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Improved Compressed Sensing Matrixes for Insulator Leakage Current Data Compressing

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Abstract

Insulator fault may lead to the accident of power network, thus the on-line monitoring of insulator is very significant. Low rates wireless network is used for data transmission of leakage current. Determination of the measurement matrix is the significant step for realizing the compressed sensing theory. This article comes up with new sparse matrices which can be used as compressed sensing matrices to make data compression and reconstruction of leakage current with the compressed sensing. This theory can achieve pretty good results. And then this article performs that the reconstitution effect is almost the same using the measurement matrix of Toeplitz matrix, circulant matrix or sparse matrix, as using a classical measurement matrix.

Keywords: *leakage current, data compression, compressed sensing, measurement matrix*

1. Introduction

High precision detection of insulator leakage current is conductive to improve the reliability of transmission lines. Wireless sensor network is a significant technology that widely applied in the leakage current monitoring of transmission line, while the communication bandwidth is limited, which make the data compressing of leakage current become essential.

The main traditional method of data compressing for leakage current includes fractal interpolation and piecewise quantization compressing. Based on the fractal theory, document [3] applied fractal interpolation method to construct the original leakage current signal. The locality of this method makes it quite difficult to reflect the overall characteristics of the leakage current signal. Document [4] puts forward and realizes the piecewise quantization compressing algorithm, putting to use the HSF code with characteristic of numerical sequence to realize highly active variable length compression. But this method leads to the instability of the leakage current frequency and makes it impossible to determine the probability of accuracy of the current value, running counter to the original intention of coding.

Since the putting forward of compressed sensing theory, data processing and compressing come into a new stage with new technology and new mentality. Document [5] raises a method of data compressing of insulator leakage current based on compressed sensing theory, which increase the compression ratio to a certain extent, attaining the compression effect for more than 10 times. And the reconstruction effect is also ideal.

The definition of the measurement matrix is the important step of achieving compressed sensing theory. The traditional measurement matrix such as the Gauss matrix and Bernoulli matrix can achieve high-accuracy reconstruction, but it's still difficult to realize with the high cost of storage. By analyzing of the principle of compressed sensing theory, this paper constructs the Top Liz matrix, cyclic matrix and sparse matrix to apply to compressed sensing of the data compression of leakage current, compares the measurement result and the efficiency with results from traditional measurement matrix to illustrate the advantages of these matrices. In consideration of the characteristics of the leakage current that being periodic and unstable, subsection compression is adopted in the experiment. Using compressed sensing to leakage current data of both pulse area recognized and stable zone, the reconstruction effect turns out to be ideal.

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2. The Characteristics of Leakage Current

Pollutants in the air deposit on the insulator surface for days and months, and finally format the pollution layer. The pollution layer reduces the electric strength of insulators greatly, leading to the accident because of the contamination flashover during the normal operation. Current leakage current is defined as the flow through the insulator surface pollution layer measured under operating voltage when filth is wet. When the operating voltage is constant, the leakage current increases with the level of pollution. Thus leakage current can be used to characterize the impact of insulator contamination, and experience shows that it is scientific to use leakage current values as characteristic value to reflect the running status of insulators [1].

The characteristics of leakage current can be summed up as periodic and unstable. The leakage current pollution flashover process is divided into 3 sections, safety zone (<20mA), forecast zone (<50mA) and danger zone (>50mA). Leakage current of safety zone is very small, and is used to represent the characteristics of leakage current that is dry and with low degree surface contamination, most of which are the sine waves. Leakage current of forecast zone is always in stage of instability, the amplitude of the current pulse increases and pulse group appears every now and then. Leakage current of danger zone is large, pulse amplitude increases rapidly and the high amplitude pulse density also increased significantly. For the frequency domain, the safety zone of leakage current is mainly fundamental and there is much high order harmonic component in the pulse area that reflects the drastic changes.

