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Optimal Vibration Controller for Vehicle Active Suspension System under Road Roughness Disturbances

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Abstract

The problem of optimal vibration control for vehicle active suspension systems under road roughness disturbance is considered. First, the models for two-degree-freedom quarter-car suspension system under road roughness disturbance are presented, and road disturbances are considered as the output of an exosystem. Then, the feedforward and feedback optimal vibration control (FFOVC) law for vehicle active suspension systems is obtained and the existence and uniqueness of the FFOVC is proved. A state observer is designed to solve the problem of the physically realizable for the feedforward compensator. Numerical simulations illustrate the effectiveness of the FFOVC law.

Keywords: *vehicle suspension system, roughness road disturbance, feedforward and feedback control, vibration control, optimal control*

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1. Introduction

With the development of machines technology and information technology, the vehicle suspension system underwent three stages: passive [1, 2], semi-active [3], and active suspension systems [4, 5, 6], in the last few decades. As is well known, vehicle active suspension systems, compared with passive and semi-active suspension system, could diminish the vibration of the vehicle body by using power sources more effectively. Compared to passive suspension system, active suspension system could meet the requirement more closely about driving safety, vehicle handling, and ride comfort. By using power sources (e.g., compressors and hydraulic pumps) active suspension systems have fewer limitations on the optimization procedures, where the suspension characteristics can be adjusted while driving to accommodate the profile of the road[7]. By using low consumption elements and minimizing the required energy level, active suspension system can compensate for the lake of higher production consumption and turn into more practical ones. Recently, a considerable amount of theoretical and experimental research effort has been aimed at improving vehicle suspension systems, such as fuzzy control [8], adaptive control [9], sliding mode theory [10], and H_∞ control [6, 11].

The essential elements in any active vehicle suspension design and control always include ride comfort, tire deflection, and suspension deflection. However, the vibration of vehicle is mainly caused by the road disturbances, which may result in deterioration of ride comfort, vehicle handling, driving safety, and even structural damage. Therefore, the influence from the road disturbances must be considered in any active suspension design. It should be noted that the road disturbances are mainly caused by road roughness and variable velocity. Then, the vibration control for vehicle active suspension system could be viewed as an optimal vibration problem where one would attempt to keep the ride comfort, tire deflection, and suspension deflection at an acceptable level [11, 12]. In order to analyze the dynamic behavior of a vehicle under road

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disturbances, road disturbances are typically considered as a random process with a ground displacement power spectral density (PSD)[13]. Based on the characters of road roughness and variable velocity, the road disturbances are formulated as an exosystem in this paper.

This paper investigates the optimal vibration control for an active vehicle suspension system under road disturbances. The model of active vehicle suspension system is built and the road disturbance is viewed as an exosystem. Then, the original vibration control is formulated as the optimal vibration control for vehicle active suspension system under road disturbances. By using the optimal control theory, the FFOVC law of the vehicle active suspension system under road disturbances is obtained, and the existence and uniqueness of the FFOVC is proved. In order to solve the physically realizable problem of the feedforward compensator, a reduced-order observer is constructed. A numerical example of the FFOVC law for an active vehicle suspension with under road disturbances is presented to demonstrate the effectiveness of the FFOVC law.

The rest of paper is organized as follows. Section 2 presents the descriptions of the vehicle active suspension system, exosystem of the road disturbance, and quadratic performance index. In Section 3, the main results of this paper are presented, in which the FFOVC law is obtained based on the optimal control theory and the existence and uniqueness of the FFOVC is proved. A reduced-order observer is constructed in Section 4 to solve the physically realizable problem. Numerical examples are given in Section 5 to demonstrate the effectiveness of the FFOVC law. Finally, we conclude our findings in Section 6.

2. Problem formulation

2.1. System formulation

The two-degree-freedom quarter-car active suspension system is shown in Fig.1 [11, 12].

