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DOA Estimation by Fourth-Order Cumulants without Source Enumeration and Eigen Decomposition

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Abstract

A new algorithm for direction of arrival (DOA) estimation is proposed.Using fourth-order cumulants and modified MUSIC (MUliplle SIgnal Classification) algorithm. However, it does not require any eigendecomposition of the cumulant matrix of the received data and source enumeration. It also eliminates the need for knowledge of the spatial characteristics of the noise and interference. This method only uses the conjugate spatial signal of different sensor positions. Computer simulation results are provided to demonstrate the performance of the proposed approach and compare them to DCI (Diagonally-loaded Conjugate correlation matrix Inverse power) method.

Keywords: *fourth-order cumulants, DCI, MUSIC, DOA estimation*

1. Introduction

Direction of arrival (DOA) estimation is one of the important contents of array signal processing, because of its important applications in radar, sonar, communications, etc.

There are many array models, and algorithms are available for estimating the DOA of sources. Among these, Multiple Signal Classification (MUSIC) [1] algorithm that belongs to subspace method of eigenstructure is the supper resolution method, and it has good performance and widely used. However, MUSIC and some modified versions of MUSIC [2-8] require the noise characteristics of the sensors be known and the total number of signals impinging on the array to be known or to be exactly estimated in advance.and also there is algorithm which is able to achieve high accuracy [11, 12]. We propose a new direction finding algorithm to overcome the aforementioned shortcomings by using high-order cumulants matrix to replace the conjugate source correlation matrix [9]. The algorithm has low computational complexity in comparison with the MUSIC algorithm and still has good performance. Both the computer simulations and the experiment of the direction finding system have been given to illustrate the performance of the algorithm.

The rest of this paper is organized as follows. Section 2 introduces the system model and DCI method. Section 3, the new algorithm is described in detail. Section 4 presents simulation results that show the effectiveness of the proposed algorithm. Finally, we conclude this paper in section 5.

Throughout the paper, lower-case boldface italic letters denote vectors; upper-case boldface italic letters represent matrices, and lower and upper-case italic letters stand for scalers. The symbol $*$ is used for conjugation operations and notation $(x)^T$ and $(x)^H$ represent transpose and conjugate transpose, respectively. We use $E(x)$, $cum(x)$ and $\otimes$ to indicate the expectation operater, the cumulants and kronecker product, separately.

2. System Model and DCI

2.1. System model

Assume there are M far-field narrowband signals $\mathbf{s}_m(t), m=1,2,..........,M$, impinging on auniform linear array (ULA) with N sensors from different directions $\theta_m, m=1,2,........,M$.

The array output vector could be represented by:

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$$\mathbf{x}(t)=\sum_{m=1}^{M}\mathbf{a}(\theta_m)s_m(t)+\mathbf{n}(t) \quad (1)$$

Where $\mathbf{a}(\theta_m)=\left[e^{j\xi_1\sin\theta_m},e^{j\xi_2\sin\theta_m},....................e^{j\xi_N\sin\theta_m}\right]^T$ is the steering vector of the *m*th signals, $\xi_n=2\pi(n-1)d/\lambda$, and λ is the wavelength of signal, d is the element space.

Then rewriting Equation (1) in matrix form, we obtain:

$$\mathbf{x}(t)=\mathbf{A}s(t)+\mathbf{n}(t) \text{ For } t=1,2,..........,K \quad (2)$$

Where:

$\mathbf{x}(t)=\left[x_1(t),x_2(t),............x_N(t)\right]^T$ is $N\times1$ observation vector (snapshot vector),

$\mathbf{A}=\left[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),...............,\mathbf{a}(\theta_M)\right]^T$ is $N\times M$ array manifold matrix,

$\mathbf{s}(t)=\left[s_1(t),s_2(t),.....................,s_M(t)\right]^T$ is $M\times1$ signal vector,

and $\mathbf{n}(t)=\left[n_1(t),n_2(t),.................n_N(t)\right]^T$ is $N\times1$ noise vector.

The covariance matrix of the data received by the sensor array, denoted by:

$$\begin{aligned}\mathbf{R}_{xx}&=E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\}\\&=\mathbf{R}_{ss}+\sigma^2\mathbf{I}\end{aligned} \quad (3)$$

Where $\mathbf{R}_{ss}=\mathbf{A\Gamma}_s\mathbf{A}^H$ and $\mathbf{\Gamma}_s=E\left\{\mathbf{s}(t)\mathbf{s}(t)^H\right\}$ denotes the covariance matrix of radiating signals and $\mathbf{I}$ is the $N\times N$ identity matrix.