3. The Compressed Sensing Theory

Compressed sensing theory mainly includes three aspects, they are the sparse representation of signals, code for measurement and reconstruction algorithm. If most of the elements are zero in the signal, then the signal is called sparse. According to the theory of harmonic analysis, a discrete time signal f with length of n can be expressed as a linear combination of the standard orthogonal basis, which is called the sparse transform. The form is as follows:

$$f = \sum_{i=1}^{N} x_i \psi_i \tag{1}$$

Or,

$$f = \Psi x \tag{2}$$

Where $\Psi = [\psi_1, \psi_2, \cdots \psi_N]$, ψ_i is a column vector. The column vector x of $N \times 1$ is the weighted coefficient sequence of f. If high coefficient of x is quite a few, then the signal f is called compressible. Substrate of transformation matrix for sparse transform can be selected according to the signal characteristics, such as substrate of the fast Fourier transform, substrate of discrete cosine transform, substrate of discrete wavelet transform, substrate of Curvelet, substrate of Gabor and redundant dictionary.

Supposing that the measurement matrix $\Phi \in \mathrm{R}^{M \times N} (M << N)$, and the measured value $y \in \mathrm{R}^{M}$, then the reconstructed signal can be calculated as follows:

$$y = \Phi f \tag{3}$$

The dimension of y is much lower than the dimension of f, thus the above equation has infinitely many solutions, so the signal f cannot be reconstructed. But if f can be expressed sparsely as $f = \Psi x$, then the expression is as follows:

$$y = \Phi f = \Phi \Psi x = \tilde{\Phi} x \tag{4}$$

Where $\tilde{\Phi} = \Phi\Psi$ is a matrix of $M \times N$, which is called the sensing matrix. Candés and Tao [8] consider that only if $\tilde{\Phi}$ satisfies restricted isometry, the signal can be reconstructed with high probability, and the signal f can be reconstructed accurately by solving the optimal norm of the measured value y. Due to Ψ have been selected usually, then the sensing matrix $\tilde{\Phi}$ can be satisfied for restricted isometry by defining measurement matrix Φ. The Gauss random matrix is usually used to meet the above requirements. Practice has proved that Gauss random matrices can achieve the accurate reconstruction of the signal f.

The matching pursuit algorithm can achieve the signal reconstruction of compressed sensing. The basic idea of classic matching pursuit algorithm is to select the best matching elements with the signal using the measurement matrix in the algorithm, go round and begin again the iterative, and make the maximum correlation from the results in each iteration with the original signal. In order to solve the problem that the number of iterations is too many during the matching pursuit algorithm, the orthogonal matching pursuit algorithm is then proposed. The algorithm speed the iteration via orthogonalization, and then to realize the faster reconstruction.

4. Measurement Matrixes in Compressed Sensing Theory

The common measurement matrices that satisfy the restricted isometry conditions include Gauss measurement matrix, Bernoulli measurement matrix and Fourier measurement matrix. In the actual application, the more random the matrix is, the more difficult is the physical implementation. In general, for matrix with random elements, the implementation will be very high price [10]. Bajwa provides two kinds of measurement matrices, Toeplitz matrix and cyclic matrix. He also suggests that random Toeplitz and cyclic matrix can be implemented in various applications easily, and proves that these two kinds of matrices have the same effect with the classical stochastic matrix in compressed sensing and reconstruction process.

Toeplitz matrix has the following form, which in addition to the first row and the first column, each element is the same as the element on its upper left corner, that is $a_{i,j} = a_{i+1,j+1}$.

$$A = \begin{pmatrix} a_N & a_{N-1} & \cdots & a_1 \\ a_{N+1} & a_N & \cdots & a_2 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N+M-1} & a_{N+M-2} & \cdots & a_M \end{pmatrix} \tag{5}$$

If meeting additional properties $\forall i, a_i = a_{N+i}$, then the matrix is also a cyclic matrix. Cyclic matrix has the following form.

$$C = \begin{pmatrix} a_N & a_{N-1} & \cdots & a_1 \\ a_1 & a_N & \cdots & a_2 \\ \vdots & \vdots & \ddots & \vdots \\ a_{M-1} & a_{M-2} & \cdots & a_M \end{pmatrix} \tag{6}$$

Bajwa [13] proves that for the Toeplitz matrix, when the measured frequency satisfy $M \geq C_\delta K^2 \log(N/K)$, then the Toeplitz matrix can basically satisfies the restricted isometry condition in very high probability.

In order to further reduce the number of independent random variables in the measurement matrix, Yang put forward the concept of sparse banded and sparse columns in the study [10], to further generate a sparse banded matrix and sparse columns matrix, reduce the steps of matrix multiplication and to improve the efficiency of compression and recovery.