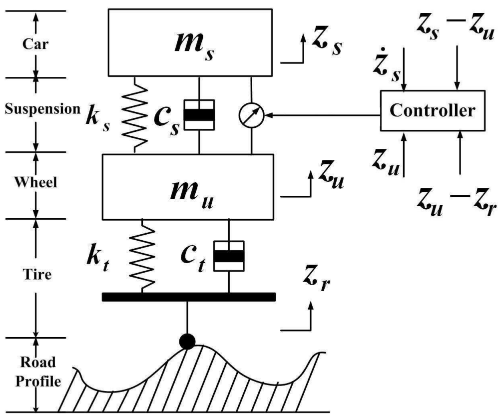

Figure 1. The two-degree-freedom quarter-car active suspension system

The dynamic equation for vehicle active suspension system is described as:

$$\begin{aligned} m_s\ddot{z}_s(t) + b_s[\dot{z}_s(t) - \dot{z}_u(t)] + k_s[z_s(t) - z_u(t)] = u(t), \\ m_u\ddot{z}_u(t) + b_s[\dot{z}_u(t) - \dot{z}_s(t)] + k_s[z_u(t) - z_s(t)] + k_t[z_u(t) - z_r(t)] = -u(t), \end{aligned} \tag{1}$$

where m_s is the sprung mass; m_u is the unsprung mass; k_s and b_s are the stiffness and damping of the passive vehicle suspension system, respectively; k_t stands for compressibility of the pneumatic tire; $z_s(t)$ and $z_u(t)$ are the displacements of the vehicle sprung mass and unsprung masses, respectively; $z_r(t)$ is the road displacement input; $u(t)$ represents the active control force of the vehicle suspension system, which is produced by hydraulic or other shock absorber.

Defining the following state variables:

$$x_1(t) = z_s(t) - z_u(t), \quad x_2(t) = z_u(t) - z_r(t), \quad x_3(t) = \dot{z}_s(t), \quad x_4(t) = \dot{z}_u(t), \tag{2}$$

$x_1(t)$ is the suspension deflection, $x_2(t)$ denotes the tire deflection, $x_3(t)$ denotes the speed of sprung mass, and $x_4(t)$ is the speed of unsprung mass. Then, the state variable $x(t)$ of the vehicle suspension system is introduced as:

$$x(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]^{\mathrm{T}}. \tag{3}$$

Riding comfort is the key performance criteria in any vehicle suspension system design. Ride comfort is usually evaluated by the sprung mass acceleration $\ddot{z}_s(t)$ in the vertical direction, the dynamic travel of suspension system usually by the amount of suspension defection $z_s(t) - z_u(t)$ and the road holding ability usually by the tire defection $z_u(t) - z_r(t)$.

In order to satisfy the requirements of performance criteria, the controlled output $y_c(t)$ is defined as:

$$y_c(t) = \begin{bmatrix} y_{c1}(t) \\ y_{c2}(t) \\ y_{c3}(t) \end{bmatrix} = \begin{bmatrix} \ddot{z}_s(t) \\ z_s(t) - z_u(t) \\ z_u(t) - z_r(t) \end{bmatrix} = \begin{bmatrix} \ddot{z}_s(t) \\ x_1(t) \\ x_2(t) \end{bmatrix}. \tag{4}$$

It is unnecessary and uneconomical to output all of the variables, so the measured output $y_m(t)$ can be expressed by:

$$\bar{y}_m(t) = \begin{bmatrix} z_s(t) - z_u(t) & \dot{z}_s(t) \end{bmatrix}^T \tag{5}$$

Then, the active vehicle suspension equation of a continuous-time system in the state space form can be expressed by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Dv(t) \\ y_c(t) &= Cx(t) + Eu(t) \\ y_m(t) &= \bar{C}x(t) \end{aligned} \tag{6}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ \frac{-k_s}{m_s} & 0 & \frac{-b_s}{m_s} & \frac{b_s}{m_s} \\ \frac{k_s}{m_u} & \frac{-k_t}{m_u} & \frac{b_s}{m_u} & \frac{-b_s}{m_u} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{b_s}{m_s} & -\frac{b_s}{m_s} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} \frac{1}{m_s} \\ 0 \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{7}$$

$v(t) = \dot{z}_r(t)$ is the road disturbance. By using ground height sensor to predict the road surface shape [14], the road disturbance $v(t)$ could be measurable.