The conjugate covarianve matrix of the array, denoted by:

$$\mathbf{R}_{xx^*}=E\left\{\mathbf{x}(t)\mathbf{x}^T(t)\right\}=\mathbf{A}R_{ss^*}\mathbf{A}^H \quad (4)$$

Where $\mathbf{R}_{ss^*}=E\left\{\mathbf{s}(t)\mathbf{s}^T(t)\right\}$ is called the conjugate source correlation matrix.

2.2. Diagonally Loaded Conjugate Correlation Matrix Inverse Power Method (DCI)

This method can estimate DOA of narrowband noncircular signals without eigendecomposition and source enumeration.This method exploits the conjugate correlation between the received signals at different sensor positions.

Here we use only conjugate array correlation matrix $\mathbf{R}_{xx^*}=E\left\{\mathbf{x}(t)\mathbf{x}^T(t)\right\}=\mathbf{A}R_{ss^*}\mathbf{A}^H$

First we square the above eguation to obtain Hermitian matrix.

$$\mathbf{C}_x=\mathbf{R}_{xx^*}\mathbf{R}^H{}_{xx^*}=\mathbf{A}\mathbf{R}_{ss^*}\mathbf{A}^T\mathbf{A}^*\mathbf{R}^H_{ss^*}\mathbf{A}^H=\mathbf{A}\mathbf{C}_s\mathbf{A}^H \quad (5)$$

The DOAs can be obtained by searching the peaks of the following spatial spectrum.

$$\mathbf{S}_{DCI}=\gamma^{-L}\left[\frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)(D_x)^L\mathbf{a}(\theta)}\right] \quad (6)$$

Where:

$$\mathbf{D}_x=\left(\mathbf{C}_x+\gamma\mathbf{I}_N\right)^{-1} \quad (7)$$

When $L \rightarrow \infty$, the DCI method approaches MUSIC [9].

3. The Proposed Algorithm.

The problem is that given the array output $\{\mathbf{x}(t), t = 1,2,.......K\}$ where K denotes the number of snapshots, estimate the DOA parameter $\theta_m, m = 1,2,........,M$ of the impinging signals. We shall work with a fourth order cumulant of received array output which can be expressed as:

$$cum(x_1,x_2,x_3,x_4) = E\{x_1x_2x_3x_4\} - E\{x_1x_2\}E\{x_3x_4\} - E\{x_1x_3\}E\{x_2x_4\} - E\{x_1x_4\}E\{x_1x_3\} \quad (8)$$

We can use matrices to express cumulants by using Kronecker products.

$$\mathbf{C}_x = E\left\{(\mathbf{x}\otimes\mathbf{x}^*)(\mathbf{x}\otimes\mathbf{x}^*)^H\right\} - E\left\{(\mathbf{x}\otimes\mathbf{x}^*)\right\}.E\left\{(\mathbf{x}\otimes\mathbf{x}^*)^H\right\} - E\left\{(\mathbf{x}.\mathbf{x}^H)\right\}\otimes E\left\{(\mathbf{x}.\mathbf{x}^H)^*\right\} \quad (9)$$

Where $\mathbf{C}_x$ is the fourth order cumulants of signal vector $\mathbf{x}$.
Considering the properties of Kronecker products we have.

$$\mathbf{C}_x = \mathbf{b}(\theta)\mathbf{C}_s\mathbf{b}^H(\theta) \quad (10)$$

Where $\mathbf{C}_s$ is fourth order cumulant matrix of signal,

$$C_s = E\left\{(\mathbf{s}\otimes\mathbf{s}^*)(\mathbf{s}\otimes\mathbf{s}^*)^H\right\} - E\left\{(\mathbf{s}\otimes\mathbf{s}^*)\right\}.E\left\{(\mathbf{s}\otimes\mathbf{s}^*)^H\right\} - E\left\{(\mathbf{s}.\mathbf{s}^H)\right\}\otimes E\left\{(\mathbf{s}.\mathbf{s}^H)^*\right\} \quad (11)$$

and $\mathbf{b}(\theta) = [\mathbf{b}(\theta_1)\mathbf{b}(\theta_2)..............\mathbf{b}(\theta_N)]$

$$= [\mathbf{a}(\theta_1)\otimes\mathbf{a}^*(\theta_1)...................\mathbf{a}(\theta_N)\otimes\mathbf{a}^*(\theta_N)] \quad (12)$$

We construct further the following fourth order cumulants matrices [10].