The form of sparse banded random matrix is as follows:

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1l} & & & & \\ & a_{21} & a_{22} & \cdots & a_{2l} & 0 & & \\ 0 & & \ddots & & & \ddots & & \\ & & & \cdots & & & & \\ a_{M,j+1} & \cdots & a_{Ml} & \cdots & a_{M1} & \cdots & a_{Mj} & \end{pmatrix} \tag{7}$$

Kinds of sparse banded matrix have certain randomness. Compared to the classical stochastic matrix, Toeplitz matrix and cyclic matrix, the independent random elements of sparse matrix reduces. According to Yang [10], compared to the Gauss random matrix of which the multiplication operation requires $M \times N$ steps, the multiplication operation of Toeplitz matrix only requires $\mathrm{O}(\log_2 N)$ steps.

Yang Hairong [10] also introduces the concept of sparse column matrix and sparse cyclic matrix. On the basis of keeping the operation steps the same order with the Toeplitz matrix, the method further reduces the number of independent random element and improves operation efficiency. The form of sparse column random matrix is as follows:

$$\begin{pmatrix} a_{11} & 0 & & a_{1p} & & a_{p\lfloor N/M \rfloor+1,1} & \\ a_{21} & a_{22} & & a_{2p} & & a_{p\lfloor N/M \rfloor+2,2} & \\ \vdots & \vdots & & \vdots & \cdots & \vdots & \cdots \\ a_{p1} & a_{p2} & \ddots & 0 & \cdots & a_{p\lfloor N/M \rfloor+p,p} & \cdots \\ 0 & a_{p+1,2} & \ddots & 0 & & 0 & \\ \vdots & & & \vdots & & \vdots & \\ 0 & & & a_{M,p} & & 0 & \end{pmatrix} \tag{8}$$

5. Experimental Results

Select a section of leakage current data with flashover process, and then use compressed sensing algorithm to compress and reconstruct the leakage current data. In view of the characteristics of periodicity and non-stationary of the leakage current data, each step of the experiment takes the way of pulse's automatic recognition first, separate the pulse zone and non-pulse zone of leakage current, form several sections, and then make compressed sensing for data of each section, compare the regenerated data group with the original signal and finally get the recovery effect with pretty high accuracy.

First of all, choose the Gauss measurement matrix and the Bernoulli measurement matrix as the measurement matrix for compressed sensing. Then construct the Toeplitz matrix and cyclic matrix. Compare the recovery capability with classical measurement matrix and finally use the sparse banded treatment and sparse column treatment methods to do the recovery of Gauss measurement matrix. The recovery effect is ideal.

All the measurement matrices selected in the experiment have $M \times N$ elements, where N is the number of collected signals, and M is the measuring number of the compressed sensing algorithm. Toeplitz matrix and cyclic matrix are shaped separately like the formula (5) and the formula (6), where each element $a_{i,j}$ satisfies the Bernoulli distribution. Sparse banded matrix is shown as formula (7), where non-zero elements obey the Gauss distribution and the subscript parameter l satisfies $l = [2*N/3] > N - M$ [8]. Sparse column matrix is shown as formula (8), where nonzero elements obey the Gauss distribution and the number of nonzero elements in the first column also satisfies $p = [2*N/3] > N - M$ [8].

The recovery effect of original leakage current data and data measured from different measurement matrices are shown in the images below.