2.2. Disturbance analysis

In order to improve the ride comfort and vehicle operation, the effect of road disturbance must be considered in active suspension design. In general, vibrations in vehicle suspension system are caused by the existence of the road disturbances. The road disturbances are typically specified as a stochastic process with a ground displacement power spectral density (PSD):

$$G_d(\Omega) = \begin{cases} G_d(\Omega_0)\left(\frac{\Omega}{\Omega_0}\right)^{-n_1}, & \Omega \le \Omega_0 \\ G_d(\Omega_0)\left(\frac{\Omega}{\Omega_0}\right)^{-n_2}, & \Omega > \Omega_0 \end{cases} \tag{8}$$

where Ω is a spatial frequency and it is the reciprocal of the wavelength, which donates the wave numbers per meter, it's dimension is m^{-1} . $\Omega_0 = 1/2\pi$ is a reference frequency. n_1 and n_2 are road roughness constants, in general, $n_1 = 2$ and $n_2 = 1.5$.

In this paper, the road disturbances for vehicle active suspension systems are mainly caused by the road roughness. Assume that the speed of the car is v_0 and the road displacement input $z_r(t)$ is an approximately periodic function. Since the wheel and suspension systems have the characteristic of low pass filtering (LPF), the road disturbances with low frequency are considered . Therefore, the road displacement input from the road irregularities can be approximately simulated by the following finite sum of Fourier series:

$$z_r(t) = \sum_{j=1}^{p} \xi_j(t) = \sum_{j=1}^{p} \phi_j \sin(j\omega_0 t + \theta_j), \quad j = 1,\ 2,\ \ldots,\ p, \tag{9}$$

where p is used to restrict the range of frequency, $\omega_0 = 2\pi v_0/l$, l is the length of the road segment. $\phi_j = \sqrt{2G_d(j\Delta\Omega)\Delta\Omega}$, $\Delta\Omega = 2\pi/l$, and the initial phase $\theta_j \in [0,\ 2\pi)$ is a random variable following a uniform disturbance.

In order to facilitate design of the optimal control law, the definition of road disturbance state vector is

$$\begin{aligned} w(t) &= [w_1(t),\ \cdots,\ w_{2p}(t)]^{\mathrm{T}} \\ &= [\xi_1(t),\ \cdots,\ \xi_p(t),\ \dot{\xi}_1(t),\ \cdots,\ \dot{\xi}_p(t)]^{\mathrm{T}} \end{aligned} \tag{10}$$

The road disturbance vector $v(t)$ can be given by:

$$\begin{aligned} \dot{w}(t) &= Gw(t), \\ v(t) &= Fw(t), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \bar{G} &= \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \tilde{G} & \mathbf{0} \end{bmatrix}, \\ F &= [\ \underbrace{0,\ \cdots,\ 0,}_{p}\ \ \underbrace{1,\ \cdots,\ 1}_{p}\ ] \\ \tilde{G} &= \mathrm{diag}\{\ -\omega_0^2,\ \ \cdots,\ \ -(p\omega_0)^2\ \} \end{aligned} \tag{12}$$

One can see that the derivatives of the system (9) and (11) are equivalent. Noting that, the rank of $\begin{bmatrix} F^{\mathrm{T}} & (FG)^{\mathrm{T}} & \cdots & (FG^{2p-1})^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} = 2p$, the pair $(F,\ \bar{G})$ is observable.

2.3. Optimal performance index formulation

In view of the limited power of the actuator, a smaller active force $u(t)$ for the active suspension system should be chosen to reduce energy consumption in practical applications. Due to the persistent effect from road disturbance, the state vector and the control vector of the active suspension system will not converge to zero synchronously. Therefore, the traditional quadratic optimal control performance index will not be available. In this case, the following quadratic average performance index is chosen as:

$$J(u(\cdot)) = \lim_{T\to\infty} \frac{1}{T} \int_0^T [y_c^{\mathrm{T}}(t) Q_0 y_c(t) + u^2(t)]dt, \tag{13}$$

where $Q_0 = \mathrm{diag}(q_1,\ q_2,\ q_3)$ and it is positive definite matrix. q_i can be determined by different vehicle's explicit requirements for performance indexes of ride comfort, road holding ability, suspension deflection, and energy-saving in the active suspension system design. Substituting (6) and (7) into (13), the quadratic average performance index (13) is reformulated as:

