needed model as MGM (1,2), MGM (1,3) and MGM (1,4).

3. Precision Inspecting for the Model

The inspecting means contain residual analysis, correlation degree analysis, and post-error analysis [1], [19-22]. The displacement relative degree, the speed related degree, the acceleration degree, and the total related degree are calculated simultaneity. These kinds of related degrees are called related degrees of C-type [22], which can be used to both of the whole analysis and the dynamic analysis. The following related degree inspection of C-type is employed in this paper.

1) To calculate the three-layer related degrees

Displacement related degree $d^{(0)}(t_k)$

$$d^{(0)}(t_k) = x_{t_k}^{(0)} / \hat{x}_{t_k}^{(0)}, k = 1, 2, \cdots, m \tag{19}$$

Speed related degree $d^{(1)}(t_k)$

$$d^{(1)}(t_k) = \frac{x_{t_{k+1}}^{(0)} - x_{t_k}^{(0)}}{\hat{x}_{t_{k+1}}^{(0)} - \hat{x}_{t_k}^{(0)}}, k = 1, 2, \cdots, m - 1. \tag{20}$$

Acceleration related degree $d^{(2)}(t_k)$

$$d^{(2)}(t_k) = \frac{x_{t_{k+1}}^{(0)} - 2x_{t_k}^{(0)} + x_{t_{k-1}}^{(0)}}{\hat{x}_{t_{k+1}}^{(0)} - 2\hat{x}_{t_k}^{(0)} + \hat{x}_{t_{k-1}}^{(0)}} \tag{21}$$
$$(k = 1, 2, \cdots, m - 1).$$

2) To calculate the three-layer related comprehensive degree at t_k

$$D(t_1) = \frac{d^{(1)}(t_1) + d^{(0)}(t_1)}{2}, D(t_m) = d^{(0)}(t_m) \tag{22}$$

$$D(t_k) = \frac{d^{(0)}(t_k) + d^{(1)}(t_k) + d^{(2)}(t_k)}{3} \tag{23}$$
$$(k = 2, 3, \cdots, m - 1)$$

3) To calculate the total related degree of the model $\hat{x}^{(0)}(t_k), k = 1, 2, \cdots, m$

$$D = \frac{1}{n} \sum_{k=1}^{m} D(t_k) \tag{24}$$

When $0.60 \le D \le 1$ or $D > 1$ and $\dfrac{1}{D} > 0.60$, the precision of the model is "Good". When $-0.3 \le D \le 0.60$, the precision of the model is "better". When $D < -0.30$ or $\dfrac{1}{D} < -0.30$, the

precision of the model is "bad" [22].

4. Examples

Example 1: The affecting data of water absorption to mechanical properties of pure PA66 are seen in [10]. After PA66 samples with the different water absorption were tested in mechanical property, the following experimental data of PA66 can be obtained, as shown in Table 1, where $X_1^{(0)}$ is bending strength (Mpa), $X_2^{(0)}$ is bending elastic modulus (Gpa) and $X_3^{(0)}$ is tensile strength (Mpa).

Table 1. The Affecting Data of Water Absorption to Mechanical Properties of Pure PA66

No.	1	2	3	4	5	6	7	8	9
water absorption t_j / %	0	0.0607	0.1071	0.1662	0.2069	0.4344	0.5243	0.8524	0.9756
$X_1^{(0)}$	83.4	84.9	84.5	84.2	84.4	78.4	75.4	59.5	54.1
$X_2^{(0)}$	2.63	2.64	2.61	2.65	2.66	2.52	2.32	1.90	1.72
$X_3^{(0)}$	84.2	84.4	86.3	84.3	81.3	74.9	75.7	73.2	66.9

The non-equidistant new information MGM(1,3) model was established by using the proposed method in this paper. The parameters of this model are as follows:

$$\mathbf{A} = \begin{vmatrix} 0.0468 & -15.6730 & -0.0620 \\ 0.0069 & -0.6888 & -0.0011 \\ 0.0954 & -0.7070 & -0.2612 \end{vmatrix}, \ \mathbf{B} = \begin{vmatrix} 80.5759 \\ 2.5492 \\ 81.7480 \end{vmatrix}$$

The fitting value of $X_3^{(0)}$:

$$\hat{X}_3^{(0)} = [82.5775, 82.1106, 81.2965, 80.497, 79.743, \quad 77.7316, 75.3726, 72.3237, 69.0904]$$

The absolute error of $X_3^{(0)}$:

$$q = [-1.6225, -2.2894, -5.0035, -3.803, -1.557, 0.8316, -0.32741, -0.87626, 2.1904]$$

The relative error of $X_3^{(0)}$ (%):

$$e = [-1.9269, -2.7125, -5.7978, -4.5113, -1.9152, 3.7804, -0.43251, -1.1971, 3.2742]$$

The mean of the relative error of $X_3^{(0)}$ is 2.8387%, so this model has higher precision and the precision of $X_3^{(0)}$ is "Good". The maximum relative error of $X_3^{(0)}$ -5.7978% is smaller than the one -6.1048% in [10].

Example 2: In the conditions of the load 600N and the relative sliding speed 0.314m/s, 0.417m/s, 0.628m/s, 0.942m/s and 1.046m/s respectively, the test data of the thin film with TiN coat are shown as in Table 2 [18].

Table 2. Test Data of the thin Film with TiN Coat

No.	1	2	3	4	5
Sliding speed (m/s)	0.314	0.471	0.628	0.942	1.046
Friction coefficient μ	0.251	0.258	0.265	0.273	0.288
Wear rate $\omega * 10^{-5}$ (mg/m)	7.5	8	8.5	9.5	11

Assumed sliding speed t_j, friction coefficient $x_1^{(0)}$ and wear rate $x_2^{(0)}$, non-equidistant new information MGM (1,2) model was established by using the proposed method in this paper. The parameters of this model are as follows:

$$\mathbf{A} = \begin{vmatrix} -1.1618 & 0.0407 \\ -101.2603 & 3.6243 \end{vmatrix}, \; \mathbf{B} = \begin{vmatrix} 0.1817 \\ 0.8559 \end{vmatrix}$$

The fitting value of $x_1^{(0)}$: $\hat{x}_1^{(0)} = [0.25494, 0.25621, 0.25994, 0.27114, 0.28825]$

The absolute error of $x_1^{(0)}$: $q = 10^{-3} \times [3.9425, -1.794, -5.0558, -1.8565, \; 0.24543]$

The relative error of $x_1^{(0)}$ (%): $e = [1.5707, -0.69535, -1.9078, -0.68004, 0.085217]$

The mean of the relative error of $x_1^{(0)}$ is 0.98783% and the one of this model is 1.4981%. So this model has higher precision. When equidistant MGM (1,3) model was used in [18], the mean of the relative error of $x_1^{(0)}$ is 1.6225%.

4. Conclusion

Aimed to non-equidistant multivariable sequence with mutual influence and restriction among multiple variables, based on index characteristic of grey model, the characteristic of integral and new information principle, the new information background value in non-equidistant multivariable new information MGM (1,n) was researched and the discrete function with non-homogeneous exponential law was used to fit the accumulated sequence and the formula of new information background value was given. The new MGM (1,n) model can be used in equidistance & non-equidistance and it extents the application scope of grey model. New model has the characteristic of high precision as well as easy to use. Examples validate the practicability and the reliability of the proposed model. There are important practical and theoretical significance and this model should be worthy of promotion.

Acknowledgements

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