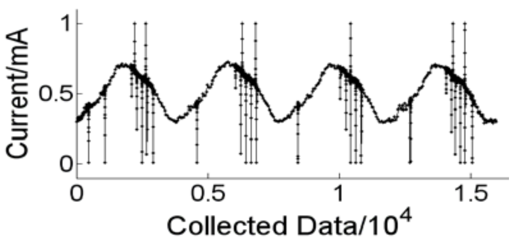

Figure 1. Raw Data

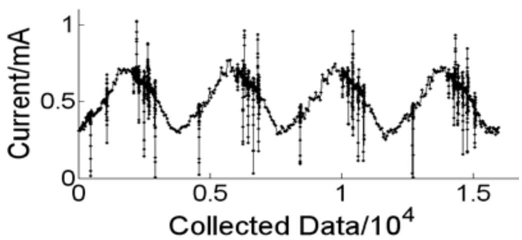

Figure 2. Reconstruction Effect of Gauss Measurement Matrix

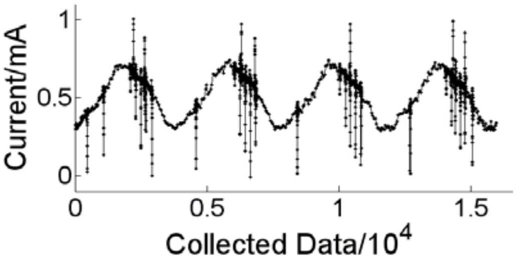

Figure 3. Reconstruction Effect of Bernoulli Measurement Matrix

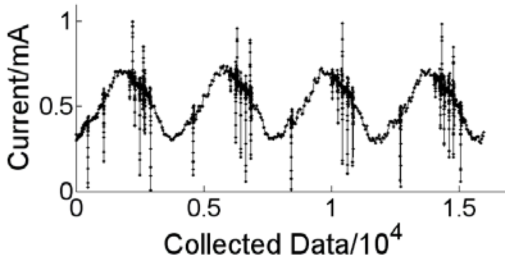

Figure 4. Reconstruction Effect of TopeLiz Measurement Matrix

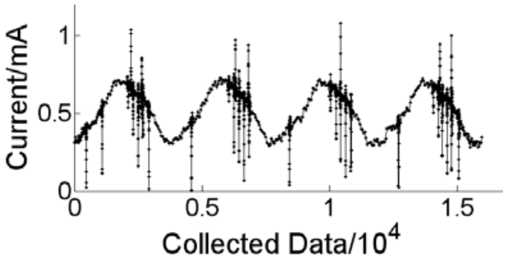

Figure 5. Reconstruction Effect of Cyclic Matrix

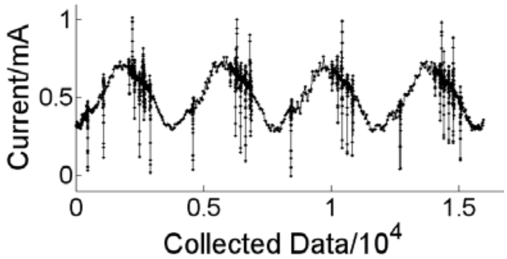

Figure 6. Reconstruction Effect of Gauss Measurement Matrix
with Sparse Banded Processing

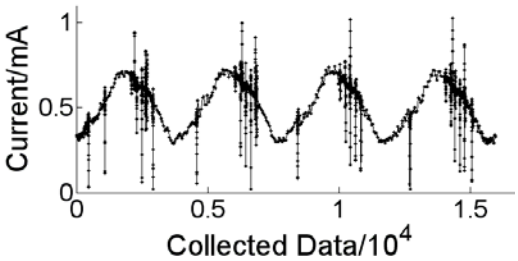

Figure 7. Reconstruction Effect of Gauss Measurement Matrix with Sparse Columns Processing

Due to the varying degrees of randomness of measurement matrices, thousands of times experiments have been carried out for the selection of every measurement matrix. The relative error formula of the matrix is shown as follows:

$$\delta = \frac{\|\delta A\|}{A} = \frac{\|\tilde{A} - A\|}{A} \tag{9}$$

Where $\tilde{A}$ represents the measured value of the matrix, and A represents the true value. Record every experimental error and take the average, then the reconstruction error of different matrix during signal recovering is obtained. It is obvious that the reconstruction error of different matrix is in the range 6% to 9%, and the reconstruction effects of Bernoulli measurement matrix, Toeplitz measurement matrix as well as cyclic matrix are better than the traditional matrix.

Table 1. Reconstruction Error of Different Measurement Matrices

Measurement matrix	Reconstruction error(%)
Gauss measurement matrix	8.53
Bernoulli measurement matrix	6.65
Toeplitz matrix	6.90
Cyclic matrix	6.68
Gauss matrix with Sparse banded processing	8.66
Gauss matrix with Sparse columns processing	8.89

6. Conclusion

The experimental results show that when the Toeplitz matrix and cyclic matrix are used as the measurement matrices of compressed sensing, the matching degree of the compression and restoration of insulator leakage current signal is pretty high in the view of signal form in the time domain, and the error values are stable in a rather limited range in the view of the numerical calculation results of recovery errors. The restoring effects are nearly the same ideal when the sparse measurement matrices are used in the signal recovery. Compared to the traditional random measurement matrix, the optimal measurement matrix has less independent random elements, which makes it easy to be implemented in engineering.

Acknowledgements

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