$$J(u(\cdot)) = \lim_{T\to\infty} \frac{1}{T}\int_0^T [x^{\mathrm{T}}(t)Qx(t) + 2x^{\mathrm{T}}(t)Nu(t) + Ru^2(t)]dt, \tag{14}$$

where

$$\begin{aligned} Q = C^{\mathrm{T}}Q_0C &= \begin{bmatrix} \frac{k_s^2 q_1}{m_s^2} + q_2 & 0 & \frac{k_s b_s q_1}{m_s^2} & \frac{k_s b_s q_1}{m_s^2} \\ * & q_3 & 0 & 0 \\ * & * & \frac{b_s^2 q_1}{m_s^2} & \frac{b_s^2 q_1}{m_s^2} \\ * & * & * & \frac{b_s^2 q_1}{m_s^2} \end{bmatrix}, \\ N = C^{\mathrm{T}}Q_0E &= \begin{bmatrix} -\frac{k_s q_1}{m_s^2} & 0 & -\frac{b_s q_1}{m_s^2} & -\frac{b_s q_1}{m_s^2} \end{bmatrix}^{\mathrm{T}}, \quad R = \frac{q_1}{m_s^2} + 1, \end{aligned} \tag{15}$$

where $*$ denoted as the symmetry elements.

Then, the problem of optimal vibration control for vehicle active suspension system is reformulated to find a control law $u^*(t)$ for the system (6) with respect to the performance index (14) that makes the performance index (14) obtain the minimum value.

3. FFOVC law

By using the following variable transformation,

$$\bar{u}(t) = u(t) + R^{-1}N^{\mathrm{T}}x(t), \tag{16}$$

System (6) and the performance index (14) are reformulated to equivalent forms as followed, respectively

$$\dot{x}(t) = (A - BR^{-1}N^{\mathrm{T}})x(t) + B\bar{u}(t) + Dv(t), \tag{17}$$

$$J(\bar{u}(\cdot)) = \lim_{T\to\infty} \frac{1}{T}\int_0^T [x^{\mathrm{T}}(t)(Q - NR^{-1}N^{\mathrm{T}})x(t) + R\bar{u}^2(t)]dt, \tag{18}$$

where

$$Q - NR^{-1}N^{\mathrm{T}} = \begin{bmatrix} \frac{k_s^2 q_1}{q_1+m_s^2} + q_2 & 0 & \frac{k_s b_s q_1}{q_1+m_s^2} & \frac{k_s b_s q_1}{q_1+m_s^2} \\ * & q_3 & 0 & 0 \\ * & * & \frac{b_s^2 q_1}{q_1+m_s^2} & \frac{b_s^2 q_1}{q_1+m_s^2} \\ * & * & * & \frac{b_s^2 q_1}{q_1+m_s^2} \end{bmatrix} \tag{19}$$

One can see that $Q - NR^{-1}N^{\mathrm{T}}$ is positive semidefinite matrix. Let $Q - NR^{-1}N^{\mathrm{T}} = \bar{D}^{\mathrm{T}}\bar{D}$, by means of the Cholesky matrix decomposition function in Matlab, we can get:

$$\bar{D} = \begin{bmatrix} \sqrt{\frac{k_s^2 q_1}{q_1+m_s^2} + q_2} & 0 & \frac{\frac{k_s b_s q_1}{q_1+m_s^2}}{\sqrt{\frac{k_s^2 q_1}{q_1+m_s^2}+q_2}} & \frac{\frac{k_s b_s q_1}{q_1+m_s^2}}{\sqrt{\frac{k_s^2 q_1}{q_1+m_s^2}+q_2}} \\ 0 & \sqrt{q_3} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{b_s^2 q_1 q_2}{k_s^2 q_1 + q_1 q_2 + q_2 m_s^2}} & \sqrt{\frac{b_s^2 q_1 q_2}{k_s^2 q_1 + q_1 q_2 + q_2 m_s^2}} \end{bmatrix}. \tag{20}$$

It's easy to verify that $(A - BR^{-1}N^{\mathrm{T}},\ B)$ is controllable and $(\bar{D},\ A - BR^{-1}N^{\mathrm{T}})$ is observable.