$$\mathbf{C}_x = E\left\{\tilde{\mathbf{Z}}(t)\tilde{\mathbf{Z}}^H(t)\right\} - vec(\mathbf{R}^*_{xx})vec^T(\mathbf{R}_{xx}) - \mathbf{R}_{xx}\otimes\mathbf{R}^*_{xx} - (\mathbf{I}^T_Q\otimes\mathbf{R}_{xx'})\circ(\mathbf{R}^*_{xx'}\otimes\mathbf{I}^T_Q) \quad (13)$$

Where $\tilde{\mathbf{Z}}(t) = \mathbf{Z}(t)\otimes\mathbf{Z}^*(t)$, $\mathbf{Z}(t) = (\mathbf{X}(t)\otimes\mathbf{X}^*(t))$, "$vec(M)$" denotes $PG\times 1$ vector formed from the elements of a $P\times G$ matrix $\mathbf{M}$ by stacking its columns, and $\mathbf{I}_Q = [1,1,............,1]^T$ which is a $Q\times 1$ vector with all elements being one.In addition "$\circ$" denotes the khatri-Rao product (element Kronecker product) i.e:

$$M\circ H = [m_1\otimes h_1,.............m_G\otimes h_G] \quad (14)$$

In which $M = [m_1, m_2,...........,m_G]$ and $H = [h_1, h_2,..............,h_G]$.

We then form the following matrix:

$$\mathbf{D}_x = (\mathbf{C}_x + \gamma\mathbf{I}_N)^{-1} \quad (15)$$

Where γ is a scalar which is smaller than the least eigenvalue of $\mathbf{C}_x$, $\mathbf{I}_N$ denotes a $N^2\times N^2$ identity matrix. Let $\{\mathbf{v}_m\}_{m=1}^M$ be the dominant eigenvectors of $\mathbf{C}_x$ according to M largest eigenvalues $\{\mu_m\}_{m=1}^M$. Wheres $\{\mathbf{u}_m\}_{m=1}^{N-M}$ be the remaining $N-M$ eigenvectors associated with the zero eigenvalues, it then follow that:

$$\mathbf{D}_x = \sum_{m=1}^{M}(\mu_m+\gamma)\mathbf{v}_m\mathbf{v}_m^H + \sum_{m=1}^{N-M}\gamma^{-1}\mathbf{u}_m\mathbf{u}_m \quad (16)$$

From the subspace orthogonality we know that $\mathbf{u}_m^H \mathbf{B}(\theta_k) = 0$ for $m = 1, ..., N-M$ and $k = 1, ..., M$. Thus the direction of arrival can be obtained by searching the peaks of the following spatial spectrum expression.

$$\mathbf{S}_{FOCDCI}(\theta) = \gamma^{-1}\left[\frac{\mathbf{b}^H(\theta)\mathbf{b}(\theta)}{\mathbf{b}^H(\theta)(\mathbf{D}_x)^L\mathbf{b}(\theta)}\right] = \frac{\gamma^{-L}\mathbf{b}^H(\theta)\mathbf{b}(\theta)}{\sum_{m=1}^{M}\varphi_m^{-L}\left|\mathbf{b}^H(\theta)\mathbf{v}_m\right|^2 + \sum_{m=1}^{N-M}\gamma^{-L}\left|\mathbf{b}^H(\theta)\mathbf{u}_m\right|^2} \tag{17}$$

Where $(\mathbf{D}_x)^L$ denotes the L-fold power of $\mathbf{D}_x$ and L is an integer, and $\varphi_m = \mu_m + \gamma$.
Let $\eta_m = \varphi_m / \gamma$ then:

$$S_{FOCDCI}(\theta) = \frac{\mathbf{b}^H(\theta)\mathbf{b}(\theta)}{\sum_{m=1}^{M}\eta_m^{-L}\left|\mathbf{b}^H(\theta)\mathbf{v}_m\right|^2 + \sum_{m=1}^{N-M}\left|\mathbf{b}^H(\theta)\mathbf{u}_m\right|^2} \tag{18}$$

When $L \to \infty$, then $\eta_m^{-L} \to 0$, and (18) to be MUSIC algorithm using fourth order cumulants $\mathbf{C}_x$.

4. Simulation Results and Analysis

In this section, computer simulatins are presented to demonstrate the performance of fourth order cumulants DCI method. The following performance measures are used to investigate the performance of DOA estimation technique.

Firrstly the power spectrum plots, when there were five sources impinging on the ULA with directions [-40 -20 0 20 40], the SNR is 10dB and the snapshot number is 1000. The spatial spectrum of FOC-DCI can be estimated correctly as shown in the Figure 1.