Based on above analysis, we obtain the following results.

Theorem 1: Consider the optimal vibration control for vehicle active suspension system (1) under the persistent effect (1) from road disturbance vector with respect to the quadratic performance indexes (13). The optimal control law uniquely exists and can be formulated as:

$$u^*(t) = -R^{-1}[(B^{\mathrm{T}}P + N^{\mathrm{T}})x(t) + B^{\mathrm{T}}P_1w(t)], \tag{21}$$

where P is the unique positive definite solution of the Riccati matrix equation,

$$(A - BR^{-1}N^{\mathrm{T}})^{\mathrm{T}}P + P(A - BR^{-1}N^{\mathrm{T}}) - PBR^{-1}B^{\mathrm{T}}P + Q - NR^{-1}N^{\mathrm{T}} = 0 \tag{22}$$

and P_1 is the unique solution of the Sylvester matrix equation

$$(A - BR^{-1}B^{\mathrm{T}}P - BR^{-1}N^{\mathrm{T}})^{\mathrm{T}}P_1 + P_1G + PDF = 0 \tag{23}$$

Proof: According to the above analysis, the optimal vibration control for vehicle active suspension system (1) with respect to the performance index (13) is equivalent to the optimal control for system (17) with respect to the performance index (18). According to the optimal control theory, the optimal control for system (17) with respect to the performance index (18) can induce the following two-point boundary value (TPBV) problem.

Applying the necessary condition of the optimal control, the optimal control law can be described as:

$$u^*(t) = -R^{-1}[B^{\mathrm{T}}\lambda(t) + N^{\mathrm{T}}x(t)]. \tag{24}$$

Let

$$\lambda(t) = Px(t) + P_1w(t). \tag{25}$$

By calculating the derivatives of (26) and substituting the second equation of (24) and (11) into (26), one gets:

$$\begin{aligned} \dot{\lambda}(t) &= P\dot{x}(t) + P_1\dot{w}(t) \\ &= P[Ax(t) + Bu(t) + DFw(t)] + P_1Gw(t) \\ &= (PA - PBR^{-1}N^{\mathrm{T}} - PBR^{-1}B^{\mathrm{T}}P)x(t) + (PBR^{-1}B^{\mathrm{T}}P_1 + B^{\mathrm{T}}P_1)w(t). \end{aligned} \tag{26}$$

By comparing (27) and the first equation of (24), one gets:

$$\begin{aligned} &\Big((A - BR^{-1}N^{\mathrm{T}})^{\mathrm{T}}P + P(A - BR^{-1}N^{\mathrm{T}}) - PBR^{-1}B^{\mathrm{T}}P + Q - NR^{-1}N^{\mathrm{T}}\Big)x(t) + \\ &\Big((A - BR^{-1}B^{\mathrm{T}}P - BR^{-1}N^{\mathrm{T}})^{\mathrm{T}}P_1 + P_1G + PDF\Big)w(t) = 0 \end{aligned} \tag{27}$$

Since (28) can be proved for all $x(t)$ and $w(t)$, we can obtain the Riccati matrix equation (22), the Sylvester matrix equation (23) and the optimal control law (21). Since $(A - BR^{-1}N^{\mathrm{T}},\ B,\ \bar{D})$ is controllable and observable, according to the linear optimal regulator theory, P is the unique positive definite solution of the Riccati matrix equation (22) and $(A - BR^{-1}B^{\mathrm{T}}P - BR^{-1}N^{\mathrm{T}})$ is the Hurwitz matrix equation described as:

$$\mathrm{Re}[\lambda_i(A - BR^{-1}B^{\mathrm{T}}P - BR^{-1}N^{\mathrm{T}})] < 0. \tag{28}$$

According to (12),$\mathrm{Re}[\lambda_i(G)] = 0$, therefore

$$\begin{array}{l} \lambda_i(A - BR^{-1}B^{\mathrm{T}}P - BR^{-1}N^{\mathrm{T}}) + \lambda_j(G) \neq 0, \\ i = 1,\ 2,\ \cdots,\ n; \quad j = 1,\ 2,\ \cdots,\ 2p, \end{array} \tag{29}$$

So P_1 is the unique solution of Sylvester matrix equation (23)[15].

4. Physically realizable problem

In System (11), the feedforward compensator of the optimal vibration control law $u^*(t)$ in (21) contains state vector $w(t)$'s information. As $w(t)$ itself is unknown and unmeasured as shown in Fig.2, the feedforward compensator is physically unrealizable. Furthermore, as we have already pointed out in section 2.1 that we choose $y_m(t)$ in System (5) to estimate the state of the measured output, it is unnecessary and uneconomical to output the state feedback of the optimal vibration control law $u^*(t)$. Therefore, we try to propose a state observer to reconstruct system state vector $x(t)$ and disturbance state vector $w(t)$.

For the simplicity of statement, we construct the state observer to solve the physically realizable problem of the optimal control law. The design of the state observer is described as:

$$\begin{array}{l} \dot{z}_1(t) = (A - L_1\bar{C})z_1(t) + L_1 y_m(t) + Bu(t) + Dv(t), \\ \dot{z}_2(t) = (G - L_2F)z_2(t) + L_2 v(t), \end{array} \tag{30}$$

where L_1 and L_2 are determined by eigenvalues of the prescribed formulas $(A - L_1\bar{C})$ and $(G - L_2F)$, respectively. Hence, we can get the physically realizable feedforward and feedback dynamic optimal vibration control law:

$$\begin{array}{l} \dot{z}_1(t) = (A - L_1\bar{C})z_1(t) + L_1 y_m(t) + Bu(t) + Dv(t) \\ \dot{z}_2(t) = (G - L_2F)z_2(t) + L_2 v(t) \\ u(t) = -R^{-1}[(B^{\mathrm{T}}P + N^{\mathrm{T}})z_1(t) + B^{\mathrm{T}}P_1 z_2(t)]. \end{array} \tag{31}$$

5. Simulation

In this section, simulation experiments are shown to illustrate the effectiveness of the FFOVC law for the active suspension systems. The parameters of vehicle active suspension system model are listed as [16]: the sprung mass $m_s = 180$ kg, the unsprung mass $m_u = 25$ kg, the stiffness of the active suspension system $k_s = 16000$ N/m, the compressibility of the pneumatic tire $k_t = 190000$ N/m, the damping of the active suspension system $b_s = 1000$ N/m, and the dimension of the control force is N. Hence, the matrix values of the active suspension system (6) are given by:

$$\begin{array}{l} A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -88.89 & 0 & -5.556 & 5.556 \\ 640 & -7600 & 40 & -40 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0.00556 \\ -0.04 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \\ C = \begin{bmatrix} -88.89 & 0 & -5.556 & -5.556 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0.00556 \\ 0 \\ 0 \end{bmatrix}. \end{array} \tag{32}$$

In (9), $v_0 = 20\mathrm{m/s},\ l = 200m,\ p = 200$ are selected. The average value of PSD is $G_d(\Omega_0) = 64 \times 10^{-6}\mathrm{m}^3$. The comparison between the road disturbance vector $v(t)$ in (9) and the estimation (11) is shown in Fig. 2. Assume that sprung mass acceleration $\ddot{z}_s(t)$, suspension deflection $z_s(t) - z_u(t)$ and tire deflection $z_u(t) - z_r(t)$ are of equal importance in ride comfort. So we select $q_1 = q_2 = q_3 = 10^6$ in performance index (13).

The comparison between the open-loop system and the FFOVC law for active suspension system is shown in this paper. The curves of sprung mass acceleration, suspension deflection, and tire deflection are presented in Figs. 3-5, respectively. In order to show clearly the comparison results between the closed-loop system and the open-loop system, the root-mean-square (RMS) values are compared in Table 1 for sprung mass acceleration, suspension deflection, and tire deflection.