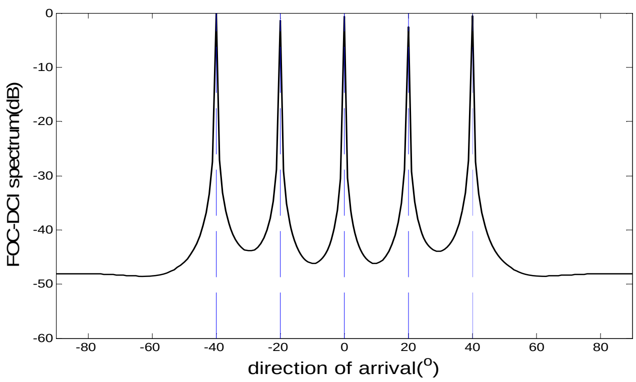

Figure 1. Estimate Five Sources using FOC-DCI Algorithm, SNR=10 dB, Number of Snapshots=500

Secondly the estimation accuracy, assume the array is illuminated by two signals from [-30 50]. All results are averaged via 1000 Monte-carlo simulation runs. The root-mean-square error (RMSE) versus signal–to-noise ratio (SNR) of FOC-DCI, SOC-DCI and FOC-DCI, NC-Root MUSIC, NC-standard ESPRIT as shown in Figure 3 and Table 1 respectively. And the RMSE versus the number of snapshots are depicted in Figure 2 and Table 2. We can see from Figure 2-3 and Table 1-2, that generally the FOC-DCI method has better estimation precision compared with SOC-DCI, NC-Root MUSIC and NC-standard ESPRIT.

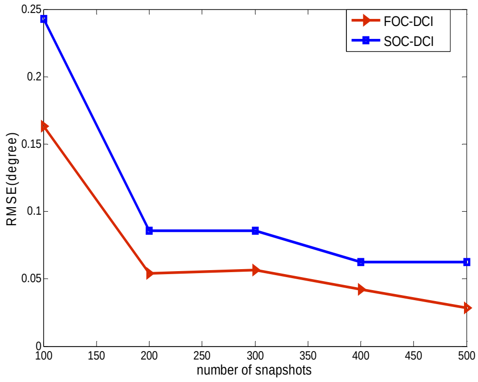

Figure 2. RMSE of the FOC-DCI and DCI Estimator versus Number of Snapshots

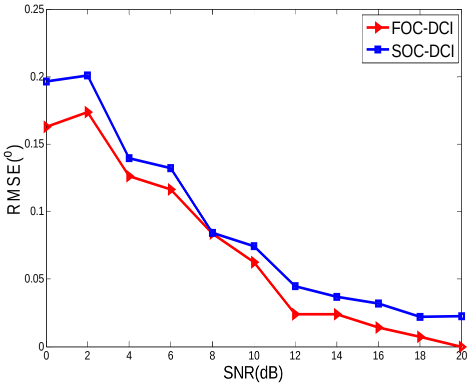

Figure 3. RMSE of the FOC-DCI and DCI Estimator versus SNR

Table 1. The RMSE of FOC-DCI, NC-standart ESPRIT and NC-Root MUSIC versus SNR

SNR in (dB)	RMSE in degrees		
	NC-standart ESPRIT	NC-ROOT MUSIC	Proposed method (FOC-DCI)
5	0.1368	0.1301	0.1252
7	0.1037	0.08497	0.09739
9	0.06714	0.0537	0.04995
11	0.05659	0.05036	0.03828
13	0.03853	0.03666	0.03121
15	0.02694	0.02876	0.02121
17	0.03182	0.02538	0.007071
19	0.02476	0.02138	0.007071

Table 2. The RMSE of FOC-DCI, NC-standart ESPRIT and NC-Root MUSIC versus Number of Snapshots

Number of snapshots	RMSE in degrees		
	NC-standart ESPRIT	NC-ROOT MUSIC	Proposed method (FOC-DCI)
50	0.09845	0.06974	0.07364
150	0.05843	0.05525	0.05525
250	0.047	0.03367	0.02828
350	0.03567	0.03466	0.03466
450	0.03685	0.02931	0.02121
550	0.02256	0.02079	0.01932
650	0.0328	0.02526	0.007071
750	0.02059	0.01678	0.01932
850	0.02623	0.02022	0.007071
950	0.02388	0.02254	0.01236

4. Conclusion

We presented new direction finding algorithm for non circular signals, which based on the fourth order cumulants of the data received by the array. It's a fast DOA estimation method, which does not require detecting the number of incident signals and performing eigendecomposition to achieve signal noise separation. An important topic which remains unsolved is on the optimum selection of γ. Both the theoretical formulation and computer simulation show that the FOC-DCI has agood performance

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