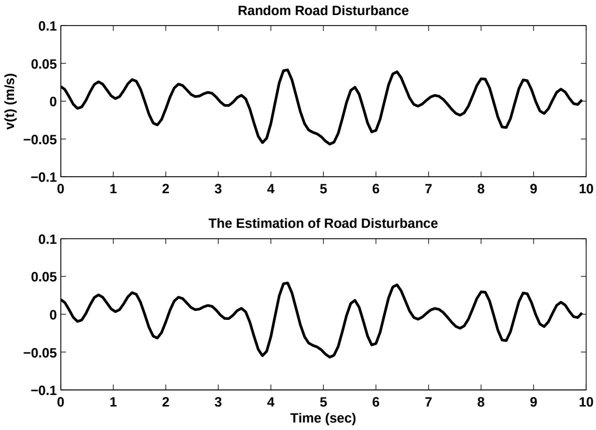

Figure 2. Displacement of road disturbance

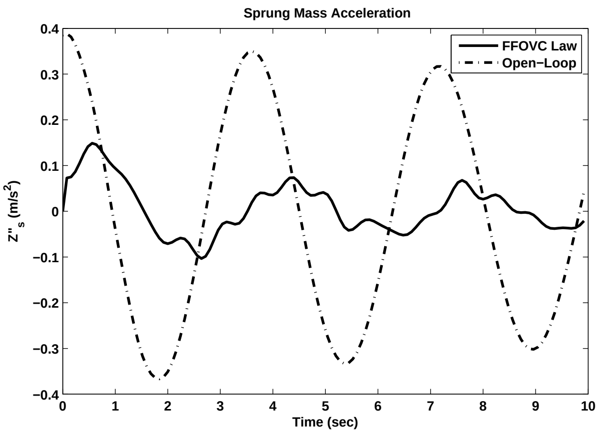

Figure 3. The curve of sprung mass acceleration

It can be seen from Figs. 3-5 and Table 1 that the optimal vibration control law we have designed in this article for vehicle active suspension system is able to effectively control the sprung mass acceleration, suspension deflection, and tire deflection in lower values. Therefore, the designed controller is efficient to improve the performance index of ride comfort.

6. Conclusion

This paper has been concerned with the development of optimal vibration control for the vehicle active suspension under road disturbances. This paper has presented that the original vibration control is formulated as the optimal vibration control for vehicle active suspension system under road disturbances. Another significant improvement is on the FFOVC law. FFOVC law can eliminate the negative effects of the road disturbances and maintain economical operation in an optimal fashion.

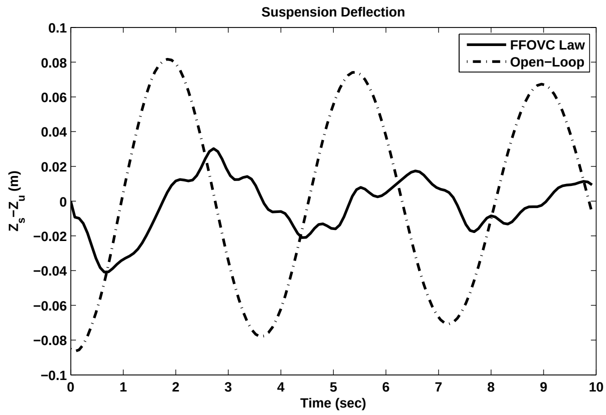

Figure 4. The curve of suspension deflection

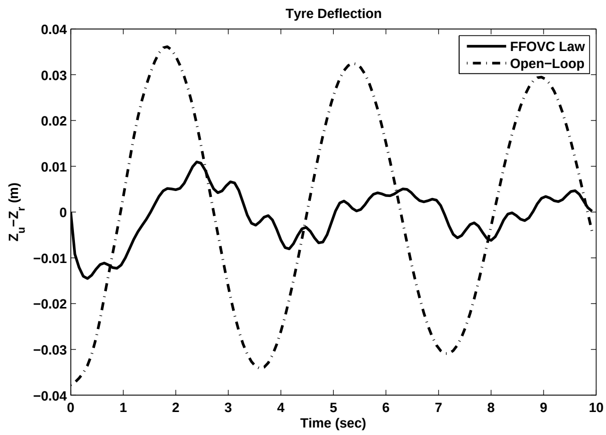

Figure 5. The curve of tire deflection

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Table 1. Comparison of RMS values of performance criteria

Coefficient	$\ddot{z}_s(m^2/s)$	$z_s - z_u(m)$	$z_u - z_r(m)$
Optimal Control	0.3022	0.0682	0.0283
Open-Loop	0.6903	0.1622	0.0719
Reduced Rate (%)	56.22	57.95	60.77

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